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Impact of Pointing Errors and Fading in a Pre-Amplified Pulse Position Modulation Optical Receiver

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***Abstract*—We present analytical results on the average bit error probability (ABEP) of an optically pre-amplified pulse-position-modulation (PPM) receiver that operates under fading and pointing errors. The analytical description enables the efficient calculation of the ABEP in Malaga-$\mathcal{M}$, $\gamma-\gamma$ and negative-exponential fading for a wide range of modulation orders and noise modes. As expected, a significant power budget gain is initially obtained by increasing the PPM modulation order, however a further improvement requires the optimization of the received beam waist. Finally, the noise modes that enter the receiver negatively affect its operation irrespective of fading and pointing errors.**

***Index Terms*—optical amplifier, pulse position modulation, Malaga-$\mathcal{M}$ fading, pointing errors**

I. Introduction

Optical wireless communication (OWC) links are adversely affected by the volatile nature of the atmosphere, which introduces power fades and surges, as well as the random misalignment of the transmitter and the receiver, owing to building sway in terrestrial systems [1]. These effects introduce a power penalty in the link budget and a number of works discuss the impact of fading and pointing errors on the performance of various modulation formats [2]–[7]. An orthogonal modulation scheme like PPM improves the receiver sensitivity with increasing modulation orders and it has been previously assessed in a fading and pointing error environment [8]. In addition, optical amplification provides a further means to reduce the introduced power penalty [9]–[14], since it reduces the receiver sensitivity and provides a power budget gain comparable to the amplifier gain.

Previous works on PPM with amplification in OWCs focus on the impact of fading or pointing errors, but not both. In this work, we consider pointing errors in conjunction with fading and derive analytical results for the ABEP of the pre-amplified PPM receiver for the generalized Malaga-$\mathcal{M}$ fading distribution, as well as the less general $\gamma-\gamma$ and negative exponential distributions. The results show that the system performance improves with increasing the modulation order, thus the available bandwidth of the system will ultimately define the performance gain that is achieved. The noise modes of the amplifier also negatively affect the ABEP and the narrowest possible optical filter should be utilized. With respect to the beam waist size at the receiver, our results show that an optimal beam waist exists in both Malaga-$\mathcal{M}$ and $\gamma-\gamma$ weak fading, an observation which agrees with previously reported results for a non-amplified on-off-keying receiver [2]. In more intense fading, the results show that the beam waist should be kept to a minimum and this holds true for all three fading distributions.

II. Malaga-$\mathcal{M}$ Fading with Pointing Errors

We consider an OWC PPM receiver whose structure is detailed in Fig. 1. Light is collected from an optical aperture and is optically amplified prior to detection, with the amplifier providing a gain equal to G and adding optical noise with a spectral density of $N_0 = n_{sp}hf(G-1)$. An optical filter is utilized to reject amplifier noise and the optical signal is then converted to electrical on a photodiode. The photodiode output is integrated over the duration of a PPM time-slot and soft decision decoding is utilized to identify the slot with the highest signal and decode the symbol.

Following a previous work [15], the ABEP of the optically pre-amplified PPM receiver is given by

$$\begin{aligned}\overline{P_e} &= \frac{Q}{2\,(Q-1)}\sum_{q=1}^{Q-1}\binom{Q-1}{q}(-1)^{q+1}\\ &\times\sum_{n=0}^{q\,(M-1)}\sum_{i=n}^{q\,(M-1)}\binom{i+M-1}{n+M-1}\frac{c_i^q}{(1+q)^{i+M}}\,\frac{w(n)}{q^n}\,, \qquad (1)\\ w(n) &= \frac{z_q^n}{n!}\int_0^\infty h^n\, f_h(h)\, e^{-z_q h}\, dh\,,\end{aligned}$$

where Q is the PPM modulation order, M are the optical noise modes, c_i^q are constants, $\lambda = E_b/N_0\,\log_2(Q)$, E_b is the optical energy per bit at the amplifier output, $z_q = \frac{q}{1+q}\,\lambda$ and h is a random variable (RV) denoting the optical signal intensity fluctuations due to fading and pointing errors. In our analysis, we assume that the intensity fluctuations are calculated from the product of two RVs $h = h_a\,h_p$, with h_a corresponding to the contribution of fading and h_p corresponding to the contribution of pointing errors.

