Bit Error Probability of an Optically Pre-amplified Pulse Position Modulation Receiver with Reed Solomon Error Correction

Antonios Aspreas
Department of Informatics and Telecommunications
University of the Peloponnese
Tripoli, Greece
aspreasant@uop.gr

Konstantinos Yiannopoulos
Department of Informatics and Telecommunications
University of the Peloponnese
Tripoli, Greece
kyianno@uop.gr

***Abstract*—We present results on the bit error probability (BEP) of Reed Solomon (RS) codes for an optically pre-amplified pulse position modulation receiver. We first derive analytical relations for the BEP calculation of the RS coded system and validate their accuracy via Monte Carlo simulations. The analytical relations are then utilized to assess the BEP performance of the system in the presence of weak and strong fading. Our simulations show that coding provides a net gain in weak fading, but no gain is observed in strong fading.**

***Index Terms*—Optical Wireless Communications, Pulse Position Modulation, Pre-amplified Optical Receiver, Reed Solomon Codes, $\mathcal{M}$-Malaga fading, $\gamma-\gamma$ fading**

I. Introduction

Optical Wireless Communications (OWC) dedicated to earth-to-space links offer a great alternative over their radio counterparts due to low cost components, low complexity receivers and high bandwidth availability. Nevertheless, power budget design limitations are introduced by atmospheric transmission phenomena, like scintillation, absorption and scattering. A plethora of methods have been proposed to improve the performance of OWC systems and reduce the impact of the aforementioned effects, including optical amplification, orthogonal modulations and channel coding [1].

Optical pre-amplification improves the receiver sensitivity and reduces the severity of fading [2]. The trade-off of using an amplifier in the receiver is the addition of the amplifier's noise to the received signal. Moreover, pulse position modulation (PPM) is a high order orthogonal modulation scheme that can be used to further improve the system sensitivity at the expense of bandwidth utilization [3]–[5]. One of the drawbacks of PPM, however, is that it is prone to burst errors since each PPM symbol is typically mapped to more than one bits. Given the orthogonality of PPM symbols, an erroneous decision on the received symbol leads to registering up to $\log_2 Q$ bit errors, where Q is the modulation order.

An efficient solution to overcome these errors is the utilization of error correction codes, and in particular codes that support burst error correction. In this direction, RS codes are a popular choice of codes for PPM links due to their high effectiveness in dealing with burst errors. This is possible since RS codes form and correct symbols of their own; as such RS codes will correct a burst of incorrect bits provided that its boundaries remain within a single RS symbol. In PPM modulation, RS burst error correction is feasible assuming that the PPM symbol length (in bits) is shorter than the RS one, which is typically the case in real-world applications.

In the current work we present analytical relations for the BEP of a pre-amplified PPM receiver with RS coding. The relations are based upon a previous analysis of the pre-amplified receiver [6] and a widely utilized approximation for RS codes [7]. The analytical results are validated via Monte Carlo (MC) simulations and it is verified that a significant coding gain can be attained for this system. Moreover, the simulations reveal that [6] provides a more accurate BEP approximation at low signal levels and this observation facilitates the performance evaluation of the coded system in the presence of fading. The corresponding simulation results reveal that the use of RS codes may provide and additional margin in the power budget design under weak fading.

The rest of the paper is structured as follows: Section II details the system model with and without RS coding, as well as analytical and simulation BEP results. Section III discusses the performance of the two systems in $\mathcal{M}$-Malaga and $\gamma-\gamma$ fading. Both weak and strong fading conditions are considered so as to explore the applicability of coding in each scenario. Finally, Section IV concludes the paper and summarizes the main results.

II. System Performance

A. System Model

The system under consideration is shown in Fig. 1. The binary information is first partitioned into RS symbols with size equal to m bits and k successive data symbols are used to generate $n-k=2t$ parity symbols, thus forming the RS(n,k) error correction block. RS codes are minimum distance codes, hence they can correct the largest possible number of errors for given code parameters. The RS minimum distance is $d_{min} =$

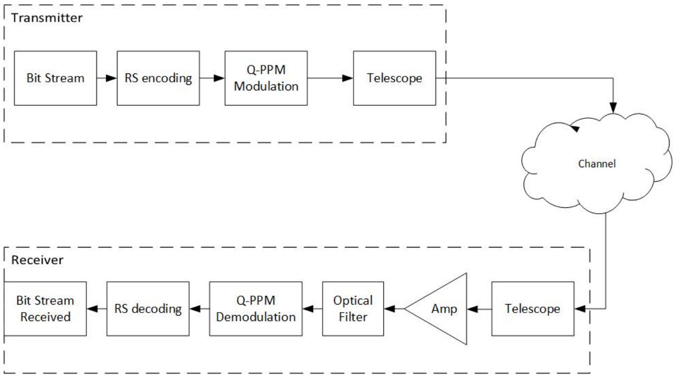

Fig. 1: System Model.

