Error Performance of Triple-Branch Generalized Selection Diversity over Nakagami Fading Channels

Kostas Peppas, Nikos C. Sagias, Fotis I. Lazarakis, Antonis A. Alexandridis, and Kostas P. Dangakis
Laboratory of Mobile Communications, Institute of Informatics and Telecommunications, National Centre for Scientific Research–"Demokritos,"
P. Grigoriou & Neapoleos, A. Paraskevi, 15310, Athens, Greece (e-mail: peppas@telecom.ntua.gr, nsagias@ieee.org, {flaz;aalex;kdang}@iit.demokritos.gr)

***Abstract*— This paper deals with a trivariate Nakagami-m distribution derived from the diagonal elements of a Wishart matrix having an arbitrary covariance matrix and integer-order fading parameters. Based on that distribution, the error rate performance of triple-branch generalized selection combining (GSC) receivers is analyzed, for which, the average bit error probability for a variety of modulation schemes is analytically obtained. The performance of GSC receivers is compared to that of conventional selection and maximal-ratio diversity ones. In order to check the accuracy of the derived formulas, various performance evaluation results are presented and compared to equivalent simulation ones.**

I. INTRODUCTION

The Nakagami-m distribution is used in modeling various propagation channels which are characterized by multipath scattering with relatively large delay-time spreads with different clusters of reflected waves [1]. Of particular interest is the multivariate Nakagami-m distribution, which plays an important role in channel modeling of wireless communication systems and to the performance analysis of digital receivers over correlated fading channels. In the research literature, there are several papers dealing with multivariate Nakagami-m distributions [1]–[4]. Various works studying the performance of generalized selection combining (GSC) receivers over correlated Nakagami-m fading channels have been presented. In two recent works for example, by assuming correlated Nakagami-m fading with positive integer-order values for fading parameters, the performance of selection combining (SC), hybrid-selection/maximal-ratio combining (H-S/MRC), and threshold-based H-S/MRC has been analyzed in [5], [6]. More specifically in [5], Green's matrix approximations have been used for studying arbitrary correlation structures, while in [6], a more general model than the equal correlation one has been considered.

Besides, various research findings have been reported in the statistics research literature, but have not been utilized by researchers working on wireless communications theory yet. Some of them concern joint chi-square distributions derived from the diagonal elements of the Wishart matrix. In [7], expansions for the probability density function (PDF) of a trivariate chi-square distribution have been presented in terms of rapidly converge infinite sums, which are simple for numerical evaluation. In the analysis follows, the average bit error probability (ABEP) of triple-branch GSC receivers over correlated Nakagami-m fading channels with integer-order fading parameters and an arbitrary covariance matrix are assessed. Performance comparisons with triple-branch MRC and SC receivers are also performed, while computer simulations validate the correctness of our findings.

Next, the following notations are used: $(\cdot)^{\dagger}$ for the transpose, $(\cdot)^{-1}$ for the inverse, $(\cdot)^{H}$ for the Hermitian transpose, $\det(\cdot)$ for the determinant, $(\cdot)^{*}$ for the complex conjugate, $\operatorname{diag}(\cdot)$ for the diagonal elements, $\mathbb{E}\langle\cdot\rangle$ for the expectation operator, and $\Re(\cdot)$ and $\Im(\cdot)$ the real and imaginary parts operators, respectively.

