The 18th Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'07)

HOUSEHOLDER-MATRICES BASED ANALYSIS OF SC RECEIVERS OVER RAYLEIGH FADING CHANNELS WITH ARBITRARY CORRELATION

George C. Alexandropoulos
and Kostas Berberidis
Department of Computer Engineering & Informatics
University of Patras
26500 Rio-Patras, Greece

Nikos C. Sagias, Fotis I. Lazarakis,
Antonis A. Alexandridis, and Kostas P. Dangakis
Institute of Informatics and Telecommunications
National Centre for Scientific Research–"Demokritos"
15310 Athens, Greece

ABSTRACT

In this paper, new results for the multivariate Rayleigh distribution with an arbitrary correlation matrix are presented. By using an efficient tridiagonalization method based on Householder matrices, closed-form union upper bounds for its joint probability density and cumulative density functions are derived. Based on the proposed mathematical analysis, a tight union upper bound and an analytical approximation for the outage probability of multibranch selection diversity receivers operating over identically distributed and arbitrarily correlated Rayleigh fading channels are obtained. Our analysis is verified by comparing numerically evaluated results with extensive computer simulation ones.

I INTRODUCTION

The theory of multivariate stochastic processes can be used as an essential mathematical tool for modeling and analyzing realistic wireless channels with correlated fading. Such fading channels are usually met in digital contemporary communications systems employing diversity receivers with not sufficiently separated antennas, where space or polarization diversity is applied (e.g. antenna arrays, handheld mobile terminals, and indoor base-stations).

In past, a lot of papers have been published in the open technical literature dealing with multivariate fading channel models and/or systems performance analysis (see [1–16] and references therein). A very generic expression for the multivariate gamma-type distribution with an arbitrary covariance matrix was derived back in 1951 [2]. That expression can be used for the derivation of generic formulas for most of the well-known multivariate distributions involved in modelling of fading channels, e.g. Rayleigh, Nakagami-m. However, its form is very complicated, and hence, simpler forms have been introduced considering specific structures of the correlation matrix. For example, an infinite series representation for the bivariate Rayleigh and Nakagami-m cumulative distribution functions (CDFs) has been presented in [3]. In a later work [4], Simon and Alouini have proposed an alternative CDF expression for the bivariate Rayleigh distribution, being in the form of a single integral with finite limits and an integrand composed of elementary functions. Mallik, in [5], has presented exact closed-form probability density function (PDF) expressions for the multivariate Rayleigh distribution with exponential and constant correlation matrices. In [6], infinite series representations for the trivariate and quadrivariate Rayleigh distributions have been proposed. Very recently, based on [2], the performance analysis of SC receivers over arbitrary correlated generalized gamma fading channels has been presented [9]. Although the derived formulas are very generic, they are computationally burden, since they are in the form of sums of Laguerre polynomials.

In this paper, we provide a simple statistical approach for the multivariate Rayleigh fading channel model with arbitrary correlation structures. By using the Householder tridiagonalization method and a standard random variables (RVs) transformation, closed-form union upper bounds for the PDF and CDF of the multivariate Rayleigh distribution are derived. As an application, a simple and tight union upper bound and an analytical approximation for the outage probability (OP) of multibranch SC receivers operating over identically distributed (id) and arbitrarily correlated Rayleigh fading channels are derived. Our analysis is also verified by extensive computer simulations.

II MULTIVARIATE RAYLEIGH FADE STATISTICS

Let $\mathbf{Y}_1 = [Y_{1,1}\, Y_{1,2} \cdots Y_{1,L}]^\dagger$, $\mathbf{Y}_2 = [Y_{2,1}\, Y_{2,2} \cdots Y_{2,L}]^\dagger$ be two L-dimensional real column vectors ($\dagger$ denotes the transpose), which are independent and id zero mean $\mathbb{E}\langle Y_{k,l}\rangle = 0$ with variance $\mathbb{E}\langle Y_{k,l}^2\rangle = \sigma^2$ ($k = 1, 2$, $l = 1, 2, \ldots, L$, and $\mathbb{E}\langle\cdot\rangle$ denotes expectation) Gaussian RVs having a symmetric and positive definite correlation matrix $\mathbf{\Sigma_G} \in \Re^{L\times L}$. Also, let $R_l = \|\mathbf{X}_l\| = \sqrt{Y_{1,l}^2 + Y_{2,l}^2}$ be the Euclidean norm of the two dimensional column vector $\mathbf{X}_l = [Y_{1,l}\, Y_{2,l}]^\dagger$ composed of the lth components of $\mathbf{Y}_k$'s. Clearly, R_l's are correlated Rayleigh RVs with marginal PDFs described by