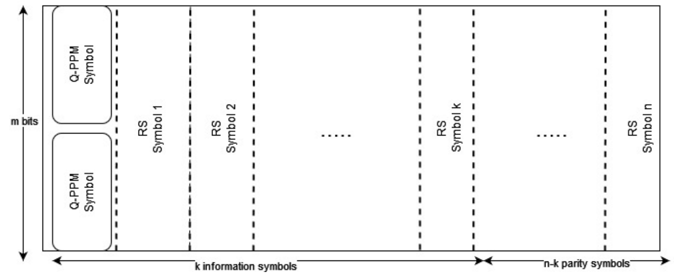

Fig. 2: Coding block structure.

$n-k+1$ and can correct up to $t = \lfloor (n-k)/2 \rfloor$ symbols errors. The coding rate is defined as $R_c = k/n$.

The RS codewords are then modulated using PPM, which utilizes Q time slots to distinguish between an equal number of PPM symbols. The PPM symbol energy E_s is contained in one of the slots and the binary representation of the slot identifies the $\log_2(Q)$ bits that correspond to the PPM symbol. As a result the RS codewords are partitioned prior to modulation and, following the discussion in [8], good performance requires that the bit-length of an RS symbol is higher than the bit-length of a Q-PPM symbol, or equivalently $m > \log_2(Q)$. This approach ensures that successive PPM errors contribute to a single RS error at the decoder, thus enhancing its burst error correction capabilities. The structure of an RS block code where Q-PPM modulation is used is presented in Fig. 2.

B. Bit Error Probabilities

The PPM symbols are transmitted over the optical wireless channel and at the receiver's end the optical signal is amplified, detected and demodulated. The demodulation process relies on identifying which slot contains the PPM symbol energy and this is achieved by selecting the slot with the highest energy (soft-decision demodulation). Due to the optical noise that is introduced from the amplifier, demodulation errors are expected and the PPM symbol error probability for the uncoded system P_{sm} has been previously calculated as [6, eq. (12)]

$$\begin{aligned} P_{sm} = & \sum_{q=1}^{Q-1} \binom{Q-1}{q} (-1)^{q+1} \\ & \times \exp\left(-\frac{q\lambda}{1+q}\right) \sum_{n=0}^{q(M-1)} \frac{c_n^q}{(1+q)^{n+M}} \\ & \times \sum_{i=0}^{n} \frac{(n+M-1)!}{(i+M-1)!(n-i)!i!} \left(\frac{\lambda}{1+q}\right)^i , \end{aligned} \tag{1}$$

where Q is the modulation order, M are the amplifier's noise modes, $\lambda = E/N_0 = E_b/N_0 \, \log_2(Q)$ is the symbol energy to noise ratio, E_b is the energy per bit after amplification, and $N_0 = n_{sp} \, h \, f \, (G-1)$ is the optical noise spectral density at the amplifier output. The coefficients c_n^q are calculated following [6, eq. (13)]. The BEP for the uncoded system, is given by

$$P_e^u = \frac{Q}{2(Q-1)} P_{sm} \, . \tag{2}$$

The bit stream at the demodulator output is then decoded using the Berlekamp–Massey algorithm, which allows for the correction of up to t RS symbol errors. Assuming that PPM symbol errors are independent, the probability that an RS symbol is in error P_s is calculated from P_{sm} as

$$P_s = 1 - (1 - P_{sm})^p \, , \tag{3}$$

where $p = m/\log_2(Q)$ is the number of PPM symbols per RS symbol. The decoder will not be able to correct the received bits whenever more than t RS symbol errors are present, and the corresponding BEP of the RS coded system is approximated by [7, eq. (31)]

$$P_e^c \approx \frac{n+1}{2n} \sum_{i=t+1}^{n} \binom{n-1}{i-1} P_s^i \, (1-P_s)^{n-i} \, . \tag{4}$$

In the last equation it is also required to include the power penalty that is introduced from the coding rate, thus $\lambda = k/n \, E_b/N_0 \, \log_2(Q)$.

