Digital Communications over Generalized-K Fading Channels

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Abstract— **In this work, a flexible distribution, called as Generalized-K, is used for fading channel modeling. Starting from its probability density function (PDF), useful closed-form expressions for the cumulative distribution function, moments-generating function, moments, and average Shannon's channel capacity are derived. These expressions are used to study important performance criteria such as the capacity, amount of fading, outage performance and bit error probability for a great variety of modulation formats. The proposed mathematical analysis is accompanied with various performance evaluation results which demonstrate the usefulness and flexibility of the proposed model.**

I. Introduction

Radiowave propagation through wireless channels is a complicated phenomenon characterized by various effects including multipath fading and shadowing. Multipath fading is introduced due to the constructive and destructive combination of randomly delayed, reflected, scattered, and diffracted signal components. Depending on the nature of the radio propagation environment, there are different models describing the statistical behavior of multipath fading [1]. Relatively new models in communications over fading channels are the K and the Generalized-K distributions [2], which in the past have been widely used in radar applications [3], [4]. The Generalized-K distribution has two shaping parameters, and as a consequence includes the K distribution as a special case. Moreover, it is sufficiently generic as it is able to incorporate most of the fading and shadowing effects observed in wireless communication channels and hence seems to be appropriate for the generic modeling of fading channels.

Representative past work concerning the K distribution can be found in [5]–[7]. In [5], Iskander *et al.* have proposed a method that combines the maximum likelihood and the method of moments for estimating the parameters of the K distribution. In [6], Abdi and Kaveh have shown that the K model provides similar performance to the well-known Rayleigh-Lognormal (R-L) model with the former being also mathematically tractable. The same authors in [7] have derived in closed-form the bit error rate of differential phase-shift keying (DPSK) and minimum-shift keying operating over K fading channel. Moreover, in the same work they presented a close agrement between the derived results and those based on the R-L distribution.

In [2], the generalized-K distribution was presented as analytically simpler than the Nakagami-lognormal or Suzuki distributions and general enough to approximate them, as well as several others including Rayleigh and Nakagami-m. Moreover, in the same work, the Amount-of-Fading (A_F) and the Average Bit-Error-Rate (ABER) for the special case of the binary phase swift keying (BPSK) modulation, were derived. However, detailed performance analysis for the SNR statistics of a receiver operating over Generalized-K fading channel has not yet been published in the open technical literature and this is the topic of the current work.

In this paper, after this introduction, the Generalized-K system and channel model is studied and closed-form expressions for its statistics are derived in Section II. In Section III the performance analysis of a single receiver is investigated, while in Section IV several numerical evaluated results are discussed.

II. The Generalized-K Fading Channel Model

The probability density function (PDF), $f_X(\cdot)$, of the envelope X under the Generalized-K fading conditions is [2]

$$f_X(x) = \frac{2\,b}{\Gamma(m)\,\Gamma(k)} \left(\frac{b\,x}{2}\right)^{k+m-1} K_{k-m}(b\,x) \quad (1)$$

where k and m are distribution's shaping parameters, $K_{k-m}(\cdot)$ is the modified Bessel function of order $(k-m)$ [8, eq. (9.6.1)], $\Gamma(\cdot)$ is the Gamma function [8, eq. (6.1.1)], $b = 2\sqrt{m/\Omega}$ and Ω is the mean power and can be derived by using [2, eq. (7)] as $\Omega = E\langle X^2\rangle/k$. Since the Generalized-$K$ is a two parameters distribution (1) is able to describe a great variety of fading and shadowed models. For example, for $m \to \infty$ and $k \to \infty$, (1) approaches hte Additive White Gaussian Noise (AWGN), no fading, channel, for $k \to \infty$, it approaches to the well-known Nakagami-m PDF, [2], while for $m = 1$, it reduces to the R-L PDF [6], [7].

The instantaneous signal-to-noise (SNR) per symbol is defined as $\gamma \triangleq X^2 E_s/N_0$, where $E_s = E\langle|s|^2\rangle$, with $E\langle\cdot\rangle$ denoting expectation and $|.|$ absolute value and N_0 is the single-sided power spectral density of the AWGN. Using the instantaneous SNR, the corresponding average SNR $\overline{\gamma} =$