$$f_{R_l}(r) = \frac{2r}{\Omega}\exp\left(-\frac{r^2}{\Omega}\right) \quad (1)$$

where $\Omega = 2\sigma^2 = \mathbb{E}\langle R_l^2\rangle$ is the average fading power. Their power correlation matrix $\mathbf{\Sigma} \in \Re^{L\times L}$ is given by $\mathbf{\Sigma}_{i,j} \equiv 1$ for $i = j$ and $\mathbf{\Sigma}_{i,j} = \mathbf{\Sigma}_{j,i} \equiv \rho_{i,j}$ for $i \neq j$, with $0 \le \rho_{i,j} < 1$ $(i, j = 1, 2, \ldots, L)$ being the power correlation coefficient between R_i^2 and R_j^2 [1, eq. (9.195)]. It can be easily proved that the correlation matrix of the underlying Gaussian processes, $\mathbf{\Sigma_G}$, is related to the power correlation matrix, $\mathbf{\Sigma}$, as $\mathbf{\Sigma_G} = \sqrt{\mathbf{\Sigma}}$ ($\sqrt{\mathbf{\Sigma}}$ stands for a matrix with elements the square root ones of $\mathbf{\Sigma}$).

Next, we review the structure of $\mathbf{\Sigma}$ for the most popular correlation models met in practical wireless systems channels.

1) *Exponential Model*

The correlation matrix of this model is defined as $\mathbf{\Sigma}_{i,j} \equiv \rho^{|i-j|}$, $\forall\, i \neq j$, with $0 \le \rho < 1$ being the correlation coef-

$$
\begin{aligned}
f_{\mathbf{R}}(\mathbf{r}) \le & |\mathbf{A}|\; |\mathbf{W}| \left(\frac{2}{\Omega}\right)^L \left(\sum_{i=1}^{L} |q_{L,i}|\; r_i\right) \exp\left[-\frac{p_{L,L}}{\Omega}\left(\sum_{i=1}^{L} |q_{L,i}|\; r_i\right)^2\right] \\
& \times \prod_{k=1}^{L-1} \left(\sum_{i=1}^{L} |q_{k,i}|\; r_i\right) \exp\left[-\frac{p_{k,k}}{\Omega}\left(\sum_{i=1}^{L} |q_{k,i}|\; r_i\right)^2\right] I_0\left(\frac{2\,|p_{k,k+1}|}{\Omega} \sum_{l_1=1}^{L}\sum_{l_2=1}^{L} |q_{k,l_2}\, q_{k+1,l_1}|\; r_{l_1}\, r_{l_2}\right)
\end{aligned} \tag{5}
$$

ficient between adjacent channels [17].

2) Constant Model

The correlation matrix of the constant model, discussed in [1] and [17], is defined as $\mathbf{\Sigma}_{i,j} \equiv \rho, \forall\, i \neq j$, with $0 \le \rho < 1$ being the correlation coefficient between any two channels.

3) Circular Model

The circular correlation matrix, presented in [18], is a Toeplitz matrix, i.e., $\mathbf{\Sigma}_{i,j} = \rho_{|i-j|}, \forall\, i \neq j$, with an Lth order symmetry, which implies that $\rho_{|i-j|} = \rho_{|L-i+j|}$.

4) Linearly Arbitrary Model

The correlation matrix of a linearly arbitrary model has a Toeplitz structure.

5) Arbitrary Model

The arbitrary correlation model is the most general one and considers arbitrary values for $\mathbf{\Sigma}_{i,j}$'s [1].

A Joint PDF

In the proposed mathematical analysis, a class of orthogonal and symmetric matrices, known as Householder matrices [19], are used for the tridiagonal decomposition [20] of the inverse of the Gaussian correlation matrix, $\mathbf{W} = \mathbf{\Sigma_G}^{-1}$.

