On the Weibull Distribution with Arbitrary Correlation

Zafiro G. Papadimitriou*‡, P. Takis Mathiopoulos*, Nikos C. Sagias†, Lazaros Merakos‡
*Institute for Space Applications and Remote Sensing, National Observatory of Athens, Metaxa & Vas. Pavlou Street, 15236 Athens, Greece, E-mail: {zpapadim;mathio}@space.noa.gr
† Institute of Informatics and Telecommunications, National Centre for Scientific Research Demokritos, Agia Paraskevi, GR-15310 Athens, Greece, E-mail: nsagias@ieee.org
‡ Department of Informatics and Telecommunications, University of Athens, GR-15784 Athens, Greece, E-mail: merakos@di.uoa.gr

***Abstract*—This paper studies the performance of the quadrivariate Weibull distribution with arbitrary correlation assuming non-identical fading parameters and average powers. Novel infinite series representations for the joint probability density function (PDF), the cumulative distribution function (CDF) and the product moments are derived. Using these theoretical results, a performance analysis study in terms of outage probability, for receivers employing selection combining (SC) diversity techniques is presented. The proposed mathematical analysis is accompanied by various numerical results, with parameters of interest the fading severity and the power decay factor. Furthermore, equivalent performance evaluation results, obtained by various computer simulations have verified the accuracy of the proposed analytical method.**

***Index Terms*—Arbitrary correlation, selection combining (SC), diversity, outage probability, Weibull fading.**

I. Introduction

One of the most efficient and widely used techniques to reduce the destructive effects of fading on the system's performance is diversity reception. When space and polarization diversity is applied in practical systems (e.g. mobile terminal, indoor base-station $etc.$), the antennas placed at the receiver are not sufficiently separated and consequently the combined signals are correlated with each other. In order to analyze and model such realistic wireless channels with correlated fading, multivariate statistics are commonly used [1].

In the open technical literature there exist many papers that concern multivariate statistics for fading channel modeling. Most of them deal with the constant and exponential correlation model. For the first one, the correlation depends on the distance among the combining antennas and as a consequence it is more suitable for equidistant antennas [1]. For the exponential correlation model, widely used for performance analysis of space diversity techniques [2]–[4] or multiple-input multiple-output (MIMO) systems [5], the correlation between the pairs of combined signals decays as the spacing between the antennas increases [1]. In [6], the authors presented useful closed-form expressions for the multivariate, exponentially correlated Nakagami-m probability density function (PDF) and the cumulative distribution function (CDF).

A more general approach would be to consider that the correlation between the pairs of combined signal can take a more general form. Thus, an arbitrary correlation model will include the two previously mentioned models as special cases. Past works concerning arbitrarily correlated multivariate distributions can be found in [7]–[10]. In [7] infinite series representations for the joint PDF and the CDF of three and four arbitrarily correlated Rayleigh random variables were presented. In [8] useful closed-form expressions for the joint Nakagami-m multivariate PDF and CDF with arbitrary correlation were derived and the correlation matrix was approximated by a Green's matrix so that the inverse matrix would be tridiagonal. Similarly, in [9], the Green's matrix was used to approximate the correlation matrix of L - branch selection combining (SC) receivers and the outage probability for log-normal fading channels has been obtained. In [10] expressions for multivariate Rayleigh and exponential PDFs generated from correlated Gaussian random variables were presented, while a general expression for the multivariate exponential characteristic function (CF), in terms of determinants, was also derived.

The Weibull distribution although it was originally used in reliability and failure data analysis, has recently received renewed interest because it exhibits a very good fit to experimental fading channel measurements for both indoor and outdoor terrestrial radio propagation environments [11], [12]. Moreover, in [13], it was argued that the Weibull distribution could also been considered as a more generic channel model for land-mobile satellite systems.

For this distribution, in a recent paper [14], the joint PDF, CDF and the moment-generating function (MGF) for the bivariate Weibull distribution have been analytically presented. The multivariate Weibull distribution has also been studied for the exponential and the constant correlation case with equal average fading powers. Finally in [15] the trivariate Weibull distribution with arbitrary correlation is studied, the joint PDF, CDF and the product moments are obtained and the outage probability of SC receivers is derived. It should be mentioned, however, that to the best of our knowledge, analyzing the performance of an arbitrary Weibull fading channel with four diversity branches has not been presented in the open technical literature.