II. THE TRIVARIATE NAKAGAMI-m DISTRIBUTION

A. Preliminaries

Let $\mathbf{Q}_p = [X_{1,p}, X_{2,p}, X_{3,p}]^{\dagger}$ be the pth sample of a three-dimensional zero-mean complex Gaussian random process ($p = 1, 2, \ldots, m$). These processes are considered to be mutually independent and identically distributed (id) having a covariance matrix

$$\mathbf{\Sigma} = 2 \begin{bmatrix} \sigma_1^2 & c_{12} & c_{13} \\ c_{12} & \sigma_2^2 & c_{23} \\ c_{13} & c_{23} & \sigma_3^2 \end{bmatrix} \qquad (1)$$

with $\sigma_\ell^2 = \mathbb{E}\langle |X_{\ell,p}|^2\rangle/2$ and $c_{\ell,\ell'} = \mathbb{E}\langle X_{\ell,p}\, X^*_{\ell',p}\rangle/2$, $\forall\, \ell \neq \ell'$ ($\ell, \ell' = 1$, 2, and 3). By defining a six-dimensional sample vector as $\mathbf{W}_p = [\Re(X_{1,p}), \Im(X_{1,p}), \Re(X_{2,p}), \Im(X_{2,p}), \Re(X_{3,p}), \Im(X_{3,p})]^{\dagger}$ and setting the crosscorrelation terms between real and imaginary parts equal to zero, i.e., $\mathbb{E}\langle \Re\{X_{\ell,p}\}\Im\{X_{\ell',p}\}\rangle = 0\ \forall \ell, \ell' = 1$, 2, 3, the joint PDF of $\mathbf{W}_p$ is

$$f_{\mathbf{W}_p}(\mathbf{W}_p) = \frac{1}{\sqrt{(2\pi)^6 \det(\mathbf{C})}} \exp\left(-\frac{1}{2}\, \mathbf{W}_p^{\dagger}\, \mathbf{C}^{-1}\, \mathbf{W}_p\right) \qquad (2)$$

where $\mathbf{C} = \mathbb{E}\langle \mathbf{W}_p\, \mathbf{W}_p^{\dagger}\rangle$ having the following structure

$$\mathbf{C} = \begin{bmatrix} \sigma_1^2 & 0 & c_{12} & 0 & c_{13} & 0 \\ 0 & \sigma_1^2 & 0 & c_{12} & 0 & c_{13} \\ c_{12} & 0 & \sigma_2^2 & 0 & c_{23} & 0 \\ 0 & c_{12} & 0 & \sigma_2^2 & 0 & c_{23} \\ c_{13} & 0 & c_{23} & 0 & \sigma_3^2 & 0 \\ 0 & c_{13} & 0 & c_{23} & 0 & \sigma_3^2 \end{bmatrix}. \qquad (3)$$

The inverse of $\mathbf{C}$ is

$$\mathbf{C}^{-1} = \begin{bmatrix} a_1 & 0 & b_1 & 0 & b_3 & 0 \\ 0 & a_1 & 0 & b_1 & 0 & b_3 \\ b_1 & 0 & a_2 & 0 & b_2 & 0 \\ 0 & b_1 & 0 & a_2 & 0 & b_2 \\ b_3 & 0 & b_2 & 0 & a_3 & 0 \\ 0 & b_3 & 0 & b_2 & 0 & a_3 \end{bmatrix} \qquad (4)$$

978-1-4244-1653-0/08/$25.00 ©2008 IEEE

where

$$a_1 = \frac{\sigma_2^2\,\sigma_3^2 - c_{23}^2}{\sqrt{\det(\mathbf{C})}}, \quad b_1 = \frac{\sigma_3^2\,c_{12} - c_{13}\,c_{23}}{\sqrt{\det(\mathbf{C})}} \tag{5a}$$

$$a_2 = \frac{\sigma_1^2\,\sigma_3^2 - c_{13}^2}{\sqrt{\det(\mathbf{C})}}, \quad b_2 = \frac{\sigma_1^2\,c_{23} - c_{12}\,c_{13}}{\sqrt{\det(\mathbf{C})}} \tag{5b}$$

$$a_3 = \frac{\sigma_1^2\,\sigma_2^2 - c_{12}^2}{\sqrt{\det(\mathbf{C})}}, \quad b_3 = \frac{\sigma_2^2\,c_{13} - c_{12}\,c_{23}}{\sqrt{\det(\mathbf{C})}} \tag{5c}$$

while the determinant of $\mathbf{C}$ is $\det(\mathbf{C}) = 2^{-6}\,[\det(\mathbf{\Sigma})]^2$.