Definition 1 (Householder matrix) *Let* $\mathbf{a}, \mathbf{b} \in \Re^{L\times 1}$, $L \ge 3$, *be two nonzero vectors having equal norms, i.e.,* $\|\mathbf{a}\| = \|\mathbf{b}\|$. *There always exists an orthogonal and symmetric matrix* $\mathbf{H} \in \Re^{L\times L}$ *of the form*

$$\mathbf{H} = \mathbf{I} - 2\,\mathbf{u}\,\mathbf{u}^{\dagger} \tag{2}$$

defined as Householder matrix, so that $\mathbf{b} = \mathbf{H}\,\mathbf{a}$, *with* $\mathbf{I} \in \Re^{L\times L}$ *being the identity matrix and*

$$\mathbf{u} = \frac{\mathbf{a}-\mathbf{b}}{\|\mathbf{a}-\mathbf{b}\|}. \tag{3}$$

By applying a similarity transformation

$$\mathbf{W}' = \mathbf{Q}^{\dagger}\,\mathbf{W}\,\mathbf{Q} \tag{4}$$

$\mathbf{W}'$ becomes real, symmetric, and tridiagonal, where $\mathbf{Q}$ is an orthogonal matrix given by $\mathbf{Q} = \prod_{k=1}^{L-2}\mathbf{H}_k$, with $\mathbf{H}_k$ being a Householder matrix which can be obtained using (2) and a computationally efficient method for tridiagonal decomposition given in the Appendix A. Moreover, some properties concerning $\mathbf{Q}$, with elements $q_{i,j} \in \Re$, are described in the Appendix B.

Theorem 1 (Joint PDF Upper Bound) *A closed-form union upper bound for the joint PDF of* $\mathbf{R} = [R_1\, R_2\, \cdots\, R_L]$ *with an arbitrary power correlation matrix is given by* (5) *(top of this page), where* $\mathbf{r} = [r_1\, r_2\, \cdots\, r_L]$, $I_0(\cdot)$ *is the zeroth-order modified Bessel function of the first kind [21, eq. (8.406/1)],* $|\mathbf{W}|$ *stands for the determinant of* $\mathbf{W}$, $p_{i,j} \in \Re$ *are the elements of* $\mathbf{W}'$, *and* $\mathbf{A} \in \Re^{L\times L}$ *is a matrix with elements* $\mathbf{A}_{i,j} = |q_{i,j}|$.

Proof: Let us consider an orthogonal transformation of RVs

$$\mathbf{Y}'_k = \mathbf{Q}^{\dagger}\mathbf{Y}_k\,. \tag{6}$$

Then, $\mathbf{Y}'_k$'s form another set of zero mean real Gaussian RVs with correlation matrix $\mathbf{\Sigma_G}' = \mathbf{Q}^{\dagger}\,\mathbf{\Sigma_G}\,\mathbf{Q}$ [22]. Also, let $R'_l = \|\mathbf{X}'_l\| = \sqrt{Y'^2_{1,l} + Y'^2_{2,l}}$ be the Euclidean norm of the two dimensional column vector $\mathbf{X}_l = [Y_{1,l}\, Y_{2,l}]^{\dagger}$ composed of the lth components of $\mathbf{Y}_k$'s. Since $\mathbf{W}' = \left(\mathbf{\Sigma_G}'\right)^{-1}$ is tridiagonal, a similar procedure such that in [11, Theorem I] can be applied, and hence, the joint PDF of $\mathbf{R}' = [R'_1\, R'_2\, \cdots\, R'_L]$ can be easily obtained as

$$
\begin{aligned}
f_{\mathbf{R}'}(\mathbf{r}) = & |\mathbf{W}| \left(\frac{2}{\Omega'}\right)^L r_L \exp\left(-\frac{p_{L,L}}{\Omega'}\, r_L^2\right) \\
& \times \prod_{k=1}^{L-1} r_k \exp\left(-\frac{p_{k,k}}{\Omega'}\, r_k^2\right) I_0\left(\frac{2\,|p_{k,k+1}|}{\Omega'}\, r_k\, r_{k+1}\right)
\end{aligned} \tag{7}
$$

with $\Omega' = \mathbb{E}\langle R'^2_l\rangle = \Omega$.

