Equalization of Negative-exponential Fading in Saturated Semiconductor Optical Amplifiers

K. Yiannopoulos, *Member IEEE*, N.C. Sagias, *Senior Member IEEE*, and A.C. Boucouvalas, *Fellow IEEE*
University of Peloponnese, Department of Informatics and Telecommunications,
Terma Karaiskaki, Tripoli 22100, Greece
e-mail: {kyianno, nsagias, acb}@uop.gr

***Abstract*— In this paper, we discuss the mitigation of negative-exponential fading in Optical Wireless Communication (OWC) systems. The mitigation technique involves the utilization of a semiconductor optical amplifier (SOA) whose gain saturates in normal link operation, but which also provides a power-dependent varying gain when the link experiences a fade. The unbalanced SOA operation serves towards the equalization of the signal power at its output and as a result the link fades become less severe and of reduced duration. Our analytical results predict that the fade probability can be reduced by over 90% under negative-exponential fading conditions, while the scintillation index at the SOA output is also reduced by 75% if an optimal level of received power, and therefore gain saturation, is observed. Finally, our results show that the average duration of fades can also be reduced by a significant percentage of 68% for the same level of gain saturation in the SOA.**

Keywords—optical wireless; negative-exponential fading; semiconductor optical amplifier

I. INTRODUCTION

The mitigation of fades is an important topic in outdoor optical wireless systems, where atmospheric turbulence induces time-varying changes to the refractive index, which in turn affect the amplitude, phase and propagation direction of the optical signal [1]-[3]. These changes ultimately manifest as time-varying power fluctuations of the received signal and when the turbulence is intense enough, the received signal decreases below the receiver sensitivity and the link is lost. This corresponds to a fade event, which affects the OWC system both in terms of capacity, as well as latency.

A number of techniques have been proposed for decreasing the fade probability and duration and for minimizing their impact. In weak (log-normal) fading conditions it is typically sufficient to utilize an aperture-averaging technique where a large-aperture receiver collects stray light [5]. The mitigation of more intense (gamma-gamma) fading, however, requires spatial or temporal diversity techniques [6]-[11]. Alternatively, more advance mitigation techniques can be implemented using aperture averaging and diversity in conjunction with coding schemes [11]-[15], which essentially adapt the transmission rate with the channel capacity.

Within the context of fade mitigation, we have previously reported a different approach that originates from optical signal processing techniques and utilizes equalization in partially saturated SOAs [16]. In this technique the SOA provides unbalanced gain to the incoming OWC signal, depending on the fade conditions. If the link is in a fade state then the SOA is not saturated by the OWC signal and provides linear gain. On the other hand, the OWC signal fully saturates the SOA at the absence of fades and experiences limited or no gain at all. The technique has been shown to be applicable in both log-normal and gamma-gamma fading and may provide a significant benefit in terms of reducing the fade probability and the average fade duration [16], [17].

In the current work we extend the analytical models we have provided for weak and medium fading to more extreme fading conditions that are typically modeled by negative-exponential statistics [2]. We further utilize the presented analytical model to assess restorative propertied of the SOA-based equalizer and demonstrate that a significant reduction of over 90% and 68% can be expected for the fade probability and duration, respectively. In addition, we investigate the quality of the signal quality in terms of the scintillation index and show that it can also be reduced by 75% at maximum. Finally, we determine the saturation level that is required at the SOA for optimal operation and establish that partial saturation of the SOA gain during the normal non-faded operation is sufficient to alleviate the most detrimental impact of negative-exponentially fading.

II. FIRST ORDER STATISTICS

In the current section we analytically calculate the fade probability and the scintillation index of the signal at the output of the SOA. These are the two most commonly considered first order metrics that describe the signal quality in the presence of fades and our goal is to demonstrate that the deployment of the SOA and its unbalanced gain saturation results in a signal that, although not fully restored, is of superior quality when compared to the original negative-exponential impaired one.