If we define vector $\mathbf{X}_\ell = [\Re(X_{\ell,1}), \Im(X_{\ell,1}), \Re(X_{\ell,2}), \Im(X_{\ell,2}), \ldots, \Re(X_{\ell,m}), \Im(X_{\ell,m})]$, then its norm is given by $R_\ell^2 = ||\mathbf{X}_\ell||^2 = \sum_{p=1}^{m}[\Re^2(X_{\ell,p}) + \Im^2(X_{\ell,p})]$, which essentially denotes the diagonal elements of the complex Wishart matrix, $\mathbf{S} = \sum_{p=1}^{m} \widetilde{\mathbf{W}}_p \widetilde{\mathbf{W}}_p^H$. It is obvious that R_ℓ is a Nakagami-m random variable (RV), with an integer-order fading parameter m and average power $\Omega_\ell = \mathbb{E}\langle R_\ell^2\rangle = 2\,m\,\sigma_\ell^2$, having PDF

$$f_{R_\ell}(r) = \frac{2\,m^m}{\Omega_\ell^m\,(m-1)!}\,r^{2m-1}\,\exp\left(-\frac{m}{\Omega_\ell}\,r^2\right). \tag{6}$$

Also, the nth-order moment of R_ℓ [1, eq. (17)] is given by $\mathbb{E}\langle R_\ell^n\rangle = (\Omega_\ell/m)^{n/2}\,\Gamma(m+n/2)/(m-1)!$, with $\Gamma(\cdot)$ being the gamma function [8, eq. (8.310/1)].

B. Joint PDF

According to [7], there are two cases which should be taken into consideration for the trivariate Nakagami-m PDF. The first one is for $m=1$ and the second one for $m = 2, 3, \ldots$ Since the first case (Rayleigh fading) has been already studied in [9], next we are exclusively interested in the second one. Using (2), the joint PDF of R_ℓ can be extracted as [7, eq. (11)]

$$f_{R_1,R_2,R_3}(r_1,r_2,r_3) = \int\limits_{|\mathbf{X}_1|=r_1}\int\limits_{|\mathbf{X}_2|=r_2}\int\limits_{|\mathbf{X}_3|=r_3} f_{\mathbf{W}_p}(\mathbf{W}_p)\,\mathrm{d}\mathbf{X}_1\,\mathrm{d}\mathbf{X}_2\,\mathrm{d}\mathbf{X}_3 \tag{7}$$

where $\int_{|\mathbf{X}_\ell|=r_\ell}$ denotes integration over the surface of a $2m$-dimensional sphere of radius r_ℓ. The triple integral in (7) has been solved in [7], and thus, the trivariate Nakagami-m PDF with integer-order fading parameters and an arbitrary covariance matrix (with elements $\mathbf{R}_{\ell,\ell'} = m\,\mathbf{\Sigma}_{\ell,\ell'}^2$) can be expressed as[1]

$$f_{R_1,R_2,R_3}(r_1,r_2,r_3) = \frac{\exp\left[-\left(a_1\,r_1^2 + a_2\,r_2^2 + a_3\,r_3^2\right)/2\right]}{(m-1)\,\left[\det\left(\mathbf{\Sigma}/2\right)\right]^m\,\left(b_1\,b_2\,b_3\right)^{m-1}} \times r_1\,r_2\,r_3 \sum_{k=m-1}^{\infty} k\,(-1)^{k-m+1}\binom{m+k-2}{2m-3} \times I_k(b_1\,r_1\,r_2)\,I_k(b_2\,r_2\,r_3)\,I_k(b_3\,r_1\,r_3) \tag{8}$$

[1]Note that (8) agrees with a recent result [10, eq. (11)].