Motivated by this observation, in this paper we present novel infinite series representations for the joint PDF, CDF and the moments for the quadrivariate Weibull distribution, assuming

978-1-4244-1688-2/08/$25.00

an arbitrary covariance matrix and non-identical fading parameters and average powers. Furthermore, the outage probability of SC receivers is derived and various performance evaluation results are presented.

The organization of this paper is as follows: After this introduction, in Section II the system model is presented and the quadrivariate Weibull distribution is analyzed. Additionally, in Section III the SNR statistics are derived while in Section IV the performance of SC receivers in terms of outage probability is studied. Various numerical results are presented in Section V showing the impact of correlation, the fading severity and the power decay factor on the system's performance. Finally, in Section VI several useful concluding remarks are given.

II. System model

Let $\mathbf{G} = \{G_1, G_2, ..., G_L\}$ be the joint complex Gaussian random variables (RVs) with zero mean $E\langle G_\ell \rangle = 0$, $\ell = 1, 2, ..., L$ and positive definite covariance matrix $\mathbf{\Psi}$, with elements $\psi_{i\kappa} = E\langle G_i G_\kappa^* \rangle$, where $E\langle \cdot \rangle$ denotes expectation. Let $\mathbf{R} = \{R_1, R_2, ..., R_L\}$ be a set of Rayleigh RVs with $R_\ell = |G_\ell|$ where $|\cdot|$ denotes absolute value. The Weibull fading channel h_ℓ can be expressed in terms of the Gaussian in-phase X_ℓ and quadrature Y_ℓ elements [14] as follows:

$$h_\ell = (X_\ell + jY_\ell)^{2/\beta_\ell} = G_\ell^{2/\beta_\ell} \tag{1}$$

where $\beta_\ell > 0$ represents the Weibull fading parameter and j the imaginary operator. Since $Z_\ell = |h_\ell|$, Z_ℓ can be expressed as a power transformation of a Rayleigh distributed RV $R_\ell = |X_\ell + jY_\ell|$ as [16]

$$Z_\ell = R_\ell^{2/\beta_\ell}. \tag{2}$$

Quadrivariate Weibull distribution

For the quadrivariate Weibull distribution ($L = 4$) we consider the inverse covariance matrix, $\mathbf{\Phi}$, given by

$$\mathbf{\Phi} = \mathbf{\Psi}^{-1} = \begin{bmatrix} \phi_{11}, & \phi_{12}, & \phi_{13}, & 0 \\ \phi_{12}^*, & \phi_{22}, & \phi_{23}, & \phi_{24} \\ \phi_{13}^*, & \phi_{23}^*, & \phi_{33}, & \phi_{34} \\ 0, & \phi_{24}^*, & \phi_{34}^*, & \phi_{44} \end{bmatrix} \tag{3}$$

with $\phi_{i\kappa} = |\phi_{i\kappa}| \exp(j\chi_{i\kappa})$ where $i, \kappa \in \{1, 2, 3, 4\}$.

Note here for the quadrivariate case, the obtained results are more general than those presented by Blumenson and Miller in [17] for the multivariate Rayleigh distribution. More specifically, the statistical properties derived in [17] hold only under the assumption that $\mathbf{\Psi}$ is tridiagonal so that $\phi_{i\kappa} = 0$ for $|i - \kappa| > 1$.

The joint PDF of $\mathbf{Z} = \{Z_1, Z_2, Z_3, Z_4\}$ can be derived using [7, eq. (16)] and applying the power transformation described in (2) as a product of the modified Bessel function of the first kind as follows

$$\begin{aligned} f_{\mathbf{Z}}(z_1, z_2, z_3, z_4) &= \beta_1\beta_2\beta_3\beta_4 \det(\mathbf{\Phi}) \\ &\times \exp\left[-\left(z_1^{\beta_1}\phi_{11} + z_2^{\beta_2}\phi_{22} + z_3^{\beta_3}\phi_{33} + z_4^{\beta_4}\phi_{44}\right)\right] \\ &\times z_1^{\beta_1-1} z_2^{\beta_2-1} z_3^{\beta_3-1} z_4^{\beta_4-1} \sum_{j=0}^{\infty}\sum_{k=-\infty}^{\infty} \epsilon_j (-1)^{j+k}\cos(A) \\ &\times I_j(2z_1^{\beta_1/2} z_2^{\beta_2/2}|\phi_{12}|) I_j(2z_1^{\beta_1/2} z_3^{\beta_3/2}|\phi_{13}|) \\ &\times I_k(2z_2^{\beta_2/2} z_4^{\beta_4/2}|\phi_{24}|) I_k(2z_3^{\beta_3/2} z_4^{\beta_4/2}|\phi_{34}|) \\ &\times I_{j+k}(2z_2^{\beta_2/2} z_3^{\beta_3/2}|\phi_{23}|). \end{aligned} \tag{4}$$