Starting from the definition of R'_l, R_l and after some algebraic manipulations with (6), the two groups of Rayleigh RVs, $\mathbf{R}'$ and $\mathbf{R}$, are related as

$$R'_1 = R_1 \tag{8a}$$

$$R'_n = \left\| \sum_{i=1}^{L} q_{n,i}\,\mathbf{X}_i \right\|\;,\; n = 2, 3, \ldots, L. \tag{8b}$$

The generalization of the triangle inequality can be applied in (8b), in order to extract union bounds between the two groups of Rayleigh RVs, as

$$R'_n \le \sum_{i=1}^{L} |q_{n,i}|\; R_i. \tag{9}$$

By using a standard method for RVs transformation, an upper bound for the joint PDF of $\mathbf{R}$ can be easily obtained as in (5), with $\mathbf{A}$ being the Jacobian matrix of the RVs transformations described by (8a) and (9). ■

$$F_{\mathbf{R}}(\mathbf{r}) \leq |\mathbf{W}| \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \frac{\prod_{i=1}^{L-1}\left(p_{i,i+1}^{k_i}/k_i!\right)^2}{p_{1,1}^{k_1+1}\left(\prod_{i=2}^{L-1} p_{i,i}^{k_{i-1}+k_i+1}\right) p_{L,L}^{k_{L-1}+1}} \gamma\left(k_1+1, \frac{p_{1,1}}{\Omega}\, r_1^2\right)$$
$$\times \left\{\prod_{j=2}^{L-1} \gamma\left[k_{j-1}+k_j+1, \frac{p_{j,j}}{\Omega}\left(\sum_{i=1}^{L} |q_{j,i}|\; r_i\right)^2\right]\right\} \gamma\left[k_{L-1}+1, \frac{p_{L,L}}{\Omega}\left(\sum_{i=1}^{L} |q_{L,i}|\; r_i\right)^2\right] \quad (10)$$

It must be mentioned that the tridiagonalization of $\mathbf{W}$, needed in (5) for the computation of $q_{i,j}$'s, can be performed using the standard function `TridiagonalForm` of the MAPLE mathematical software package.

By comparing the derived upper bound of (5) with the exact joint PDF in [2, eq. (3.7)], it is concluded that both are generic, while (5) is significantly simpler.

B Joint CDF

By using an infinite series representation for Bessel functions [21, eq. (8.445)] in (5) and after L integrations, a union upper bound for the Rayleigh joint CDF of $\mathbf{R}$ can be derived as in (10) (top of this page), where $\gamma(x, y)$ is the lower incomplete Gamma function [21, eq. (8.352/1)]. Note that in (10), the incomplete Gamma functions can be further simplified to standard functions using [21, eq.(8.352/1)].

III Outage Probablity of Multibranch SC Receivers

We consider an L-branch diversity receiver operating over id and arbitrarily correlated Rayleigh fading channels. Let a signal's transmission over the lth flat Rayleigh fading channel ($l = 1, 2, \ldots, L$) corrupted by additive white Gaussian noise (AWGN), with E_s being the transmitted symbols' energy and N_0 the single-sided noise power spectral density of the AWGN. The instantaneous signal-to-noise ratio (SNR) per symbol of the lth diversity channel can be expressed by $\gamma_l = R_l^2\, E_s/N_0$, with its corresponding average value being $\overline{\gamma}_l = \mathbb{E}\langle R_l^2\rangle\, E_s/N_0 = \Omega\, E_s/N_0 = \overline{\gamma}_s\ \forall\ l$.

A Bound for the OP

The instantaneous SNR per symbol at the output of an L-branch SC receiver will be the one with the highest instantaneous value among the L branches, i.e., $\gamma_{\mathrm{sc}} = \max\{\gamma_1, \gamma_2, \ldots, \gamma_L\}$. The OP, P_{out}, is defined as the probability that the SC output SNR falls below a given outage threshold, γ_{th}. This probability can be easily obtained as $P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = F_{\boldsymbol{\gamma}}(\gamma_{\mathrm{th}}, \gamma_{\mathrm{th}}, \ldots, \gamma_{\mathrm{th}})$, where using (10) and a standard method for RVs transformation yields as in (11) (top of the next page).