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$\Omega k\, E_s/N_0$, [9, eq. (03.04.21.0008.01)] and (1) the cumulative distribution function (CDF) of γ can be easily derived as

$$F_\gamma(\gamma) = \pi \csc[\pi(k-m)] \times \left[\left(\frac{m\,k}{\overline{\gamma}}\right)^m \frac{\gamma^m {}_1F_2(m; 1+m-k, 1+m; m\,k\,\gamma/\overline{\gamma})}{\Gamma(k)\Gamma(1+m-k)\Gamma(1+m)} - \left(\frac{m\,k}{\overline{\gamma}}\right)^k \frac{\gamma^k {}_1F_2(k; 1-m+k, 1+k; m\,k\,\gamma/\overline{\gamma})}{\Gamma(m)\Gamma(1-m+k)\Gamma(1+k)}\right] \quad (2)$$

where ${}_pF_q(\cdot)$ is the generalized hypergeometric function, [10, eq. (9.14/1)], with p, q integers. Differentiating (2) with respect to x, the PDF of γ can be derived as

$$f_\gamma(\gamma) = \frac{\pi \csc[\pi(k-m)]}{\gamma\,\Gamma(k)\Gamma(m)} \times \left[\left(\frac{m\,k}{\overline{\gamma}}\right)^m \frac{\gamma^m {}_0F_1(1+m-k; m\,k\gamma/\overline{\gamma})}{\Gamma(1+m-k)} - \left(\frac{m\,k}{\overline{\gamma}}\right)^k \frac{\gamma^k {}_0F_1(1-m+k; m\,k\gamma/\overline{\gamma})}{\Gamma(1-m+k)}\right]. \quad (3)$$

By using (3) and [10, eq. (7.522/9)], the moments-generating function (MGF) can be easily obtained as

$$\mathcal{M}_\gamma(s) = \pi \csc[\pi(k-m)] \times \left\{\left(\frac{m\,k}{\overline{\gamma}}\right)^m \frac{{}_1F_1[m; 1+m-k; m\,k/(s\overline{\gamma})]}{s^m\,\Gamma(k)\Gamma(1+m-k)} + \left(\frac{m\,k}{\overline{\gamma}}\right)^k \frac{{}_1F_1[k; 1-m+k; m\,k/(s\overline{\gamma})]}{s^k\,\Gamma(m)\Gamma(1-m+k)}\right\}. \quad (4)$$

Moreover, starting with the definition of the nth order moment [11, eq. (5.38)] and using (3), integrals of the form $\mathcal{I}_1 = \int_0^\infty x^m {}_0F_1(b; x)\,dx$ need to be solved. $\mathcal{I}_1$ can be solved by using [10, eq. (7.811/4)] and expressing the confluent hypergeometric function ${}_0F_1(\cdot)$ as [9, eq. (07.17.26.0008.01)]

$${}_0F_1(b; x) = \pi\Gamma(b)\, G_{1,3}^{1,0}\left[x\middle|{}^{1/2}_{0,1-b,1/2}\right] \quad (5)$$

where $G_{p,q}^{m,n}\left[x\middle|{}^{\alpha_1,\alpha_2,\ldots,\alpha_p}_{b_1,b_2,\ldots,b_q}\right]$ is the Meijer's G-function [10, eq. (9.301)]. Hence, the nth order moment of γ output SNR, $\mu_\gamma(n)$, can be easily derived after some straight-forward mathematical manipulations as

$$\mu_\gamma(n) = \pi^2 \left(\frac{m\,k}{\overline{\gamma}}\right)^{-n} \frac{\csc[\pi(k-m)]}{\Gamma(m)\Gamma(k)} \times \left[\frac{\Gamma(n+m)}{\Gamma(1-k-n)\Gamma(1/2-m-n)\Gamma(1/2+n+m)} - \frac{\Gamma(n+k)}{\Gamma(1-m-n)\Gamma(1/2-k-n)\Gamma(1/2+n+k)}\right]. \quad (6)$$

Using (3) in the definition of the average channel capacity $\overline{C}$ [12], in Shannon's sense, integrals of the form appear

$$\mathcal{I}_2 = \int_0^\infty x^{m-1} \log_2(1+x)\, {}_0F_1\left(1+m-k; \frac{m\,k\,x}{\overline{\gamma}}\right) dx. \quad (7)$$

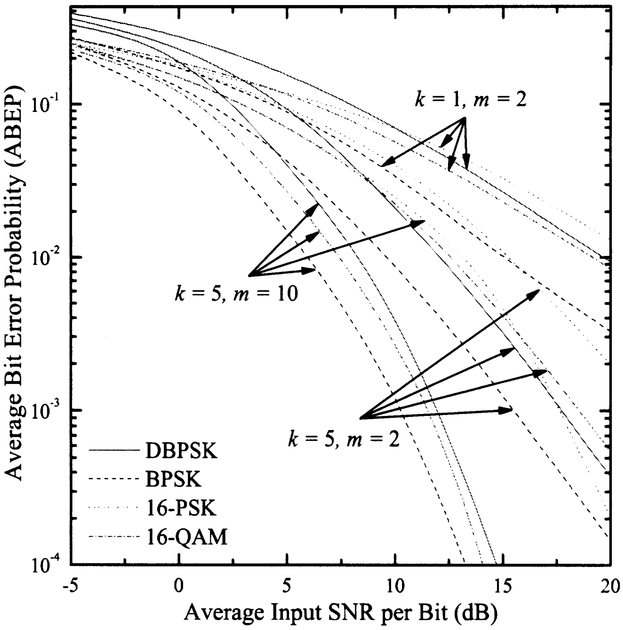

Fig. 1. ABEP of DBPSK, BPSK, Gray-encoded 16-PSK and 16-QAM signaling versus $\overline{\gamma}$ for several values of k and m.