Using the infinite series expansion of the Bessel function [18, eq. (8.447/1)]

$$I_\nu(u) = \sum_{k=0}^{\infty} \frac{1}{(k+\nu)!k!} (\frac{u}{2})^{2k+\nu} \tag{5}$$

and after some cumbersome but straightforward mathematical steps, (4) can be expressed as

$$\begin{aligned} f_{\mathbf{Z}}(z_1, z_2, z_3, z_4) &= \beta_1\beta_2\beta_3\beta_4 \det(\mathbf{\Phi}) \\ &\times \exp\left[-\left(z_1^{\beta_1}\phi_{11} + z_2^{\beta_2}\phi_{22} + z_3^{\beta_3}\phi_{33} + z_4^{\beta_4}\phi_{44}\right)\right] \\ &\times z_1^{\beta_1-1} z_2^{\beta_2-1} z_3^{\beta_3-1} z_4^{\beta_4-1} \sum_{j=0}^{\infty}\sum_{k=-\infty}^{\infty} \epsilon_j (-1)^{j+k}\cos(A) \\ &\times \sum_{\ell,m,n,p,q=0}^{\infty} C\,|\phi_{12}|^{2\ell+j}|\phi_{13}|^{2m+j}|\phi_{24}|^{2n+|k|} \\ &\times |\phi_{34}|^{2p+|k|}|\phi_{23}|^{2q+|j+k|} z_1^{\beta_1(\ell+n+j)} \\ &\times z_2^{\beta_2(\ell+n+q+\frac{j+|k|+|j+k|}{2})} z_3^{\beta_3(m+p+q+\frac{j+|k|+|j+k|}{2})} \\ &\times z_4^{\beta_4(n+p+\frac{|k|}{2})} \end{aligned} \tag{6}$$

where ϵ_j is the Neumann factor ($\epsilon_0 = 1, \epsilon_j = 2$ for $k = 1, 2, \cdots$), $A = j(\chi_{12} + \chi_{23} + \chi_{31}) + k(\chi_{23} + \chi_{34} + \chi_{42})$ and

$$C = \frac{1}{\ell!(\ell+j)!m!(m+j)!n!(n+|k|)!p!(p+|k|)!q!(q+\frac{|k+j|}{2})!}.$$

The corresponding CDF is obtained by integrating (6), as follows

$$\begin{aligned}F_{\mathbf{Z}}(z_1,z_2,z_3,z_4) &= \det(\mathbf{\Phi})\sum_{\mathbf{j=0}}^{\infty}\sum_{\mathbf{k=-\infty}}^{\infty}\epsilon_{\mathbf{j}}(-\mathbf{1})^{\mathbf{j+k}}\mathbf{cos}(\mathbf{A})\\
&\times\sum_{\ell,m,n,p=0}^{\infty}\frac{|\phi_{12}|^{2\ell+j}|\phi_{13}|^{2m+j}|\phi_{24}|^{2n+|k|}|\phi_{34}|^{2p+|k|}}{\ell!(\ell+j)!m!(m+j)!n!(n+|k|)!p!(p+|k|)!}\\
&\times\frac{\gamma\left(\ell+m+j+1,z_1^{\beta_1}\phi_{11}\right)}{\phi_{11}^{\ell+m+j+1}}\\
&\times\frac{\gamma\left(n+p+|k|+1,z_4^{\beta_4}\phi_{44}\right)}{\phi_{44}^{n+p+|k|+1}}\sum_{q=0}^{\infty}\frac{|\phi_{23}|^{2q+|j+k|}}{q!(q+\frac{|k+j|}{2})!}\\
&\times\frac{\gamma\left(\ell+n+q+\frac{j+|k|+|j+k|}{2}+1,z_2^{\beta_2}\phi_{22}\right)}{\phi_{22}^{\ell+n+q+\frac{j+|k|+|j+k|}{2}+1}}\\
&\times\frac{\gamma\left(m+p+q+\frac{j+|k|+|j+k|}{2}+1,z_3^{\beta_3}\phi_{33}\right)}{\phi_{33}^{m+p+q+\frac{j+|k|+|j+k|}{2}+1}}\end{aligned}\tag{7}$$

with $\gamma(\cdot,\cdot)$ denoting the incomplete lower Gamma function [18, eq. (3.381/1)].