We begin our analysis by considering that the average power P_{in} of each received pulse at the SOA input follows the channel response and under strong fading conditions it can be modeled as a negative-exponential variable with a probability density function (pdf) equal to

$$f_{Pin}\left(P_{in}\right)=\frac{1}{P_{av}}\cdot\exp\left(-\frac{P_{in}}{P_{av}}\right), \tag{1}$$

where P_{av} is the average received optical power. The received pulses are driven to the SOA and each pulse experiences a gain that is dependent on its energy. Assuming that the gain recovery time is limited to less than one bit period, so that the SOA fully recovers to its small signal G_0 after each incoming pulse, the input U_{in} and output U_{out} pulse energies obey [18], [19]

$$U_{out} = U_{sat} \cdot \log\left(1 + G_0 \cdot \left(\exp\left(\frac{U_{in}}{U_{sat}}\right) - 1\right)\right), \tag{2}$$

with U_{sat} being the SOA saturation energy. Input and output pulse energies are proportionally related to average optical powers

$$\begin{aligned} U_{out} &= P_{out} \cdot T_b, \\ U_{in} &= P_{in} \cdot T_b, \end{aligned} \tag{3}$$

and as a result (2) can be re-written as

$$\exp\left(\frac{P_{out}}{P_s}\right) - 1 = G_0 \cdot \left(\exp\left(\frac{P_{in}}{P_s}\right) - 1\right), \tag{4}$$

where P_s is the rate-dependent saturation parameter of the SOA

$$P_s = \frac{U_{sat}}{T_b}. \tag{5}$$

The pdf of the optical signal envelope at the output of the SOA is then calculated in (6) after combining (2) and (4). For discussion purposes, we further define the normalization parameters r (normalized input power) and u (normalized output power)

$$\begin{aligned} r &= \frac{P_{av}}{P_s} \\ u &= \frac{P_{out}}{P_{av}}, \end{aligned} \tag{7}$$

to obtain the normalized SOA output power pdf as

$$f_u(u) = \frac{1}{G_0} \cdot e^{r \cdot u} \cdot \left(\frac{G_0 - 1 + e^{r \cdot u}}{G_0}\right)^{-1-\frac{1}{r}}. \tag{8}$$

A fade occurs whenever the output power u remains below a predetermined threshold u_R and by integrating (8)

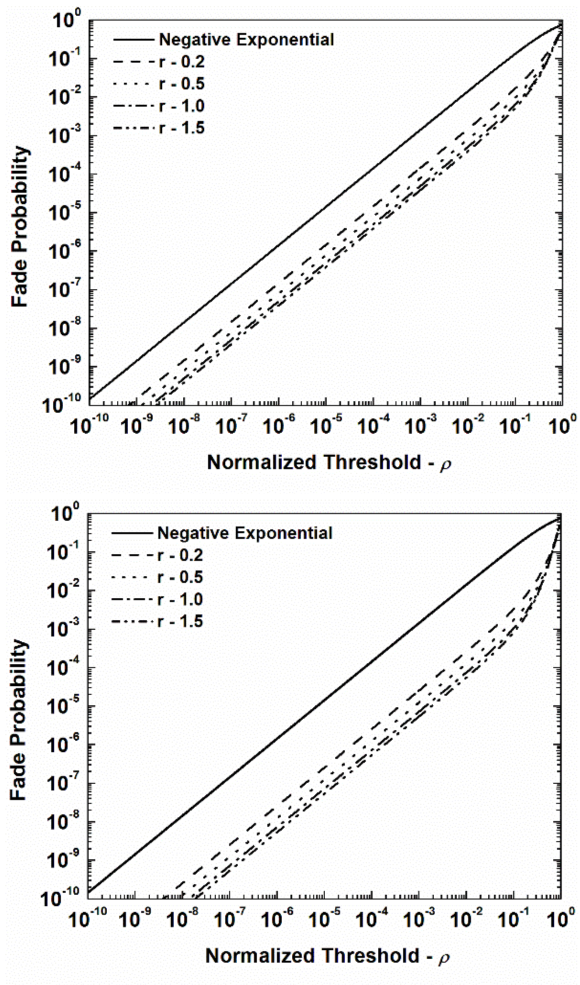

Figure 1. Fade probability at the output of the SOA versus the normalized threshold ρ. The SOA small signal gain G_0 equals 20 dB (top) and 30 dB (bottom).