By expressing $\ln(1+x) = G_{2,2}^{1,2}\left[x\middle|{}^{1,\,1}_{1,\,0}\right]$ and ${}_0F_1(\cdot)$ as in (5), $\mathcal{I}_2$ can be solved with the aid of [13] and $\overline{C}$ can be obtained in closed-form as

$$\overline{C} = \frac{\pi \csc[\pi(k-m)]\,BW}{\ln 2\,\Gamma(k)\Gamma(m)} \times \left\{\left(\frac{m\,k}{\overline{\gamma}}\right)^m G_{2,4}^{3,1}\left[-\frac{m\,k}{\overline{\gamma}}\middle|{}^{-m,\quad 1-m}_{0,-m,-m,k-m}\right] - \left(\frac{m\,k}{\overline{\gamma}}\right)^k G_{2,4}^{3,1}\left[-\frac{m\,k}{\overline{\gamma}}\middle|{}^{-k,\quad 1-k}_{0,-k,-k,m-k}\right]\right\} \quad (8)$$

where BW is signal's transmission bandwidth.

III. Performance Analysis

In this section the performance of a single receiver operating over the Generalized-K fading channel will be investigated. The received baseband signal is $z = s\,h + n$, where s is the transmitted complex symbol, n is the complex AWGN, and h is the channel complex gain. One of the most commonly used performance criterion for telecommunications systems operating over fading channels is the bit-error probability. By using (4), and following the MGF based approach, [1], the average bit-error probability (ABEP) can be readily evaluated for a variety of modulation schemes [1]. Hence, the ABEP can be calculated i) directly for non-coherent binary frequency-shift keying and differential binary phase-shift keying (DBPSK) and ii) via numerical integration for binary phase-shift keying (BPSK), M-PSK, M-ary quadrature amplitude modulation (M-QAM) and M-DPSK since single integrals composed of elementary (i.e., exponential and trigonometric) functions and with finite limits are obtained.

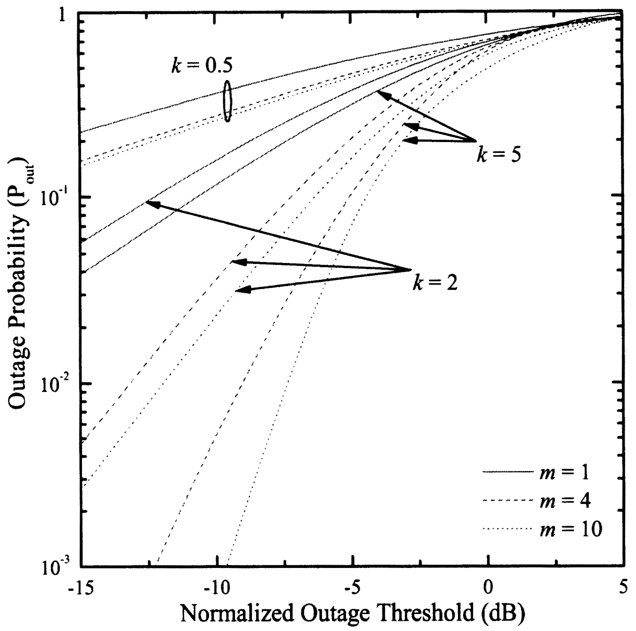

Fig. 2. P_{out} as a function of the normalized outage threshold for several values of k and m.

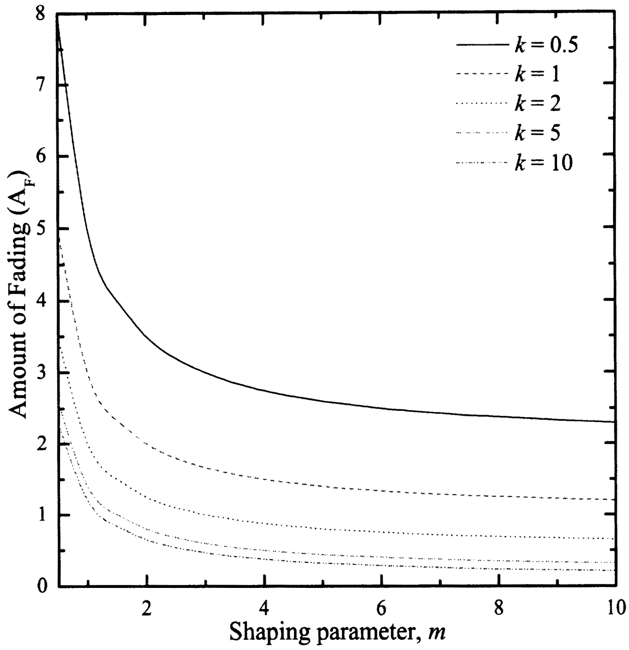

Fig. 3. A_F as a function of m for several values of k.