Moreover, the joint moments are defined as

$$\begin{aligned}E\langle z_1^{\alpha},z_2^{\eta},z_3^{\vartheta},z_4^{\zeta}\rangle &\triangleq \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} z_1^{\alpha}z_2^{\eta}z_3^{\vartheta}z_4^{\zeta}\\
&\times f_{\mathbf{Z}}(z_1,z_2,z_3,z_4)dz_1dz_2,dz_3dz_4\end{aligned}\tag{8}$$

and hence substituting (6) in (8) can be derived as follows

$$\begin{aligned}E\langle z_1^{\alpha},z_2^{\eta},z_3^{\vartheta},z_4^{\zeta}\rangle &= \det(\mathbf{\Phi})\times\sum_{\mathbf{j=0}}^{\infty}\sum_{\mathbf{k=-\infty}}^{\infty}\epsilon_{\mathbf{j}}(-\mathbf{1})^{\mathbf{j+k}}\cos(\mathbf{A})\\
&\times\sum_{\ell,m,n,p=0}^{\infty}C\,|\phi_{12}|^{2\ell+j}|\phi_{13}|^{2m+j}|\phi_{24}|^{2n+|k|}|\phi_{34}|^{2p+|k|}\\
&\times\frac{\Gamma\left(\ell+m+j+\alpha/\beta_1+1\right)}{\phi_{11}^{(\ell+m+j+1)+\alpha/\beta_1}}\frac{\Gamma\left(n+p+|k|+\zeta/\beta_4+1\right)}{\phi_{44}^{(n+p+|k|+1)+\zeta/\beta_4}}\\
&\times\sum_{q=0}^{\infty}|\phi_{23}|^{2q+|j+k|}\frac{1}{\phi_{22}^{(\ell+n+q+\frac{j+|k|+|j+k|}{2}+\frac{\eta}{\beta_2}+1)}}\\
&\times\frac{1}{\phi_{33}^{(m+p+q+\frac{j+|k|+|j+k|}{2}+\frac{\vartheta}{\beta_3}+1)}}\\
&\times\Gamma\left(\ell+n+q+\frac{j+|k|+|j+k|}{2}+\frac{\eta}{\beta_2}+1\right)\\
&\times\Gamma\left(m+p+q+\frac{j+|k|+|j+k|}{2}+\frac{\vartheta}{\beta_3}+1\right).\end{aligned}\tag{9}$$

III. SNR Statistics

In order to obtain and study performance measures for diversity receivers, the statistics of SNR are needed. Assuming a diversity receiver with L branches, the received at the ℓth branch baseband signal can be mathematically expressed as follows

$$\zeta_{\ell} = wh_{\ell} + n_{\ell} \tag{10}$$

where w is the complex transmitted symbol, with $E_s = E\langle|w|^2\rangle$ being the transmitted average symbol's energy and n_{ℓ} is the additive white Gaussian noise (AWGN) with single-sided power spectral density N_0. The instantaneous per symbol SNR of the ℓth diversity channel can be expressed as

$$\gamma_{\ell} = Z_{\ell}^2 E_s/N_0. \tag{11}$$

Moreover, since the corresponding average SNR is expressed as

$$\overline{\gamma}_{\ell} = E\langle Z_{\ell}^2\rangle\frac{E_s}{N_0} = \Gamma(d_{2,\ell})\Omega_{\ell}^{2/\beta_{\ell}}\frac{E_s}{N_0} \tag{12}$$

where $d_{\tau,\ell} = 1 + \tau/\beta_{\ell}$ with τ taking non-negative values, expressions for the statistics of γ_{ℓ} can be easily derived by replacing β_{ℓ} with $\beta_{\ell}/2$ and Ω_{ℓ} with $(\alpha_{\ell}\overline{\gamma}_{\ell})^{\beta_{\ell}/2}$, in the corresponding expressions for the fading envelope Z_{ℓ}.