where $I_k(\cdot)$ denotes the kth-order modified Bessel function of the first kind [8, Section 8.406]. The power correlation coefficient between ℓth and ℓ'th channels, defined as[2] $\rho_{\ell\ell'} = \mathrm{cov}(R_\ell^2, R_{\ell'}^2)/\,[\sqrt{\mathrm{var}(R_\ell^2)}\sqrt{\mathrm{var}(R_{\ell'}^2)}]$, can be easily expressed as a function of $c_{\ell\ell'}$ as $c_{\ell\ell'} = \sqrt{\Omega_\ell\,\Omega_{\ell'}/(4\,m^2\,\rho_{\ell\ell'})}$. Hence, all parameters given by (5) can be reexpressed in terms of parameters of interest in wireless communications, such as $\rho_{\ell\ell'}$'s, m, and Ω_ℓ's, as follows

$$a_1 = \frac{2m}{T\,\Omega_1}\,(1-\rho_{23}), \quad b_1 = \frac{-2\,m/T}{\sqrt{\Omega_1\,\Omega_2}}\,(\sqrt{\rho_{12}} - \sqrt{\rho_{23}\,\rho_{13}}) \tag{9a}$$

$$a_2 = \frac{2m}{T\,\Omega_2}\,(1-\rho_{13}), \quad b_2 = \frac{2\,m/T}{\sqrt{\Omega_2\,\Omega_3}}\,(\sqrt{\rho_{23}} - \sqrt{\rho_{12}\,\rho_{13}}) \tag{9b}$$

$$a_3 = \frac{2m}{T\,\Omega_3}\,(1-\rho_{12}), \quad b_3 = \frac{2\,m/T}{\sqrt{\Omega_1\,\Omega_3}}\,(\sqrt{\rho_{13}} - \sqrt{\rho_{12}\,\rho_{23}}) \tag{9c}$$

and $\det(\mathbf{\Sigma}) = T\,\prod_{\ell=1}^{3}\Omega_\ell/m$ with $T = 1 - (\rho_{12} + \rho_{23} + \rho_{13}) + 2\,\sqrt{\rho_{12}\,\rho_{23}\,\rho_{13}}$.

1) Uncorrelated: In case where the three channels are uncorrelated, i.e., $\rho_{\ell,\ell'} = 0\ \forall \ell \neq \ell'$ ($T=1$), $a_\ell = 2\,m/\Omega_\ell$ and $b_\ell = 0\ \forall\ell$. Based on the following power series expansion for $I_k(\cdot)$'s

$$\lim_{b_\ell \to 0}\frac{I_k(b_\ell\,r_\ell\,r_{\ell'})}{b_\ell^{m-1}} = \begin{cases} \frac{1}{(m-1)!}\left(\frac{r_\ell\,r_{\ell'}}{2}\right)^{m-1}, & k = m-1;\\ 0, & k > m-1, \end{cases} \tag{10}$$

all terms except for $k = m-1$ vanish in the sum in (8), reducing $f_{R_1,R_2,R_3}(r_1,r_2,r_3)$ to a product of three independent marginal PDFs, i.e., $f_{R_1,R_2,R_3}(r_1,r_2,r_3) = \prod_{\ell=1}^{3} f_{R_\ell}(r_\ell)$.

2) Constant correlation: In case of constant correlation among the three channels, i.e., $\rho_{\ell,\ell'} = \rho\ \forall\ell \neq \ell'$ ($T = 1 - 3\,\rho + 2\,\rho^{3/2}$), $a_\ell = 2\,m\,(1-\rho)/\Omega_\ell$ and

$$b_1 = \frac{2\,m}{\sqrt{\Omega_1\,\Omega_2}}\,\frac{\sqrt{\rho}}{2\,\rho - \sqrt{\rho} - 1} \tag{11a}$$

$$b_2 = \frac{2\,m}{\sqrt{\Omega_2\,\Omega_3}}\,\frac{\sqrt{\rho}}{2\,\rho - \sqrt{\rho} - 1} \tag{11b}$$

$$b_3 = \frac{2\,m}{\sqrt{\Omega_1\,\Omega_3}}\,\frac{\sqrt{\rho}}{2\,\rho - \sqrt{\rho} - 1} \tag{11c}$$