B Approximation for the OP

In order to evaluate $F_{\gamma_{\mathrm{sc}}}(\gamma) = F_{\boldsymbol{\gamma}}(\gamma, \gamma, \ldots, \gamma)$, which requires that $\|\mathbf{X}_l\| = \|\mathbf{X}\|$, we further approximate $\mathbf{X}_l \cong \mathbf{X}$. By using (8) and the standard method for RVs transformation, the OP can be approximated as in (12) (top of the next page).

Table 1: Number of required terms for convergence to the sixth significant digit of the union bound of the OP of SC with a linearly arbitrary model ($L = 3$).

$\gamma_{\mathrm{th}}/\overline{\gamma}_s$ (dB)	Terms
5	14
0	11
-5	7
-10	5
-15	2
-20	1

It must be mentioned that (12) is very generic and holds when $\left(\sum_{i=1}^{L} q_{n,i}\right)^2 \neq 0,\ \forall\, n$, i.e., it does not hold only for the very special cases of constant or circular structures. Also, note that (12) coincides with [5] and [12], for the special case of exponential correlation.

IV Numerical and Computer Simulation Results

The numerical evaluation of (11) requires the summation of an infinite number of terms. As an indicative example, Table 1 summarizes the number of terms in each sum needed, so as the expression for the OP to converge after the truncation of the infinite series. The findings are not very different, concerning the convergence, if (12) is used. A linearly arbitrary correlation model with $L = 3$ [13, p. 886] has been considered. As Table 1 indicates, the number of required terms depends strongly on the normalized outage threshold, $\gamma_{\mathrm{th}}/\overline{\gamma}_s$. As $\gamma_{\mathrm{th}}/\overline{\gamma}_s$ decreases, less terms are required to be summed.

Having numerically evaluated (11), in Fig. 1, an upper bound for P_{out} is plotted as a function of $\gamma_{\mathrm{th}}/\overline{\gamma}_s$, for a triple-branch SC receiver with a linearly arbitrary correlation matrix for $L = 3$ given in [13, p. 886]. It can be easily verified that P_{out} degrades with an increase of $\gamma_{\mathrm{th}}/\overline{\gamma}_s$. More importantly, the obtained results clearly show that the proposed bounds for P_{out} are tight, compared with extensive computer simulations for the exact OP. For example, for $L = 3$ and $P_{\mathrm{out}} = 10^{-3}$, the distance between the two curves is less than 0.1 dB. Also, it is interesting to be mentioned that from additional experiments that were conducted, the smaller the L is, the tighter the bounds are.

By numerically evaluating (12), in Fig. 2, P_{out} is plotted as a function of the $\gamma_{\mathrm{th}}/\overline{\gamma}_s$, for multibranch SC receivers with a linearly arbitrary correlation matrix for $L = 5$ given in [23, eq.

$$P_{\text{out}}\left(\gamma_{\text{th}}\right) \leq |\mathbf{W}| \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \frac{\prod_{i=1}^{L-1}\left(p_{i,i+1}^{k_i}/k_i!\right)^2}{p_{1,1}^{k_1+1}\left(\prod_{i=2}^{L-1} p_{i,i}^{k_{i-1}+k_i+1}\right) p_{L,L}^{k_{L-1}+1}}\, \gamma\left(k_1+1, p_{1,1}\frac{\gamma_{\text{th}}}{\overline{\gamma}_s}\right)$$
$$\times \left\{\prod_{j=2}^{L-1} \gamma\left[k_{j-1}+k_j+1, p_{j,j}\left(\sum_{i=1}^{L}|q_{j,i}|\right)^2 \frac{\gamma_{\text{th}}}{\overline{\gamma}_s}\right]\right\} \gamma\left[k_{L-1}+1, p_{L,L}\left(\sum_{i=1}^{L}|q_{L,i}|\right)^2 \frac{\gamma_{\text{th}}}{\overline{\gamma}_s}\right] \quad (11)$$