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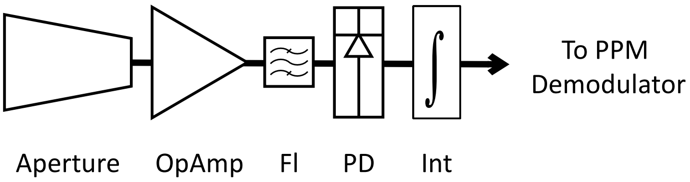

Fig. 1. Optically pre-amplified PPM receiver. OpAmp: Optical amplifier; Fl: Optical Filter; PD: Photodiode; Int: Time-slot integrator.

With respect to fading, h_a is a normalized Malaga-$\mathcal{M}$ RV distributed as [16]

$$f_{h_a}(h_a) = \frac{1}{h_a}\sum_{k=1}^{\beta} b_k\, G_{0,2}^{2,0}\left(\begin{array}{c} - \\ \alpha, k \end{array}\middle|\, \delta\, h_a\right), \quad \delta = \frac{\alpha\,\beta\,(\gamma+\Omega')}{\gamma\,\beta+\Omega'}, \quad b_k = \frac{A}{2}\,a_k\left(\frac{\alpha\,\beta}{\gamma\,\beta+\Omega'}\right)^{-\frac{\alpha+k}{2}}. \tag{2}$$

$G_{p,q}^{m,n}(\cdot)$ is the Meijer G-function [17, eq. (9.301)] and the Malaga-$\mathcal{M}$ model parameters $\alpha, \beta, \gamma, \Omega'$ are described in detail in the literature [16], along with the calculation of A, a_k. Table I summarizes the parameter values for weak, moderate and strong fading as have been obtained in [16].

TABLE I
MALAGA-$\mathcal{M}$ DISTRIBUTION PARAMETER VALUES

Parameter	**Irradiance Fluctuations**		
	Weak	**Moderate**	**Strong**
α	50	2.55	2.281
β	14	22	33
γ	0.006	0.016	0.135
Ω'	1.099	1.751	2.04

The pointing errors h_p are distributed following

$$f_{h_p}(h_p) = \frac{g^2}{A_0^{g^2}}\, h_p^{g^2-1}\,,\; 0 \le h_p \le A_0\,, \tag{3}$$

where it is assumed that the horizontal and vertical sway are independent and identical zero-mean Gaussian variables. The model parameters g and A_0 are calculated from the receiver aperture radius r, the received beam waist w_z and the radial displacement jitter σ_s [2]. The parameters values that are considered in this work are shown in Table II and correspond to $r = 10$ cm and $\sigma_s = 30$ cm, while the beam waist is calculated from $R = w_z/r$.

TABLE II
POINTING ERRORS PARAMETER VALUES

Parameter	**Values**		
R	10	20	30
A_0	0.020	0.005	0.002
g^2	2.807	11.14	25.029

Given (2) and (3), the combined intensity fluctuations h are distributed as [18]

$$f_h(h) = \frac{g^2}{h}\sum_{k=1}^{\beta} b_k \int_1^{\infty} G_{0,2}^{2,0}\left(\begin{array}{c} - \\ \alpha, k \end{array}\middle|\, \frac{\delta\, h\, z}{A_0}\right) z^{-g^2-1}\, dz\,. \tag{4}$$

We now combine (1) and (4) to find the ABEP. Using [17, eq. (7.813)], we find that

$$\int_0^{\infty} h^{n-1}\, G_{0,2}^{2,0}\left(\begin{array}{c} - \\ \alpha, k \end{array}\middle|\, \frac{\delta\, h\, z}{A_0}\right) e^{-z_q h}\, dh = \frac{1}{z_q^n}\, G_{1,2}^{2,1}\left(\begin{array}{c} 1-n \\ \alpha, k \end{array}\middle|\, \frac{\delta\, z}{A_0\, z_q}\right) \tag{5}$$

and

$$w(n) = \frac{g^2}{n!}\sum_{k=1}^{\beta} b_k \int_1^{\infty} G_{1,2}^{2,1}\left(\begin{array}{c} 1-n \\ \alpha, k \end{array}\middle|\, \frac{\delta\, z}{A_0\, z_q}\right) z^{-g^2-1}\, dz\,. \tag{6}$$

The appearing integrals are evaluated numerically and Fig. 2 presents results for $Q = 2, 16$ in weak, moderate and strong Malaga-$\mathcal{M}$ fading.