3) Exponential correlation: In case of exponential correlation among the three channels, i.e., $\rho_{\ell,\ell'} = \rho^{|\ell-\ell'|}$ $\forall\ell,\ell'$ ($T = (1-\rho)^2$), $a_i = 2\,m/[\Omega_i\,(1-\rho)]$ ($i = 1$ and 3), $a_2 = (2\,m/\Omega_2)\,(1-\rho)/(1+\rho)$ and $b_j = -(2\,m/\sqrt{\Omega_j\,\Omega_{j+1}})\,\sqrt{\rho}/(1-\rho)$ ($j = 1$ and 2), $b_3 = 0$. Note that using (10), (8) reduces to [3, eq. (3)]

C. Joint CDF

The joint CDF of R_ℓ can be calculated as

$$F_{R_1,R_2,R_3}(r_1,r_2,r_3) = \int_0^{r_1}\int_0^{r_2}\int_0^{r_3} f_{R_1,R_2,R_3}(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \tag{12}$$

[2]As it is well known, $\rho_{\ell\ell'}$ is related to the correlation coefficient of the underlying real Gaussian processes, $\varrho_{\ell\ell'}$, as $\rho_{\ell\ell'} = \varrho_{\ell\ell'}^2$.

but (8) can not be used in the current form. Using an infinite series representation for Bessel functions [8, eq. (8.445)] and after performing some straightforward algebraic manipulations, yields

$$F_{R_1,R_2,R_3}(r_1,r_2,r_3) = \frac{T^{2m}}{m-1} \times \sum_{k=m-1}^{\infty} \sum_{l_1,l_2,l_3=0}^{\infty} k\,(-1)^{k-m+1} \binom{m+k-2}{2m-3} \times \prod_{\ell=1}^{3} \frac{\omega_\ell^{2\,l_\ell+k+1-m}}{\psi_\ell^{n_\ell+1}\, l_\ell!\,(l_\ell+k)!}\, \gamma\left(n_\ell+1, \frac{m\,\psi_\ell}{T\,\Omega_\ell}\, r_\ell^2\right) \quad (13)$$

where $n_1 = l_1 + l_3 + k$, $n_2 = l_1 + l_2 + k$, $n_3 = l_2 + l_3 + k$, and $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function [8, eq. (8.350/1)]. Also, ψ_ℓ's and ω_ℓ's are defined as $\psi_1 = 1 - \rho_{23}$, $\psi_2 = 1 - \rho_{13}$, $\psi_3 = 1 - \rho_{12}$ and $\omega_1 = -\sqrt{\rho_{12}} + \sqrt{\rho_{23}\,\rho_{13}}$, $\omega_2 = -\sqrt{\rho_{23}} + \sqrt{\rho_{12}\,\rho_{13}}$, $\omega_3 = -\sqrt{\rho_{13}} + \sqrt{\rho_{12}\,\rho_{23}}$. Note that since the first argument of the gamma function in (13) is a positive integer, $\gamma(\cdot,\cdot)$ can be expressed in terms of elementary functions [8, eq. (8.352/1)].