$$P_{\text{out}}\left(\gamma_{\text{th}}\right) \cong |\mathbf{W}| \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \frac{\prod_{i=1}^{L-1}\left(p_{i,i+1}^{k_i}/k_i!\right)^2}{p_{1,1}^{k_1+1}\left(\prod_{i=2}^{L-1} p_{i,i}^{k_{i-1}+k_i+1}\right) p_{L,L}^{k_{L-1}+1}}\, \gamma\left(k_1+1, p_{1,1}\frac{\gamma_{\text{th}}}{\overline{\gamma}_s}\right)$$
$$\times \left\{\prod_{j=2}^{L-1} \gamma\left[k_{j-1}+k_j+1, p_{j,j}\left(\sum_{i=1}^{L}q_{j,i}\right)^2 \frac{\gamma_{\text{th}}}{\overline{\gamma}_s}\right]\right\} \gamma\left[k_{L-1}+1, p_{L,L}\left(\sum_{i=1}^{L}q_{L,i}\right)^2 \frac{\gamma_{\text{th}}}{\overline{\gamma}_s}\right] \quad (12)$$

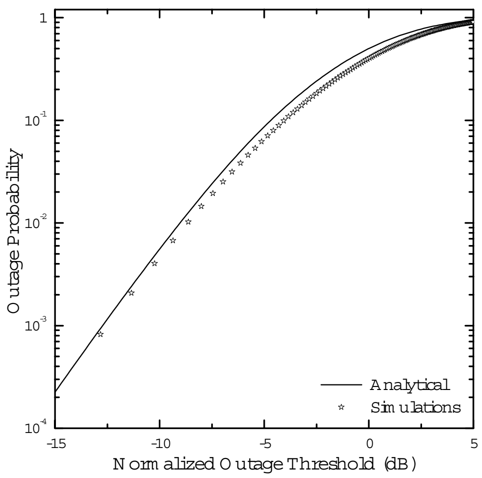

Figure 1: Upper bound for the outage probability of a triple-branch SC as a function of the normalized outage threshold.

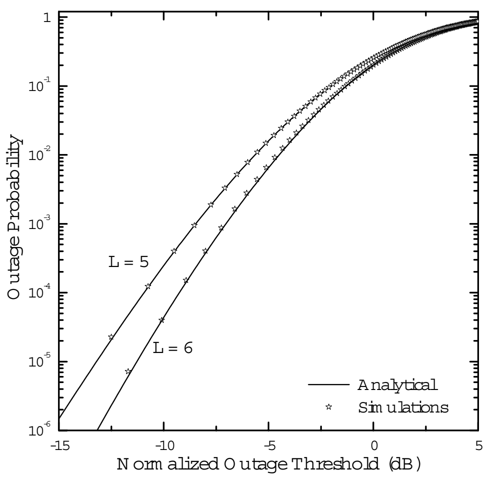

Figure 2: Approximation for the outage probability of multi-branch SC as a function of the normalized outage threshold.

(40)] and an arbitrary correlation matrix for $L = 6$ given by

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & 0.632 & 0.366 & 0.203 & 0.141 & 0.080 \\ 0.632 & 1 & 0.562 & 0.403 & 0.160 & 0.123 \\ 0.366 & 0.562 & 1 & 0.540 & 0.476 & 0.250 \\ 0.203 & 0.403 & 0.540 & 1 & 0.672 & 0.342 \\ 0.141 & 0.160 & 0.476 & 0.672 & 1 & 0.533 \\ 0.080 & 0.123 & 0.250 & 0.342 & 0.533 & 1 \end{bmatrix}. \quad (13)$$

From this figure, it can be easily verified that P_{out} degrades with an increase of $\gamma_{\text{th}}/\overline{\gamma}_s$. The obtained results clearly show that the approximate curves for P_{out} are sufficiently close to their equivalent simulation results, which are also included within Fig. 2 for comparison purposes, verifying the validity of the approximate expression.