Equations (2) and (4) are plotted in Fig. 3 for $Q = 4$, 16 and three RS codes with a different length and a similar code rate. The analytical results demonstrate that the coded system is able to correct the symbol errors that are imparted from the noise of the amplifier. The predicted gain amounts to $2-3 \, dB$ for $Q = 4$ at a $BEP = 10^{-6}$, with a longer code providing additional gain. A similar behavior is observed for $Q = 16$, where the coded system outperforms the uncoded one, even though the coding gain is somewhat reduced compared with $Q = 4$.

C. Validation via Monte Carlo Simulation

The advantage of (4) lies in the simplicity in deriving the probability of bit error since the PPM symbol error probability P_{sm} and the code parameters are the only requirements. However, its accuracy needs to be investigated, especially at low signal energies which are typically expected in a fading environment. To this end, MC simulation results are plotted in Fig. 3, along with the analytical ones that were obtained via (2) and (4). The simulations were performed by RS encoding random bit streams, which where then modulated

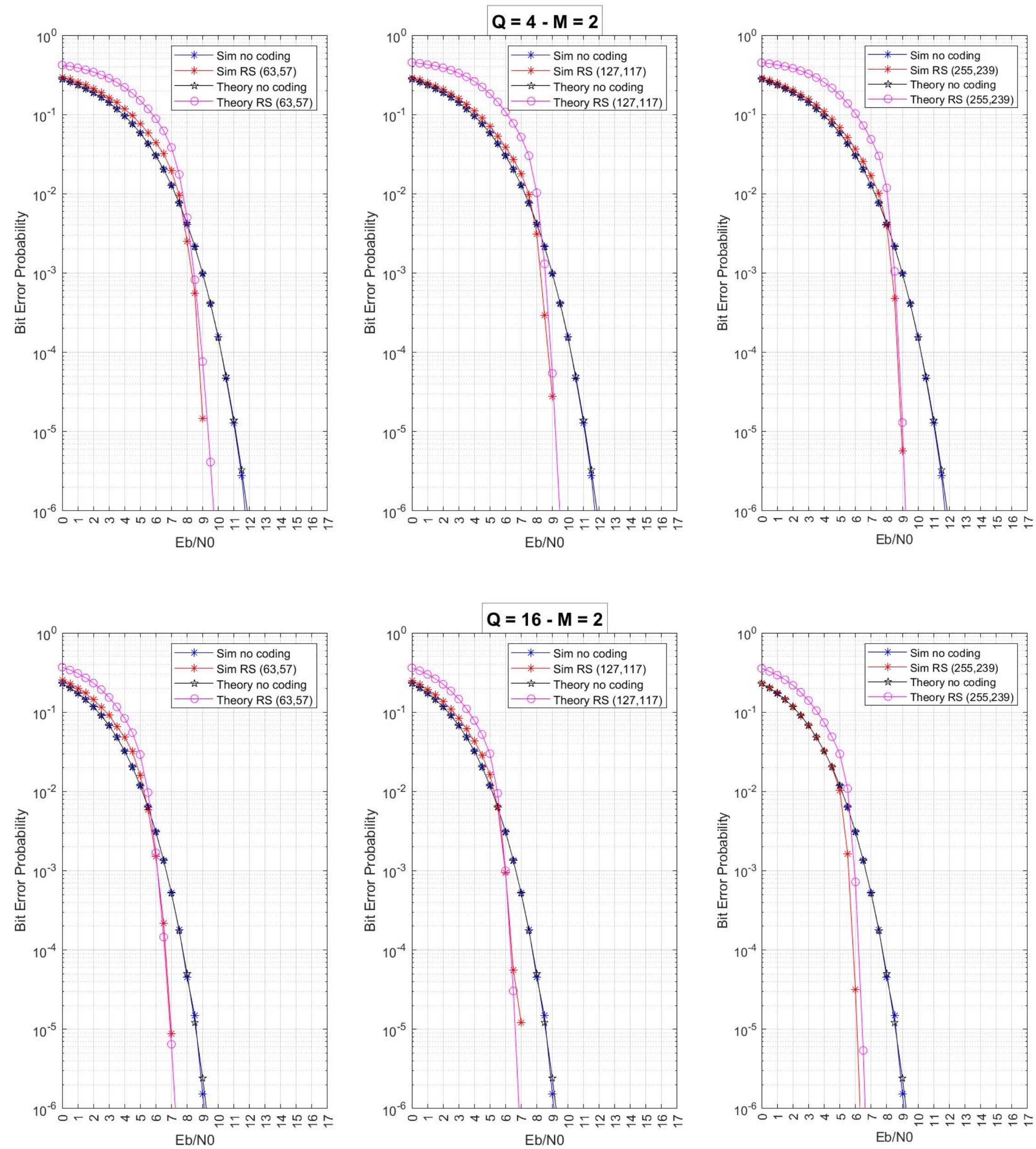

Fig. 3: Analytical and simulated BEP performance of the uncoded and coded system.

into PPM symbols. The slot signals in each PPM symbol where randomly generated from a central χ^2 distribution in the slots without any energy, and a non-central χ^2 distribution for the slot with the symbol energy [6]. The demodulator selected the slot with the highest signal value and reported the corresponding bits to the decoder, which recovered the original bit stream after possibly correcting errors. The BEP was measured by comparing the transmitted bits with the outputs of the demodulator and the decoder.