Thus using (7), the CDF of the SNR for the quadrivariate Weibull distribution with arbitrary correlation,

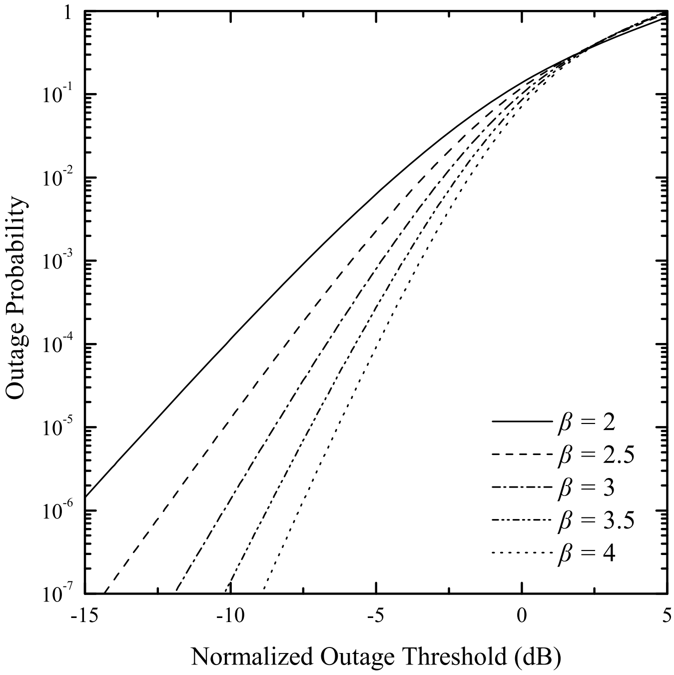

Fig. 1. Outage probability of a 4-branch SC receiver as a function of the normalized outage threshold, $\gamma_{th}/\overline{\gamma}$ for different values of β.

$F_{\boldsymbol{\gamma}}(\gamma_1,\gamma_2,\gamma_3,\gamma_4)$, can be obtained as

$$F_{\gamma}(\gamma_1,\gamma_2,\gamma_3,\gamma_4)=\det(\mathbf{\Phi}')\sum_{\mathbf{j=0}}^{\infty}\sum_{\mathbf{k=-\infty}}^{\infty}\epsilon_{\mathbf{j}}(\mathbf{-1})^{\mathbf{j+k}}\mathbf{\cos(A)}$$
$$\times\sum_{\ell,m,n,p=0}^{\infty}\frac{|\phi'_{12}|^{2\ell+j}\,|\phi'_{13}|^{2m+j}\,|\phi'_{24}|^{2n+|k|}\,|\phi'_{34}|^{2p+|k|}}{\ell!(\ell+j)!m!(m+j)!n!(n+|k|)!p!(p+|k|)!}$$
$$\times\frac{\gamma\left(\ell+m+j+1,\gamma_1^{\beta_1/2}\phi'_{11}\right)}{{\phi'}_{11}^{\ell+m+j+1}}$$
$$\times\frac{\gamma\left(n+p+|k|+1,\gamma_4^{\beta_4/2}\phi'_{44}\right)}{{\phi'_{44}}^{n+p+|k|+1}}\sum_{q=0}^{\infty}\frac{|\phi'_{23}|^{2q+|j+k|}}{q!(q+\frac{|k+j|}{2})!}$$
$$\times\frac{\gamma\left(\ell+n+q+\frac{j+|k|+|j+k|}{2}+1,\gamma_2^{\beta_2/2}\phi'_{22}\right)}{{\phi'_{22}}^{\ell+n+q+\frac{j+|k|+|j+k|}{2}+1}}$$
$$\times\frac{\gamma\left(m+p+q+\frac{j+|k|+|j+k|}{2}+1,\gamma_3^{\beta_3/2}\phi'_{33}\right)}{{\phi'_{33}}^{m+p+q+\frac{j+|k|+|j+k|}{2}+1}} \quad (13)$$

where Φ' with elements $\phi'_{i\kappa}$ is the inverse covariance matrix obtained from (3) after the substitutions mentioned in (11) and (12).