D. Truncation Error

In order to find a simple bound for the truncation error of the CDF series in (13), we follow the method presented in [9, Section II.B]. Assume that the series in (13) are limited to L_0, L_1, L_2, and L_3 terms in indexes k, l_1, l_2, and l_3, respectively. The remaining terms constitute the truncation error. Based on the fact that $\gamma(n_\ell + 1, x) \leq n_\ell!$, the truncation error of (13) can therefore be upper bounded by

$$\begin{aligned}|E_T(L_0,L_1,L_2,L_3)| \leq & \sum_{k=L_0}^{\infty}\sum_{l_1=0}^{\infty}\sum_{l_2=0}^{\infty}\sum_{l_3=0}^{\infty} H(k,l_1,l_2,l_3) \\ & + \sum_{k=m-1}^{L_0-1}\sum_{l_1=L_1}^{\infty}\sum_{l_2=0}^{\infty}\sum_{l_3=0}^{\infty} H(k,l_1,l_2,l_3) \\ & + \sum_{k=m-1}^{L_0-1}\sum_{l_1=0}^{L_1-1}\sum_{l_2=L_2}^{\infty}\sum_{l_3=0}^{\infty} H(k,l_1,l_2,l_3) \\ & + \sum_{k=m-1}^{L_0-1}\sum_{l_1=0}^{L_1-1}\sum_{l_2=0}^{L_2-1}\sum_{l_3=L_3}^{\infty} H(k,l_1,l_2,l_3)\end{aligned} \quad (14)$$

with

$$H(k,l_1,l_2,l_3) = \frac{T^{2m}}{m-1} \binom{m+k-2}{2m-3} \binom{n_1}{l_3} \binom{n_2}{l_1} \times \binom{n_3}{l_2} \prod_{\ell=1}^{3} \frac{|\omega_\ell|^{2\,l_\ell+k+1-m}}{\psi_\ell^{n_\ell+1}}. \quad (15)$$

As an example, we consider the constant correlation model, i.e., $\rho_{\ell\ell'} = \rho\ \forall \ell \neq \ell'$ ($\psi_\ell = 1 - \rho$ and $\omega_\ell = -\sqrt{\rho} + \rho$), with $m = 2$, 4 and $\rho = 0.1$, 0.5, 0.7. Setting $r_\ell^2/\Omega_\ell = r^2/\Omega$ $\forall \ell$ and assuming $L_0 = L_1 = L_2 = L_3$, Table I summarizes the number of the terms required in (13) to achieve a ratio $|E_T|/F_{R_1,R_2,R_3}(r,r,r) < 10^{-3}$. As shown, the convergence rate depends strongly on m and ρ. Specifically, the higher the m and/or ρ are, the more terms are needed. Moreover, for fixed m and ρ, as r^2/Ω increases, less terms are needed in the CDF series to achieve the target ratio.

TABLE I
NUMBER OF REQUIRED TERMS FOR CONVERGENCE OF (13) FOR THE CONSTANT CORRELATION MODEL TO ACHIEVE A TARGET RATIO $|E_T|/F_{R_1,R_2,R_3}(r,r,r) < 10^{-3}$

	$r^2/\Omega = 0.3$		$r^2/\Omega = 1$		$r^2/\Omega = 3$	
	$m = 2$	$m = 4$	$m = 2$	$m = 4$	$m = 2$	$m = 4$
$\rho = 0.1$	6	9	5	7	4	6
$\rho = 0.5$	15	21	12	15	10	13
$\rho = 0.7$	23	38	20	29	10	13

III. TRIPLE-BRANCH GSC RECEIVERS

A. System Model

We consider a triple-branch GSC(K,3) ($K = 1, 2, 3$) receiver operating over an arbitrary correlated Nakagami-m multipath fading environment with not necessarily id channel statistics. According to GSC(K,3) scheme, the K strongest branches having the highest instantaneous SNRs are selected among the three available and appropriately combined. The GSC(K,3) reception is equivalent to MRC reception if all three branches are combined (i.e., $K = 3$), while it is equivalent to SC reception if only one out of the three branches is selected (i.e., $K = 1$).

The baseband received signal at the ℓth diversity branch is $\zeta_\ell = z\,R_\ell + w_\ell$ where z is the transmitted symbol with energy $E_s = \mathbb{E}\langle|z|^2\rangle$, R_ℓ is the Nakagami-m distributed fading envelope, and w_ℓ is the additive white Gaussian noise with a single-sided power spectral density N_0. The noise components are assumed to be statistically independent of the signal and uncorrelated to each other. Moreover, all three channels are considered as slowly time varying, and thus, their characteristics are perfectly known to the receiver.