Following Fig. 2, a narrow beam waist of $R = 10$ provides the optimal ABEP for low E_b/N_0 in weak fading, however as the signal power increases it is preferable to widen the beam waist to $R = 20$. Despite the apparent loss in the received power, widening the beam reduces the power penalty that is introduced by pointing errors and a net gain is observed. A further increase in the beam waist to $R = 30$ does not improve the performance, due the additional power loss, and the optimal beam waist value equals $R \approx 20$ in weak fading. This behaviour is in agreement with the observations in previous works [2], where it has been shown that a similar beam waist size provided optimal outage probability in weak $\gamma - \gamma$ fading. In more intense fading the behavior is different and expanding the beam negatively affects the ABEP. In this regime, fading dominates the power penalty and one prefers to mitigate its impact by using a narrow beam to collect more energy rather than address the lesser impact of pointing errors. As such, an optimal value of $R \approx 10$ is observed for both moderate and strong fading.

The modulation order positively affects the system performance and the 16-PPM system exhibits a power gain of approximately 4-5 dB compared to the 2-PPM one in all fading and pointing error scenarios. Fig. 3 further details the performance of the system for modulation orders of $Q = 2, 4, 8, 16$ in the optimized $R = 20$ receiver under weak fading. An initial power gain of 2.5 dB is achieved when the modulation order is increased to $Q = 4$, but the gain gradually becomes smaller and corresponds to 1.5 and 1.0 dB for 8-PPM and 16-PPM, respectively. The figure also shows the impact of noise modes in the performance of the receiver and up to $M = 2000$ is presented. Clearly, the introduction of additional noise degrades the performance and the optical filter should be as narrow as possible. If this is not an option due to wavelength drifts or multi-wavelength operation, then a power penalty of 3-4 dB per ten-fold increase in the noise modes is to be expected. As an example, an $M = 200$ system with a number of modes comparable to the ones considered in [19] exhibits a power penalty equal to 7 dB. Similar arrangements can be explored by evaluating the presented equations.

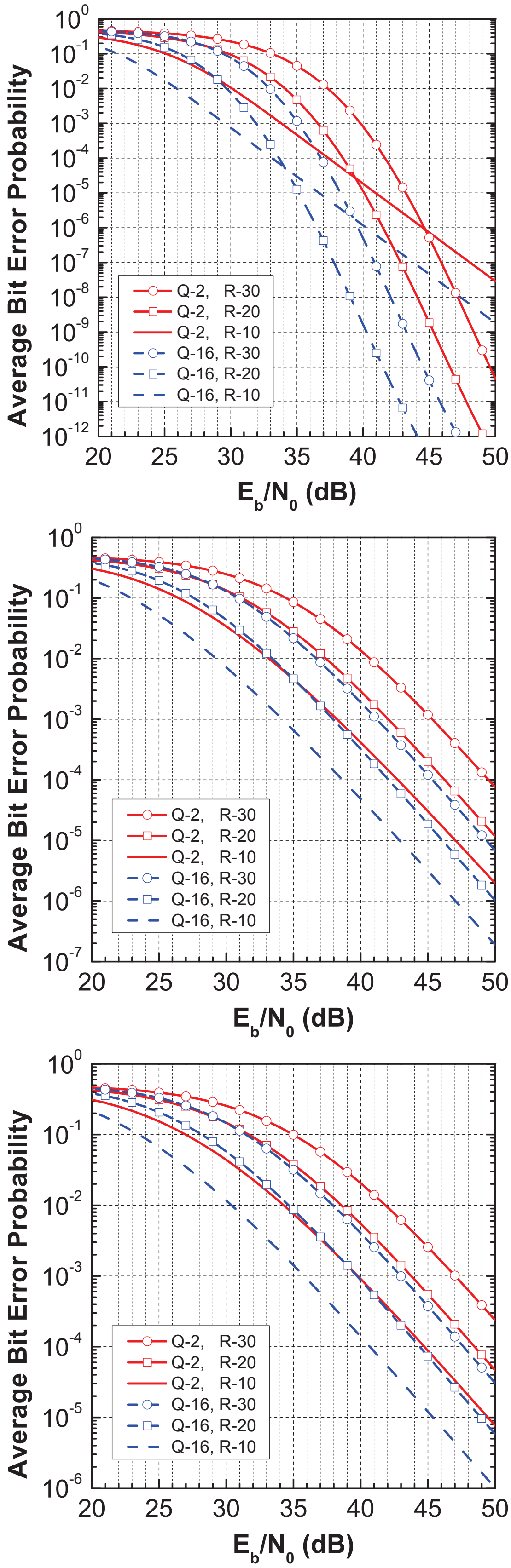

Fig. 2. PPM ABEP in Malaga-$\mathcal{M}$ fading with pointing errors. The plots correspond to weak (top), moderate (middle) and strong (bottom) fading. The noise modes are equal to $M = 2$.