V CONCLUSIONS

In this paper, new results for the multivariate Rayleigh fading channel model with arbitrary correlation structures were presented. By using an efficient tridiagonalization method based on Householder matrices and a standard method for RVs trasformation, closed-form union upper bounds for the joint Rayleigh PDF and CDF were derived. As an application, a tight union upper bound and an analytical approximation for the OP of multibranch SC operating over id and arbitrarily correlated Rayleigh fading channels were obtained. Comparisons between numerically evaluated results and extensive computer simulation ones verified the validity of our approach.

ACKNOWLEDGMENT

This work has been partly supported by IST NoE ACE Smart Antennas Activity A2.5.

APPENDICES

A EFFICIENT ALGORITHMIC COMPUTATION OF (4)

Let $\mathbf{W} \in \Re^{L\times L}$ be a real symmetric matrix. Starting with $\mathbf{W}_0 = \mathbf{W}$, the kth ($k = 1, 2, \ldots, L-2$) Householder transformation can be obtained using the following recursive formula

$$\mathbf{W}_k = \mathbf{H}_k \, \mathbf{W}_{k-1} \, \mathbf{H}_k \tag{A-1}$$

which alternatively, can be efficiently computed performing only vector multiplications as

$$\mathbf{W}_k = \mathbf{W}_{k-1} - 2\mathbf{u}_k \, \mathbf{z}_k^\dagger - 2\mathbf{z}_k \, \mathbf{u}_k^\dagger \tag{A-2}$$

where $\mathbf{z}_k = \mathbf{v}_k - c_k \, \mathbf{u}_k$ with $\mathbf{v}_k = \mathbf{W}_{k-1} \, \mathbf{u}_k$ and $c_k = \mathbf{u}_k^\dagger \, \mathbf{v}_k$. In order to derive the vector $\mathbf{u}_k$, a useful property of the Householder matrix is used. A similar vector, $\mathbf{b}_k$, to $\mathbf{a}_k = \mathbf{W}_{k-1}(:,k)$ ($\mathbf{W}_{k-1}(:,k)$ denotes the kth column of $\mathbf{W}_{k-1}$) may be constructed as

$$\begin{aligned}\mathbf{b}_k = \mathbf{H}_k \, \mathbf{a}_k &= \mathbf{H}_k \left[a_1 \;\; a_2 \;\; \cdots \;\; a_k \;\; a_{k+1} \;\; a_{k+2} \;\; \cdots \;\; a_L \right]^\dagger \\ &= \left[a_1 \;\; a_2 \;\; \cdots \;\; a_k \;\; -s_k \;\; \underbrace{0 \; \cdots \; 0}_{L-(k+1)} \right]^\dagger \end{aligned} \tag{A-3}$$

where $s_k = \text{sign}(a_{k+1}) \sqrt{\sum_{i=k+1}^{L} a_i^2}$, so as vectors $\mathbf{b}_k$ and $\mathbf{a}_k$ to have identical norms. Note that the sign of s_k is chosen to be equal to that of a_{k+1} for less round-off error propagation [20]. By using (A-3), $\mathbf{u}_k$ can be efficiently obtained based on (3), avoiding the computation of $\mathbf{H}_k$, as

$$\mathbf{u}_k = \frac{1}{d_k} \left[\underbrace{0 \;\, 0 \;\, \cdots \;\, 0}_{k} \;\; s_k + a_{k+1} \;\; a_{k+2} \;\; \cdots \;\; a_L \right]^\dagger \tag{A-4}$$

where $d_k = \|\mathbf{a}_k - \mathbf{b}_k\| = \sqrt{2a_{k+1}s_k + 2s_k^2}$. After using (A-2) $L-2$ times, $\mathbf{W}' = \mathbf{W}_{L-2}$ is finally formed.

B SOME PROPERTIES FOR THE TRANSFORMATION MATRIX

The orthogonal matrix used for the transformation in (4) (and in (6)) is of the form

$$\mathbf{Q}^\dagger = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & q_{2,2} & \cdots & q_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & q_{L,2} & \cdots & q_{L,L} \end{bmatrix} \tag{A-5}$$

having the following properties:

i) for any row i, the sum of the squared elements is unity, i.e. $\sum_{j=1}^{L} q_{i,j}^2 = 1$,

ii) for any two rows i and k with $i \neq k$, the sum of products of corresponding elements equals to zero, i.e. $\sum_{j=1}^{L} q_{i,j} \, q_{k,j} = 0$, and

iii) $\mathbf{Q}^{-1} = \mathbf{Q}^\dagger$ and $|\mathbf{Q}^\dagger| = \pm 1$.