IV. Outage Probability of SC Receivers

Consider a SC receiver with four branches operating over an arbitrarily correlated Weibull fading environment and in the presence of an AWGN channel. The instantaneous per symbol SNR at the output of a receiver of this type, will be the one with the highest instantaneous value between the branches [19]

$$\gamma_{sc}=max\{\gamma_1,\gamma_2,\gamma_3,\gamma_4\}. \quad (14)$$

The outage probability, P_{out}, is defined as the probability that the instantaneous error probability exceeds a specified value or equivalently the probability that the receiver output SNR, γ_{sc}, falls below a certain specified threshold, γ_{th} [1]. As a consequence and since the CDF of γ_{sc} equals $F_{\boldsymbol{\gamma}}(\gamma_{sc},\gamma_{sc},...,\gamma_{sc})$, P_{out} can be derived as follows

$$P_{out}(\gamma_{th})=F_{\gamma_{sc}}(\gamma_{th}). \quad (15)$$

Thus, with the aid of (13), P_{out} can be directly obtained.

V. Numerical Results

In this section, we use the previous mathematical analysis in order to present numerical results for the performance of SC receivers, where without loss of generality, it is assumed that $\beta_\ell=\beta\ \forall\ell$.

Consider an antenna array with normalized covariance matrix

$$\mathbf{\Psi}=\begin{bmatrix}1, & 0.4975, & 0.2998, & 0.1121\\ 0.4975, & 1, & 0.1912, & 0.1585\\ 0.2998, & 0.1912, & 1, & 0.1868\\ 0.1121, & 0.1585, & 0.7868, & 1\end{bmatrix} \quad (16)$$

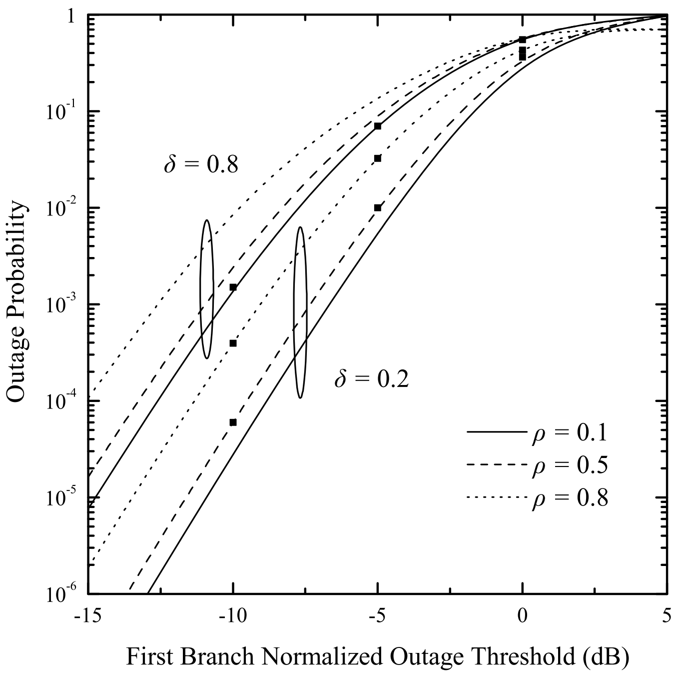

Fig. 2. Outage probability of a 4-branch SC receiver as a function of the first branch normalized outage threshold, $\gamma_{th}/\overline{\gamma}_1$ for different values of ρ and δ.

The inverse covariance matrix now satisfies (3) and thus, using (15) the outage probability can be evaluated as shown in Fig. 1. More specifically, assuming $\overline{\gamma}_1=\overline{\gamma}_2=\overline{\gamma}_3=\overline{\gamma}_4=\overline{\gamma}$, the outage probability is plotted as a function of the normalized outage threshold, $\gamma_{th}/\overline{\gamma}$, for different values of β. It is evident that the system's performance degrades with a decrease of β and/or an increase on $\gamma_{th}/\overline{\gamma}$.

Moreover, Fig. 2 and Fig. 3 show the impact of an exponentially decaying power factor δ on the outage probability of SC receiver with exponentially correlated branches, assuming non-identical distributed Weibull channels, i.e., $\overline{\gamma}_\ell=\overline{\gamma}_1\exp[-(\ell-1)\delta]$. In Fig. 2 the outage probability is plotted as a function of the first branch normalized outage threshold $\gamma_{th}/\overline{\gamma_1}$ assuming $\beta=2.5$ and for different values of δ and ρ while in Fig. 3 the outage probability is plotted for $\rho=0.3$ and for different values of δ and β. In order to verify the proposed mathematical analysis some computer simulation results are also included for comparison purposes. The obtained results show clearly that the systems performance degrades with an increase of δ and ρ.