we find that the fade probability equals

$$p_R = p(u < u_R) = 1 - \left(\frac{G_0 - 1 + e^{r \cdot u_R}}{G_0}\right)^{-\frac{1}{r}}. \tag{9}$$

The fade probability is plotted in Fig. 1 against the normalized receiver power threshold ρ

$$\rho = \frac{u_R}{u_{rms}}, \tag{10}$$

where u_{rms} is numerically evaluated using

$$f_{Pout}(P_{out}) = \frac{1}{P_{av}} \cdot \exp\left(\frac{P_{out}}{P_s}\right) \cdot \left(\frac{G_0 - 1 + \exp\left(\frac{P_{out}}{P_s}\right)}{G_0}\right)^{-\frac{P_s}{P_{av}} - 1}. \tag{6}$$

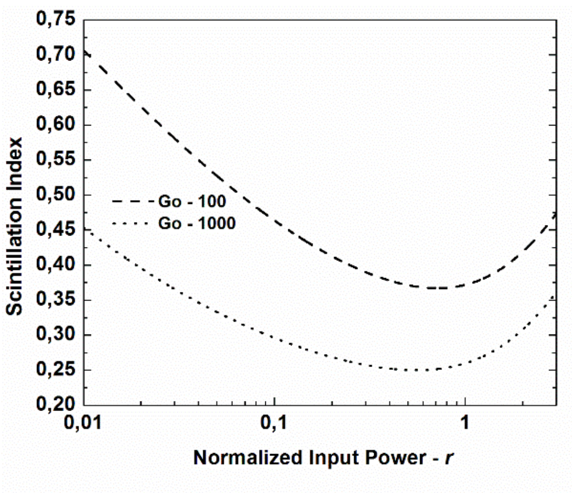

Figure 2. Scintillation index the output of the SOA versus the normalized input power r.

$$u_{rms}^2 = \frac{1}{G_0} \cdot \int_0^\infty u^2 \cdot e^{r \cdot u} \cdot \left(\frac{G_0 - 1 + e^{r \cdot u}}{G_0} \right)^{-1-\frac{1}{r}} du. \quad (11)$$

This normalization is performed so as to take into account the average static gain which is provided by the SOA and thus compare pdfs that correspond to different average output powers in a fair manner.

Fig. 1 shows that the fade probability is considerably decreased when the SOA is deployed in comparison with the incoming negative-exponential signal. A fade probability decrease of over 90% is predicted for a 20 dB small-signal gain SOA, while a 30 dB gain device further reduces the fade probability by 95%. This is expected from the SOA response as detailed (2), since higher gain devices lead to better equalization of the output pulse energies. The results also suggest that increasing the average input power r is always beneficial, still the attained benefit saturates and input powers of over ~1.0 do not yield a significant improvement. This last observation can be further explored by numerically evaluating the scintillation index (SI) of the signal at the SOA output

$$c_u = \frac{\sigma}{\mu} = \frac{\sqrt{u_{rms}^2 - u_{mean}^2}}{u_{mean}}, \quad (12)$$

with

$$u_{mean} = \frac{1}{G_0} \cdot \int_0^\infty u \cdot e^{r \cdot u} \cdot \left(\frac{G_0 - 1 + e^{r \cdot u}}{G_0} \right)^{-1-\frac{1}{r}} du. \quad (13)$$

The SI is plotted in Fig. 2 against the normalized input power r for SOAs with 20 and 30 dB small-signal gains. The figure shows that the output signal SI values are significantly lower than those of the original negative-exponential (SI equal to 1), which serves to verify the beneficial impact of the equalization process. Similar to the fade probability results, high gain SOAs are also capable of attaining lower SIs as compared to lower gain ones and are therefore preferable. Finally, it is noteworthy to mention that the SI is minimized for input powers approximately equal to the saturation parameter P_s of the SOA. This is expected from the SOA operation, since low average input powers lead to a non-saturated device that always provides linear gain, thus no equalization, to both fade impaired and non-impaired pulses. In the totally opposite regime, a fully saturated device will provide unity gain to all pulses irrespective of their energy and as a result fades are not equalized, as well. The optimal point of operation corresponds to a partially saturated SOA (r~1.0) that provides increasing gain to weaker pulses and thus restores the input signal to some extent.