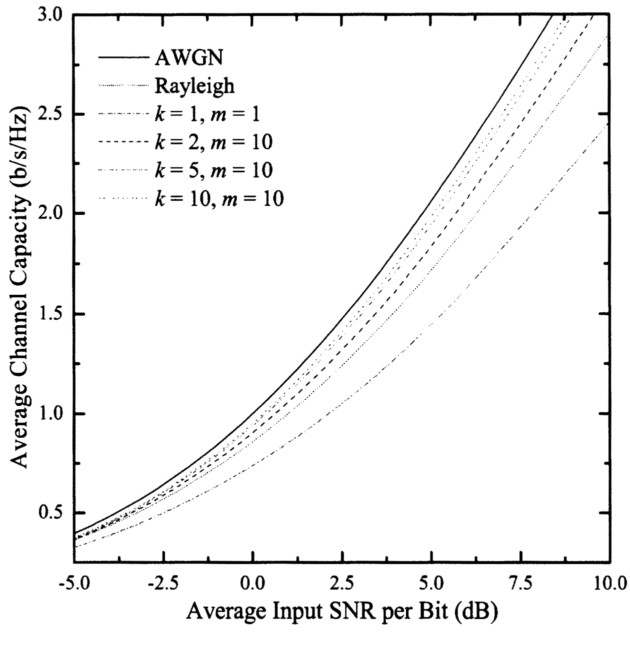

Fig. 4. $\overline{C}$/BW versus average SNR per Bit, for several values of m and k.

Another standard performance criterion is the outage probability, P_{out}, which is defined as the probability that the output SNR falls below a given threshold, γ_{th}. Hence, $P_{out}(\gamma_{th})$ can be simply obtained as

$$P_{out}(\gamma_{th}) = F_{\gamma}(\gamma_{th}). \tag{9}$$

Moreover, A_F, which is another important statistical characteristic of fading channels, can be easily obtained by using, [1],

$$A_F = \frac{\text{var}(\gamma)}{\overline{\gamma}^2} = \frac{\mu_{\gamma}(2)}{\overline{\gamma}^2} - 1. \tag{10}$$

Finally, the normalized average channel capacity is directly related to the bandwidth efficiency and denotes the amount of data transmitted in a given spectrum allocation. It can be easily obtained, by using (8), as $\overline{C}/BW$, in terms of $b/s/Hz$.

IV. Numerical Results

In order to demonstrate the usefulness of the theoretical value and based on the proposed formulation, the performance of a receiver operating over the Generalized-K fading channel is presented in Figs. 1-4.

More specifically, in Fig. 1, the ABEP is plotted for DBPSK, BPSK, 16-PSK and 16-QAM signaling with Gray encoding, as a function of the average input SNR per bit, $\overline{\gamma}_b = \overline{\gamma}/\log(M)$, for several values of m and k. As expected, the ABEP improves as $\overline{\gamma}_b$ increases, while for a fixed value of $\overline{\gamma}_b$, ABEP also improves with an increase of k and/or m. Fig. 2 shows P_{out} versus normalized outage threshold for several values of k and m. P_{out} decreases (i.e., the outage performance improves) with an increase of k and/or m. However the gap among the curves decreases as k and/or m increase. Notice that for $m = 1$, the Generalized-K distribution approaches the R-L one.

The A_F is presented as a function of the m parameter and for several values of k. Note that as m and/or k increase, the A_F decreases in an exponential way. Moreover, it is more clear in Fig. 3 that the variation of m, k has greater influence at the channel's performance when they have small values, i.e., $m \in [0, 2]$ and $k \in (0.5, 2)$. Finally, in Fig. 4 the normalized average channel capacity is plotted as a function of the average SNR per bit for several values of m, k. For comparison purposes, the normalized channel capacity for the AWGN, i.e., $\overline{C}_{awgn}/BW = \log_2(1+\overline{\gamma}_b)$, and the Rayleigh channels are also plotted. As it was expected, the average capacity of the Generalized-K fading channel is always less than the capacity provided by the AWGN channel. Moreover, for $m = k = 1$ the provided capacity is less than Rayleigh's, and it improves as m and/or k increase.

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