B. Order Statistics

The instantaneous SNR per symbol $\gamma_\ell = R_\ell^2\,E_s/N_0$ in the ℓth input branch is an Erlang RV with $\overline{\gamma}_\ell = \Omega_\ell\,E_s/N_0$ being the corresponding average input SNR per symbol. By applying the RVs transformation $R_\ell = \sqrt{\Omega_\ell\,\gamma_\ell/\overline{\gamma}_\ell}$ in (8), the joint PDF of $\gamma_1, \gamma_2, \gamma_3$ becomes

$$f_{\gamma_1,\gamma_2,\gamma_3}(\gamma_1,\gamma_2,\gamma_3) = \frac{2^{3\,(m-1)}\exp\left(-\frac{1}{2}\sum_{\ell=1}^{3}\widetilde{a}_\ell\,\gamma_\ell\right)}{(m-1)\left[\det\left(\widetilde{\boldsymbol{\Sigma}}\right)\right]^m\left(\widetilde{b}_1\,\widetilde{b}_2\,\widetilde{b}_3\right)^{m-1}} \times \sum_{k=m-1}^{\infty} k\,(-1)^{k-m+1}\binom{m+k-2}{2m-3} I_k\left(\widetilde{b}_1\sqrt{\gamma_1\,\gamma_2}\right) \times I_k\left(\widetilde{b}_2\sqrt{\gamma_2\,\gamma_3}\right) I_k\left(\widetilde{b}_3\sqrt{\gamma_1\,\gamma_3}\right). \quad (16)$$

The parameters $\widetilde{a}_\ell$ and $\widetilde{b}_\ell$ as well as the determinant $\det(\widetilde{\boldsymbol{\Sigma}})$ can be easily derived from corresponding a_ℓ and b_ℓ, and $\det(\boldsymbol{\Sigma})$, just replacing Ω_ℓ with $\overline{\gamma}_\ell$ $\forall \ell$ in (9).

The instantaneous SNR per symbol at the output of a GSC(K,3) receiver can be expressed as $\gamma_{\rm gsc} = \sum_{k=1}^{3} \xi_k\, \gamma_{(k)}$, where $\xi_k = 1$, if $k = 1$, 2, K, and $\xi_k = 0$, if $k = K+1$, 3, while $\gamma_{(\ell)}$'s are the descending ordered γ_ℓ's, i.e., $\gamma_{(1)} \geq \gamma_{(2)} \geq \gamma_{(3)}$ (by default, $\xi_1 = 1$). Based on [11, Appendix], the joint PDF of $\gamma_{(\ell)}$'s can be expressed as

$$f_{\gamma_{(1)},\gamma_{(2)},\gamma_{(3)}}(\gamma_1,\gamma_2,\gamma_3) = \sum_{e_i \in \mathcal{S}_3} f_{\gamma_1,\gamma_2,\gamma_3}\left(\gamma_{e_i[1]}, \gamma_{e_i[2]}, \gamma_{e_i[3]}\right) \tag{17}$$

with $e_i \in \mathcal{S}_3$ denoting $e_\ell = \{e_i[1], e_i[2], e_i[3]\}$, one specific permutation of the integers $\{1,2,3\}$. The MGF of the GSC output SNR per symbol can be obtained from the above equation as $\mathcal{M}_{\gamma_{\rm gsc}}(s) = \mathbb{E}\langle \exp(-s\,\gamma_{\rm gsc})\rangle$. Using an infinite series representation for Bessel functions [8, eq. (8.445)], this MGF yields