III. Second Order Statistics

The fade mitigation capabilities of the SOA-based equalizer are also demonstrated by the second order statistics of the signal at the output of the SOA. Even though the SOA drastically reduces the fade probability, as we have demonstrated in the previous section, fades are still expected to occur and it is of practical importance to have an estimation of the average fade duration (AFD). The AFD at the output of the SOA is calculated from the output signal level crossing rate (LCR), which in turn requires the knowledge of the joint pdf between the output signal and its time derivative. The joint pdf at the SOA output is calculated from the joint pdf of the exponentially faded input signal, given by [20]

$$f\left(P_{in}, \dot{P}_{in}\right) = f_{Pin}(P_{in}) \cdot \frac{\exp\left(-\frac{\dot{P}_{in}^2}{4 \cdot P_{in} \cdot P_{av} \cdot r(0)} \right)}{\sqrt{4\pi \cdot P_{in} \cdot P_{av} \cdot r(0)}}. \quad (14)$$

$r(\tau)$ is the normalized covariance function that describes the rapidity of fading and the dot operator corresponds to the time derivative. In order to calculate the joint pdf at the SOA output we utilize (4) to obtain the relations between the input and output signals and their derivatives, resulting in

$$P_{in} = P_s \cdot \log\left(1 + \frac{1}{G_0}\left(\exp\left(\frac{P_{out}}{P_s} \right) - 1 \right) \right)$$
$$\dot{P}_{in} = \frac{\exp\left(\frac{P_{out}}{P_s} \right)}{\exp\left(\frac{P_{out}}{P_s} \right) + G_0 - 1} \cdot \dot{P}_{out}. \quad (15)$$

After applying the variable transform of (15) to (14), we find that the joint pdf at the SOA equals

$$f\left(P_{out}, \dot{P}_{out}\right) = f_{Pout}(P_{out}) \cdot \frac{\exp\left(-\frac{\dot{P}_{out}^2}{2 \cdot \sigma_{SOA}^2} \right)}{\sqrt{2\pi} \cdot \sigma_{SOA}}, \quad (16)$$

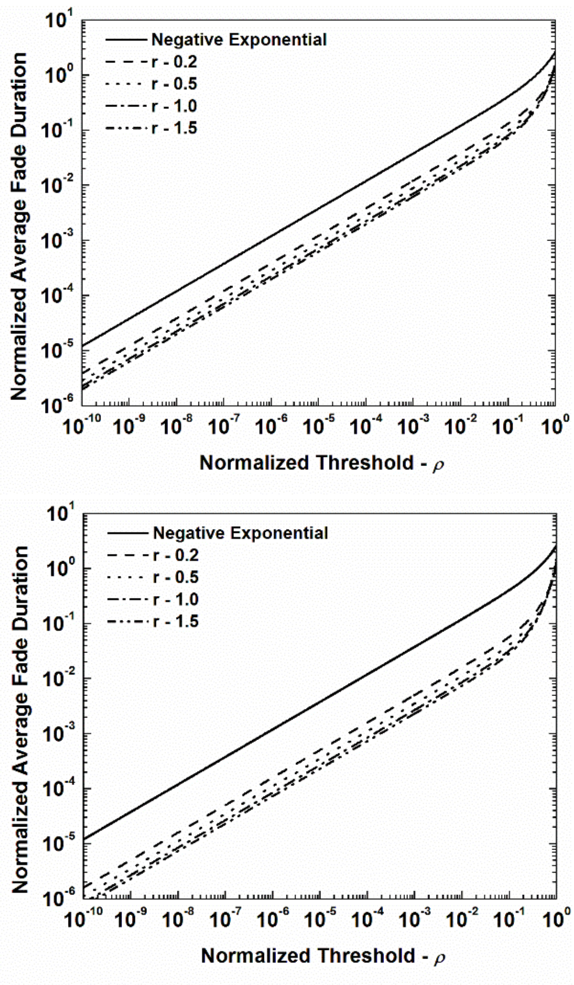

Figure 3. Average fade duration at the output of the SOA versus the normalized threshold ρ. The SOA small signal gain G_0 equals 20 dB (top) and 30 dB (bottom).

with variance σ_{SOA} being defined in (17). By integrating (16) with respect to the time derivative, we calculate the LCR at the output of the SOA in (18), and the AFD is finally derived after combining the fade probability and LCR in (19).