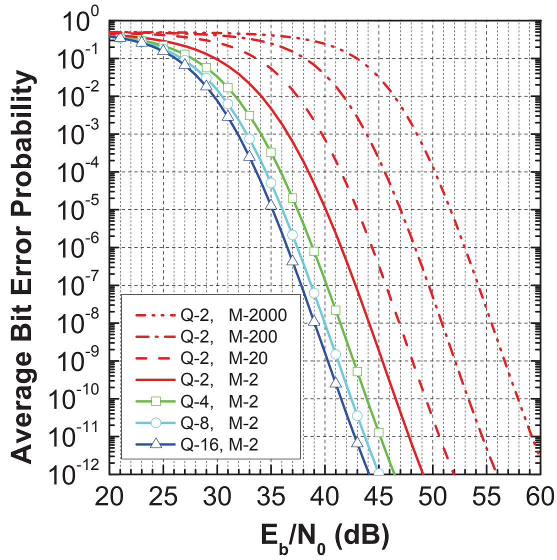

Fig. 3. PPM ABEP in weak Malaga-$\mathcal{M}$ fading with pointing errors for increasing modulation orders and noise modes.

III. Results for Other Fading Distributions

In $\gamma-\gamma$ fading h_a is distributed as

$$f_{h_a}(h_a) = \frac{1}{\Gamma(\alpha)\,\Gamma(\beta)\,h_a}\, G_{0,2}^{2,0}\left(\begin{matrix} - \\ \alpha, \beta \end{matrix} \middle| \alpha\,\beta\,h_a \right) \tag{7}$$

where the distribution parameters α and β are calculated from [20, eq. (5.15, 9.41, 9.46, 9.138)]. Following the same procedure with the Malaga-$\mathcal{M}$ case we find that the ABEP is given by (1) with the weight function being modified to

$$\begin{aligned} w(n) = {} & \frac{g^2}{\Gamma(\alpha)\,\Gamma(\beta)\,n!} \\ & \times \int_1^{\infty} G_{1,2}^{2,1}\left(\begin{matrix} 1-n \\ \alpha, \beta \end{matrix} \middle| \frac{\alpha\,\beta\,z}{A_0\,z_q} \right) z^{-g^2-1}\,dz\,. \end{aligned} \tag{8}$$

Fig. 4 summarizes the performance of the system in $\gamma-\gamma$ fading for the parameters that are shown in Table III and have been obtained for 100 m and 500 m links that operate at the wavelength of 1550 nm and for a structure constant equal to $C_n^2 = 4.58 \cdot 10^{-13}$ $\mathrm{m}^{-2/3}$. The pointing error parameters are shown in Table II. The results are similar to the Malaga-$\mathcal{M}$ case and an optimal beam waist of $R \approx 20$ is observed in weaker fading, while $R \approx 10$ in stronger fading. The modulation order increase provides a 4-5 dB gain in $\gamma-\gamma$ fading, as well.