The plots in Fig. 3 show that the simulation and analytical results for the uncoded system coincide, which verifies the validity of the simulation. It can also be seen that (4) presents a good fit only for the high E_b/N_0 regime. As E_b/N_0 decreases and the system is dominated by noise, the simulation shows that the coded and uncoded systems exhibit a similar BEP and (2) is a better approximation than (4). This can be explained by the fact that the number of RS symbol errors that occur per block are more than t, hence the decoder can not correct them and reports the original erroneous bit stream. Given these results, a more accurate approximation that describes the performance of the coded system for both the high and low E_b/N_0 regions is

$$P_e = \min\{P_e^c,\, P_e^u\}\,. \tag{5}$$

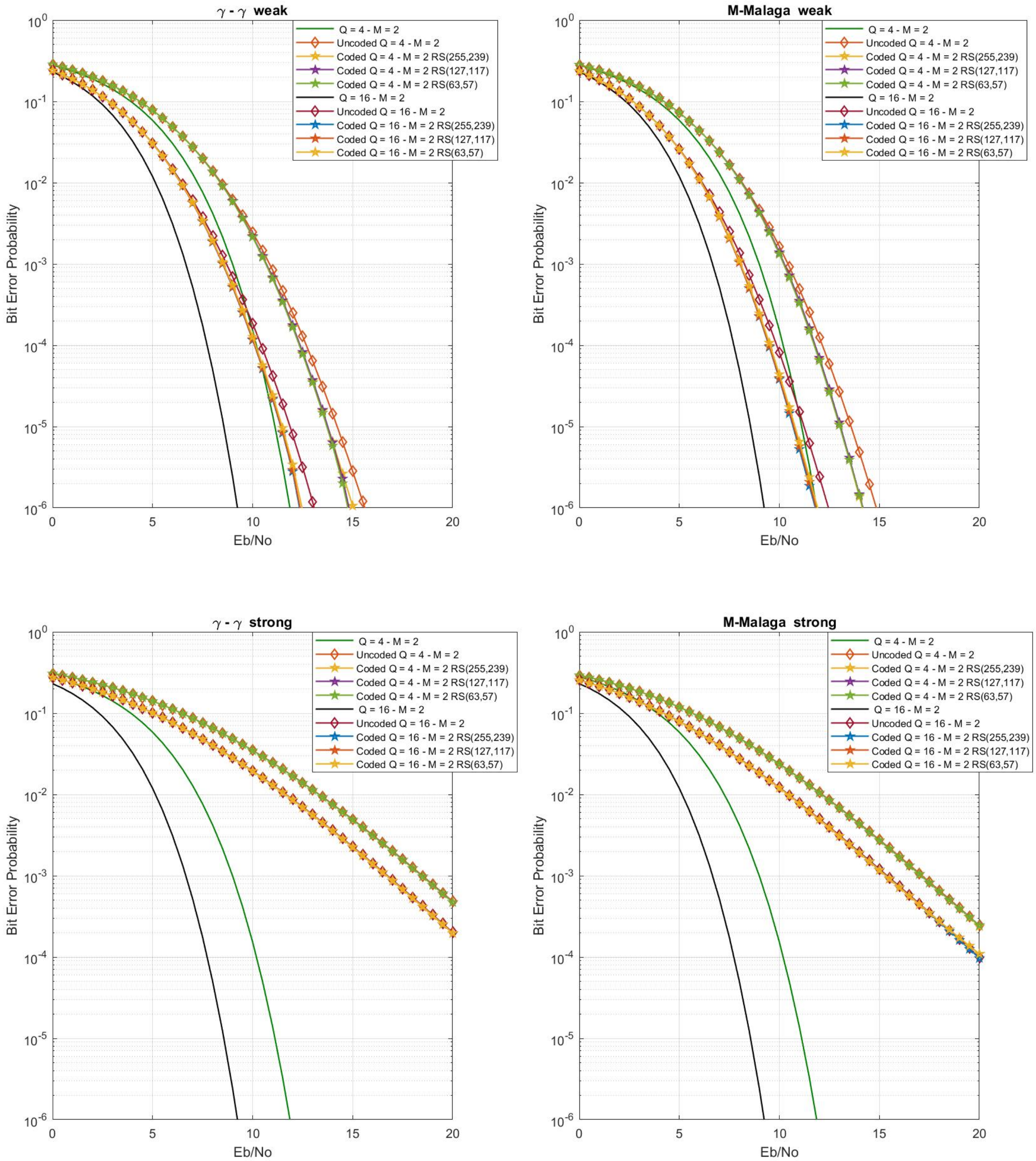

Fig. 4: ABEP performance of the uncoded and coded system in $\gamma - \gamma$ and $\mathcal{M}$-Malaga fading.

III. System Performance under Fading

In an OWC system the optical signal is transmitted through the atmosphere and the time varying inhomogeneities of the atmospheric refractive index introduce fluctuations at the received energy. The energy fluctuations are modelled as a random variable h and, assuming that the channel does not change significantly within the PPM symbol duration, the instantaneous BEP is calculated from (2), (4) and (5) by replacing λ with λh. The average BEP (ABEP) of the system is then obtained via MC simulation after evaluating the instantaneous BEP over a large number of possible channel states h_n following

$$P_e^{ave} = \frac{1}{N} \sum_{n=1}^{N} P_e(\lambda h_n) . \tag{6}$$

In our simulations, we randomly generated $N = 10^6$ channel amplitudes h_n using the $\gamma - \gamma$ [9] and $\mathcal{M}$-Malaga channel

TABLE I: $\gamma - \gamma$ Channel Model Parameters

Parameter	Weak	Strong
α	16.5347	14.9057
β	5.50966	1.1138

TABLE II: $\mathcal{M}$-Malaga Channel Model Parameters

Parameter	Weak	Strong
α	50	2.2814
β	14	33
γ	0.06	0.1354
Ω'	1.4847	3.7270

models [10]. The corresponding pdfs are equal to

$$f_h(h) = \frac{1}{\Gamma(\alpha)\,\Gamma(\beta)\,h}\, G_{0,2}^{2,0}\left(\alpha\,\beta\,h \middle| \begin{matrix} - \\ \alpha, \beta \end{matrix}\right) \tag{7}$$

and