Eq. (19) is plotted in Fig. 3 against the normalized threshold ρ. The figure suggests that the 20 dB-gain SOA reduces the duration of fades by 68%-84%, while the 30 dB-gain SOA achieves an even better reduction of 87%-94%. As a result the duration of fades, and therefore the system latency, can be decreased by up to two orders of magnitude by utilizing the SGM-based SOA equalizer, provided that the SOA is saturated to the appropriate level. Similar to the first order statistics, a signal power approximately equal to the saturation parameter is adequate to attain maximum AFD reduction.

IV. Conclusion

We have analytically evaluated a fade mitigation technique that utilizes SOAs in the negative-exponential fading regime. We have demonstrated that the power equalization properties of the SOA are adequate to alter the statistics of the incoming OWC signal and provide an output signal that exhibits less intense and shorter-lived fluctuations. To this end, we derived both first order (fade probability, scintillation index) and second order (average fade duration) statistics and showed that all metrics can be reduced by a sizable percentage provided that the SOA gain is partially saturated in the normal (non-faded) link operation.

Acknowledgment

This work has been supported by COST Action IC1101 “Optical Wireless Communications - An Emerging Technology,” and University of Peloponnese research support project “Optical Signal Processing for Fade Mitigation in Outdoor Optical Wireless Systems.”

References

[1] R.S. Lawrence, and J.W. Strohbehn, “A survey of clear-air propagation effects relevant to optical communications,” in *Proc. IEEE* 58, 1523-1545, 1970.

[2] L. C. Andrews, and R. L. Phillips, Laser beam propagation through random media, 2nd edition 2005, SPIE Press, Bellingham, Washington.

[3] V.W.S. Chan, “Free-Space Optical Communications,” *IEEE/OSA J. Lightwave Technol.*, vol. 24, no. 12, pp. 4750 – 4762, December 2006.

$$\sigma_{SOA}^2 = \frac{2 \cdot P_s \cdot P_{av} \cdot r(0) \cdot \left(G_0 - 1 + e^{\left(\frac{Pout}{Ps}\right)}\right)^2 \cdot \log\left(\frac{G_0 - 1 + e^{\left(\frac{Pout}{Ps}\right)}}{G_0}\right)}{e^{\left(\frac{2 \cdot Pout}{Ps}\right)}} \tag{17}$$

$$LCR = \int_0^{\infty} f\left(P_{out}, \dot{P}_{out}\right) \cdot \dot{P}_{out} \cdot d\,\dot{P}_{out} = f_{Pout}(P_{out}) \cdot \frac{\sigma_{SOA}}{\sqrt{2\pi}} = \sqrt{\frac{r(0)}{\pi}} \cdot \left(\frac{1}{r} \cdot \log\left(\frac{G_0 - 1 + e^{r \cdot u}}{G_0}\right)\right)^{\frac{1}{2}} \cdot \left(\frac{G_0 - 1 + e^{r \cdot u}}{G_0}\right)^{-\frac{1}{r}} \tag{18}$$

$$AFD = \frac{p(u < u_R)}{LCR} = \sqrt{\frac{\pi}{r(0)}} \cdot \frac{\left(\frac{G_0 - 1 + e^{r \cdot u_R}}{G_0}\right)^{\frac{1}{r}} - 1}{\left(\frac{1}{r} \cdot \log\left(\frac{G_0 - 1 + e^{r \cdot u_R}}{G_0}\right)\right)^{\frac{1}{2}}} \tag{19}$$

[4] H. Yuksel, S. Milner, and C.C. Davis, “Aperture averaging for optimizing receiver design and system performance on free-space optical communication links,” *OSA J. Opt. Net.*, 4 (8), pp. 462 - 475, 2005.