TABLE III
$\gamma-\gamma$ Distribution Parameter Values

Parameter	l-100 m	l-500 m
α	16.53	4.04
β	14.91	1.53

Finally, in negative exponential fading h_a is distributed as

$$f_{h_a}(h_a) = e^{-h_a} \tag{9}$$

and the intensity fluctuations are distributed as

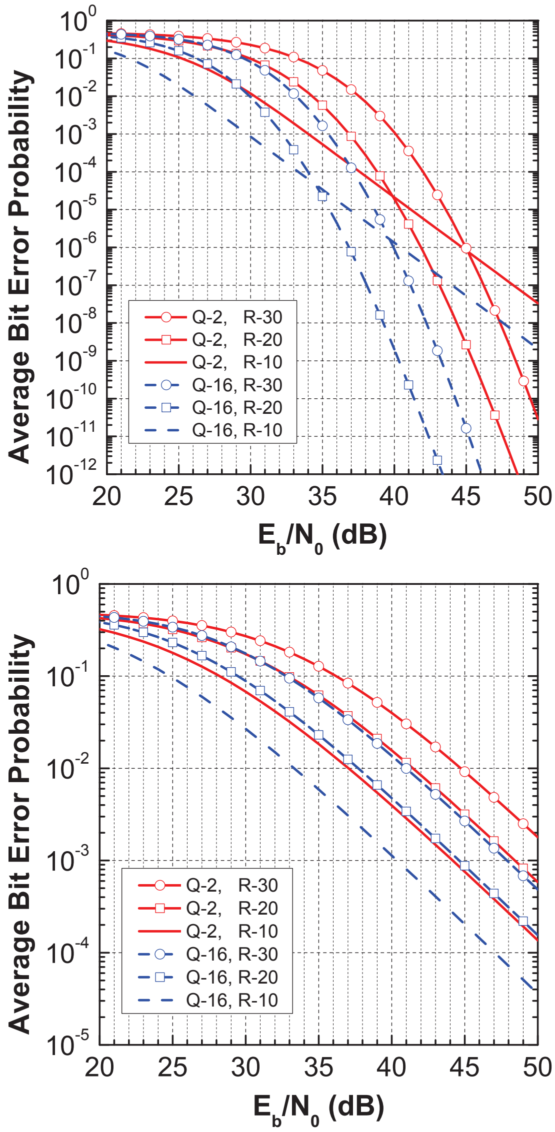

Fig. 4. PPM ABEP in $\gamma-\gamma$ fading with pointing errors. The plots correspond to an 100 m (top) and 500 m (bottom) link. The noise modes are equal to $M=2$.

$$f_h(h) = \frac{g^2}{A_0} \int_1^\infty e^{-h\,z/A_0}\, z^{-g^2}\, dz\,. \tag{10}$$

The weight function becomes

$$\begin{aligned} w(n) &= \frac{z_q^n}{n!}\,\frac{g^2}{A_0} \int_0^\infty \int_1^\infty h^n\, e^{-(z_q+z/A_0)\,h}\, z^{-g^2}\, dz\, dh \\ &= \frac{g^2}{A_0\, z_q} \int_1^\infty \left(\frac{z}{A_0\, z_q}+1\right)^{-n-1} z^{-g^2}\, dz \end{aligned} \tag{11}$$

and the corresponding results are plotted in Fig. 5. The system performance in negative exponential fading is in agreement with what is expected from the previous results for strong Malaga-$\mathcal{M}$ and $\gamma-\gamma$ fading.

IV. Conclusion

We have presented analytical results for the impact of fading and pointing errors in a pre-amplified PPM receiver. We have considered the generalized Malaga-$\mathcal{M}$ fading distribution,

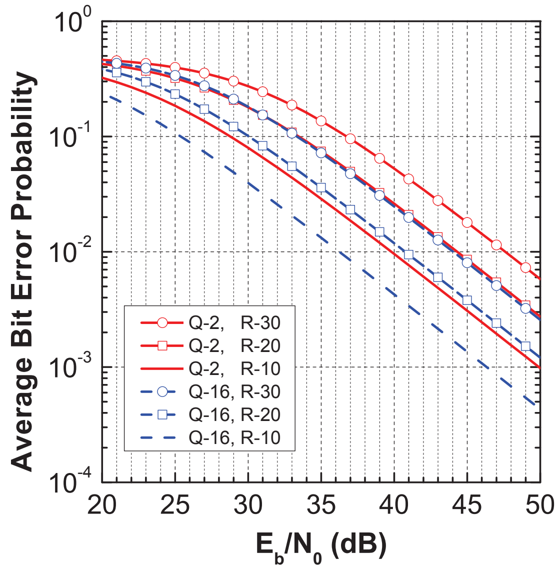

Fig. 5. PPM ABEP in negative exponential fading with pointing errors. The noise modes are equal to $M=2$.

therefore a number of distributions that arise from the Malaga-$\mathcal{M}$ distribution can be treated in a straight-forward manner by adjusting the distribution parameters. We have also provided analytical results for the well-established $\gamma-\gamma$ and negative exponential distributions. The results were utilized to assess the performance in weak, moderate and strong fading and demonstrate that the performance improves with the modulation order and worsens with the number of noise modes irrespective of the fading regime and the distribution that is utilized to describe it. Similar to other modulation formats, it was shown that the beam waist plays an important role in the ABEP of the PPM receiver, especially in the weak fading regime where the impact of pointing errors can be reduced by expanding the beam.

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