$$\begin{aligned} f_h(h) &= \frac{1}{h} \sum_{j=1}^{\beta} b_j\, G_{0,2}^{2,0}\left(\delta\,h \middle| \begin{matrix} - \\ \alpha, j \end{matrix}\right), \\ \delta &= \frac{\alpha\,\beta\,(\gamma+\Omega')}{\gamma\,\beta+\Omega'}, \\ b_j &= \frac{A}{2}\,a_j \left(\frac{\alpha\,\beta}{\gamma\,\beta+\Omega'}\right)^{-\frac{a+j}{2}}, \end{aligned} \tag{8}$$

respectively, and have been normalized so that the random fluctuations do not provide any loss or gain ($E\{h\} = 1$). $G_{p,q}^{m,n}(\cdot)$ is the Meijer G-function [11, eq. (9.301)] and the distribution parameters are described in detail in the literature [9], [10]. The parameter values that were used in the simulations are summarized in Tables I and II.

The simulation results are presented in Fig. 4 for weak and strong fading and the three different RS codes. The results show that the coded ABEP performance surpasses the uncoded one in weak fading and a gain of approximately 1 dB is observed at high E_b/N_0. Longer and shorter codes also provide similar gains due to their comparable code rates. Moreover, the uncoded 16-PPM system performs better than the RS coded 4-PPM one for all ABEPs up to 10^{-6}, while the 8-PPM system (not shown for brevity) has an intermediate performance. Thus, given the available bandwidth and E_b/N_0 it is possible to select the combination of the modulation order and the code length that achieves the optimal ABEP in weak fading. In contrast, the results show that the RS codes do not improve the ABEP performance under strong fading, since the system operates constantly at a low E_b/N_0 and the two systems (coded and uncoded) exhibit an almost identical instantaneous BEP following the results of Fig. 3.

IV. Conclusion

In this paper, the error correction capabilities of RS coding were investigated for a pre-amplified PPM OWC system. Analytical expressions for the coded BEP were derived and compared with simulation results, leading to an accurate BEP approximation of the coded system in both the low and high E_b/N_0 regimes. The approximation was then utilized to assess the ABEP performance of the coded system and show that RS codes are beneficial in weak fading scenarios provided that an adequate power margin is available, while they offer practically no gain in strong fading conditions.

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