[5] S.G. Wilson, M. Brandt-Pearce, Q. Cao, and M. Baedke, "Optical repetition MIMO transmission with multi-pulse PPM," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 9, pp. 1901-1910, Sept. 2005.

[6] S.M. Navidpour, M. Uysal, and M. Kavehrad, "BER performance of free-space optical transmission with spatial diversity," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2813-2819.

[7] A. Goldsmith, S.A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 5, pp. 684-702, June 2003.

[8] M.-A. Khalighi, N. Schwartz, N. Aitamer, and S. Bourennane, "Fading Reduction by Aperture Averaging and Spatial Diversity in Optical Wireless Systems," *OSA J. Opt. Commun. Net.*, 1, 580-593 (2009).

[9] A. García-Zambrana, C. Castillo-Vázquez, and B. Castillo-Vázquez, "Outage performance of MIMO FSO links over strong turbulence and misalignment fading channels," *OSA Opt. Exp.*, 19, 13480-13496 (2011).

[10] S. Trisno, I.I. Smolyaninov, S.D. Milner, and C.C. Davis, “Delayed diversity for fade resistance in optical wireless communication system through simulated turbulence,” in *Proc. SPIE Optical Transmission Systems and Equipment for WDM Networking III*, vol. 5596, pp. 385-393, 2005.

[11] J.A. Greco, “Design of the High-Speed Framing, FEC, and Interleaving Hardware Used in a 5.4 km Free-Space Optical Communication Experiment,” in *Proc. SPIE Free-space Laser Communications IX,* vol. 7464, 2009.

[12] X. Zhu and J. M. Kahn, “Performance Bounds for Coded Free-Space Optical Communications through Atmospheric Turbulence Channels,” *IEEE Trans. Commun.*, 51, 1233-1239, 2003.

[13] I.B. Djordjevic, “LDPC-coded MIMO optical communication over the atmospheric turbulence channel using Q-ary pulse-position modulation,” *OSA Opt. Exp*, 15, 10026-10032 (2007).

[14] H. Henniger, F. David, D. Giggenbach, and C. Rapp, “Evaluation of FEC for the atmospheric optical IM/DD channel,” *in Proc. SPIE Free-Space Laser Communication Technologies XV*, vol. 4975, 2003.

[15] A. Anguita, M. A. Neifeld, B. Hildner, and B. Vasic, "Rateless Coding on Experimental Temporally Correlated FSO Channels," *IEEE/OSA J. Lightwave Technol.*, vol. 28, no. 7, pp. 990-1002, April 2010.

[16] K. Yiannopoulos, N.C. Sagias, and A.C. Boucouvalas, "Fade Mitigation Based on Semiconductor Optical Amplifiers," *IEEE/OSA J. Lightwave Technol.*, vol. 31, no. 23, pp. 3621-3630, December 1, 2013.

[17] A.C. Boucouvalas, N.C. Sagias and K. Yiannopoulos, “First Order Statistics of Semiconductor Optical Amplifier Assisted Optical Wireless Systems under Log-normal Fading,” in *Proc. International Workshop on Optical Wireless (IWOW) 2013, Newcastle, UK.*

[18] G. P. Agrawal et al., “Self-phase modulation and spectral broadening of optical pulses in semiconductor laser amplifiers,” *IEEE J. Quantum Electron.*, vol. 25, no. 11, pp. 2297-2306, Nov. 1989.

[19] M. Eiselt, W. Pieper, and H. G. Weber, “SLALOM: Semiconductor Laser Amplifier in a Loop Mirror,” *IEEE/OSA J. Lightwave Technol.*, vol. 13, pp. 2099–2112, Oct. 1995.

[20] S. Primak, and V. Kontorovich, "On the second order statistics of generalized Gamma process," *IEEE Trans. Commun.*, vol.57, no.4, pp. 910-914, April 2009.