In case the normalized covariance matrix is given as [8]

$$\mathbf{\Psi}=\begin{bmatrix}1, & 0.7, & 0.538, & 0.3777\\ 0.7, & 1, & 0.769, & 0.538\\ 0.538, & 0.769, & 1, & 0.7\\ 0.3777, & 0.538, & 0.7, & 1\end{bmatrix} \quad (17)$$

following a similar procedure, the outage probability can be derived as shown in Fig. 4, for different values of β. Once again the obtained results show that the system's performance improves with an increase of β. The theoretical results are again in excellent agreement with the computer simulation

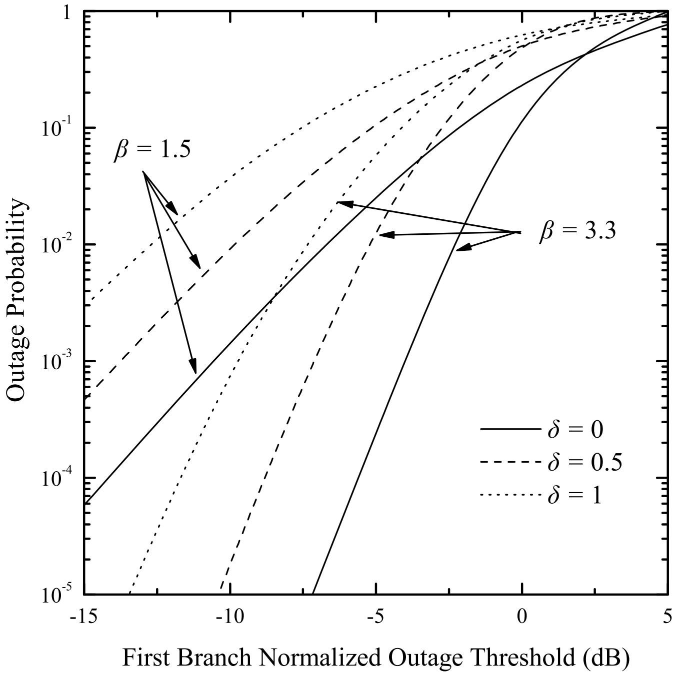

Fig. 3. Outage probability of a 4-branch SC receiver as a function of the first branch normalized outage threshold, $\gamma_{th}/\overline{\gamma}_1$ for different values of β and δ.

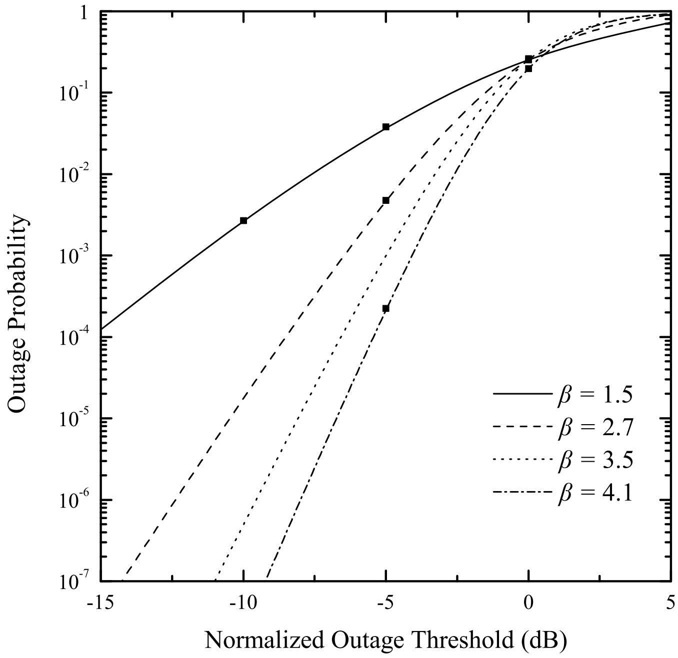

Fig. 4. Outage probability of a 4-branch SC receiver as a function of the normalized outage threshold, $\gamma_{th}/\overline{\gamma}$ for different values of β.

results.

VI. Conclusions

A performance analysis study for the quadrivariate Weibull distribution with arbitrary correlation was presented. Initially, infinite series representations for the joint PDF, CDF and the product moments were derived. These theoretical results were applied to analyze the performance of SC receivers with four branches, operating in an arbitrarily correlated fading environment. The outage probability, one of the most useful performance criteria, was studied. Various performance evaluation results were presented showing the effects of correlation, fading severity and the power decay factor on the system's performance.