REFERENCES

[1] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[2] A. S. Krishnamoorthy and M. Parthasarathy, "A multivariate gamma-type distribution," *Ann. Math. Statist.*, vol. 22, pp. 549–557, 1951.

[3] C. C. Tan and N. C. Beaulieu, "Infinite series representations of the bivariate Rayleigh and Nakagami-m distributions," *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1159–1161, Oct. 1997.

[4] M. K. Simon and M.-S. Alouini, "A simple integral representation of the bivariate Rayleigh distribution," *IEEE Commun. Lett.*, vol. 2, no. 5, pp. 128–130, May 1998.

[5] R. K. Mallik, "On multivariate Rayleigh and exponentials distributions," *IEEE Trans. Inf. Theory*, vol. 49, no. 6, pp. 1499–1515, Jun. 2003.

[6] Y. Chen and C. Tellambura, "Infinite series representation of the trivariate and quadrivariate Rayleigh distribution and their applications," *IEEE Trans. Commun.*, vol. 53, no. 12, pp. 2092–2101, Dec. 2005.

[7] ——, "Performance analysis of three-branch selection combining over arbitrarily correlated Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 861–865, May 2005.

[8] ——, "Performance analysis of L-branch equal gain combiners in equally correlated Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 8, no. 3, pp. 150–152, Mar. 2005.

[9] R. A. A. de Souza, C. Fraidenraich, and M. D. Yacoub, "On the multivariate $\alpha - \mu$ distribution with arbitrary correlation," in *Proc. VI International Telecommunications Symposium (ITS2006)*, Fortaleza-CE, Brazil, Sep. 2006.

[10] M. D. Yacoub, "The $\alpha - \mu$ distribution: A physical fading model for the Stacy distribution," *IEEE Trans. Veh. Technol.*, vol. 56, no. 1, pp. 27–34, Jan. 2007.

[11] L. E. Blumenson and K. S. Miller, "Properties of generalized Rayleigh distributions," *Ann. Math. Statist.*, vol. 34, no. 3, pp. 903–910, Sep. 1963.

[12] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoulos, "On the multivariate Nakagami-m distribution with exponential correlation," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1240–1244, Aug. 2003.

[13] ——, "An efficient approach to multivariate Nakagami-m distribution using Green's matrix approximation," *IEEE Trans. Wireless Commun.*, vol. 2, no. 5, pp. 883–889, Sep. 2003.

[14] G. C. Alexandropoulos, N. C. Sagias, and K. Berberidis, "On the multivariate Weibull fading model with arbitrary correlation matrix," *IEEE Antennas Wireless Propag. Lett.*, vol. 6, pp. 93–95, 2007.

[15] X. Zhang and N. C. Beaulieu, "SER of threshold-based hybrid selection/maximal-ratio combining in correlated Nakagami fading," *IEEE Trans. Commun.*, vol. 53, no. 9, pp. 1423–1426, Sep. 2005.

[16] ——, "Performance analysis of generalized selection combining in generalized correlated Nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, no. 11, pp. 2103–2112, Nov. 2006.

[17] V. A. Aalo, "Performance of maximal-ratio diversity systems in a correlated Nakagami-fading environment," *IEEE Trans. Commun.*, vol. 43, no. 8, pp. 2360–2369, Aug. 1995.

[18] M.-S. Alouini, A. Abdi, and M. Kaveh, "Sum of gamma variates and performance of wireless communication systems over Nakagami-fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1471–1480, Nov. 2001.

[19] A. S. Householder, "Unitary triangularization of a nonsymmetric matrix," *Journal of the ACM*, vol. 5, no. 4, pp. 339–342, Oct. 1958.

[20] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD:The Johns Hopkins University Press, 1996.

[21] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.

[22] J. G. Proakis, *Digital Communications*, 3rd ed. NY, USA: McGraw-Hill, 1995.

[23] Q. T. Zhang, "Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1141–1150, Jul. 1999.