Acknowledgments

This paper is part of the 03ED910 research project, implemented within the framework of the "Reinforcement Programme of Human Research Manpower" (PENED) and co-financed by National and Community Funds (25% from the Greek Ministry of Development-General Secretariat of Research and Technology and 75% from E.U.- European Social Fund).

References

[1] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[2] V. V. Veeravalli, "On the performance analysis for signaling on correlated fading channels," *IEEE Transactions on Communications*, vol. 49, no. 11, pp. 1879–1883, Nov. 2001.

[3] Y.-K. Ko, M.-S. Alouini, and M. K. Simon, "Outage probability of diversity systems over generalized fading channels," *IEEE Transactions on Communications*, vol. 48, no. 11, pp. 1783–1787, Nov. 2000.

[4] P. Lombardo, G. Fedele, and M. M. Rao, "MRC performance for binary signals in Nakagami fading with general branch correlation," *IEEE Transactions on Communications*, vol. 47, no. 1, pp. 44–52, Jan. 1999.

[5] S. L. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Communication Letters*, vol. 5, no. 9, pp. 369–371, Sep. 2001.

[6] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoloulos, "On the multivariate Nakagami-m distribution with exponential correlation," *IEEE Transactions on Communications*, vol. 51, no. 8, pp. 1240–1244, Aug. 2003.

[7] Y. Chen and C. Tellambura, "Infinite series representation of the trivariate and quadrivariate Rayleigh distribution and their applications," *IEEE Transactions on Communications*, vol. 53, no. 12, pp. 2092–2101, Dec. 2005.

[8] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoloulos, "An efficient approach to multivariate Nakagami-m distribution using Green's matrix approximation," *IEEE Transactions on Wireless Communications*, vol. 2, no. 5, pp. 883–889, Sep. 2003.

[9] T. Piboongungon and V. A. Aalo, "Outage probability of L-branch selection combining in correlated Lognormal fading channels," *IEEE Electronic Letters*, vol. 40, no. 14, pp. 886–888, Jul. 2004.

[10] R. K. Mallik, "On the multivariate Rayleigh and exponential distributions," *IEEE Transactions on Information Theory*, vol. 49, no. 6, pp. 1499–1515, Jun. 2003.

[11] F. Babich and G. Lombardi, "Statistical analysis and characterization of the indoor propagation channel," *IEEE Transactions on Communications*, vol. 48, no. 3, pp. 455–464, Mar. 2000.

[12] N. H. Shepherd, "Radio wave loss deviation and shadow loss at 900 MHz," *IEEE Transactions on Vehicular Technology*, vol. VT-26, pp. 309–313, 1977.

[13] P. S. Bithas, G. K. Karagiannidis, N. C. Sagias, P. T. Mathiopoulos, S. A. Kotsopoulos, and G. Corazza, "Performance analysis of a class of GSC receivers over nonidentical Weibull fading channels," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 6, pp. 1963–1970, Nov. 2005.

[14] N. C. Sagias and G. K. Karagiannidis, "Gaussian class multivariate Weibull distributions: Theory and applications in fading channels," *IEEE Transactions on Information Theory*, vol. 51, no. 10, pp. 3608–3619, Oct. 2005.

[15] Z. G. Papadimitriou, N. C. Sagias, P. S. Bithas, P. T. Mathiopoulos, and L. Merakos, "The trivariate Weibull distribution with arbitrary

correlation," in *Proc. of the IEEE Int. Workshop on Satellite and Space Communications*, Madrid, Spain, 2006, pp. 249–253.

[16] M. D. Jacoub, "The $\alpha-\mu$ distribution: A general fading distribution," in *Proc. IEEE Int. Symp. Personal, Mobile Radio Communications*, Lisbon, Portugal, Sep. 2002.

[17] L. E. Blumenson and K. S. Miller, "Properties of generalized rayleigh distributions," *Ann.Math. Statist.*, vol. 34, pp. 903–910, 1963.

[18] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic Press, 2000.

[19] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoulos, "Performance analysis of triple selection diversity over exponentially correlated Nakagami-m fading channels," *IEEE Transactions on Communications*, vol. 51, no. 8, pp. 1245–1248, Aug. 2003.