$$\begin{aligned}\mathcal{M}_{\gamma_{\rm gsc}}(s) &= \frac{2^{3\,(m-1)}}{(m-1)\left[\det\left(\widetilde{\mathbf{\Sigma}}\right)\right]^m} \\ &\times \sum_{k=m-1}^{\infty} \sum_{l_1,l_2,l_3=0}^{\infty} k\,(-1)^{k-m+1} \binom{m+k-2}{2m-3} \\ &\times \left[\prod_{\ell=1}^{3} \frac{2^{-k-2l_\ell}\, \widetilde{b}_\ell^{2\,l_\ell+k+1-m}}{l_\ell!\,(l_\ell+k)!}\right] \\ &\times \sum_{e_i \in \mathcal{S}_3} G\left[s;\, n_{e_i[1]}, \widetilde{a}_{e_i[1]};\, n_{e_i[2]}, \widetilde{a}_{e_i[2]};\, n_{e_i[3]}, \widetilde{a}_{e_i[3]}\right]\end{aligned} \tag{18}$$

with $G(s;\, n_1, a_1;\, n_2, a_2;\, n_3, a_3) = \int_0^\infty \int_{\gamma_3}^\infty \int_{\gamma_2}^\infty \{\prod_{i=1}^{3} \gamma_i^{n_i} \exp[-(a_i/2 + s\,\xi_i)\,\gamma_i]\} \mathrm{d}\gamma_1\, \mathrm{d}\gamma_2\, \mathrm{d}\gamma_3$. Using [8, eqs. (8.350/2) and (8.352/2)], this triple integral can be solved in closed form as

$$\begin{aligned}&G(s;\, n_1, a_1;\, n_2, a_2;\, n_3, a_3) \\ &= n_1! \sum_{p_1=0}^{n_1} \sum_{p_2=0}^{n_2+p_1} (p_1)_{n_2}\, (p_2)_{n_3} \prod_{\ell=1}^{3} \left[(B_\ell + s) \sum_{i=1}^{\ell} \xi_i\right]^{-u_\ell}\end{aligned} \tag{19}$$

with $B_\ell = 0.5 \sum_{i=1}^{\ell} a_i / \sum_{i=1}^{\ell} \xi_i$, $u_1 = 1 + n_1 - p_1$, $u_2 = 1 + p_1 + n_2 - p_2$, and $u_3 = n_3 + p_2 + 1$.

Note that for $\xi_1 = \xi_2 = \xi_3 = 1$ ($K = 3$: MRC scheme), (18) numerically agrees with [12, eq. (11)] $\mathcal{M}_{\gamma_{\rm mrc}}(s) = [\det(\mathbf{I}_3 + s\,\widetilde{\mathbf{\Sigma}})]^{-m}$, with $\mathbf{I}_3$ being the 3×3 identity matrix.

C. Error Probability

Using the MGF of triple-branch GSC output SNR per symbol, given by (18), the ABEP for non-coherent binary frequency shift keying (NBFSK) and binary differential phase shift keying (BDPSK) modulation signalling can be directly calculated (e.g. for BDPSK, $\overline{P}_{be} = 0.5\,\mathcal{M}_{\gamma_{\rm gsc}}(1)$). For other types of modulation formats, including binary phase shift keying (BPSK), M-ary-phase shift keying (M-PSK), quadrature amplitude modulation (M-QAM), amplitude modulation (M-AM), and differential phase shift keying (M-DPSK), single integrals with finite limits and integrands composed of elementary (exponential and trigonometric) functions, have to be readily evaluated via numerical integration [2].

TABLE II

NUMBER OF REQUIRED TERMS FOR CONVERGENCE OF (18) FOR THE CONSTANT CORRELATION MODEL

	$\rho = 0.1$		$\rho = 0.5$		$\rho = 0.7$	
$\overline{\gamma}$ (dB)	$m=2$	$m=4$	$m=2$	$m=4$	$m=2$	$m=4$
-5	4	6	7	10	10	19
0	3	6	6	8	7	13
5	2	4	4	6	5	8
10	1	3	3	4	4	7

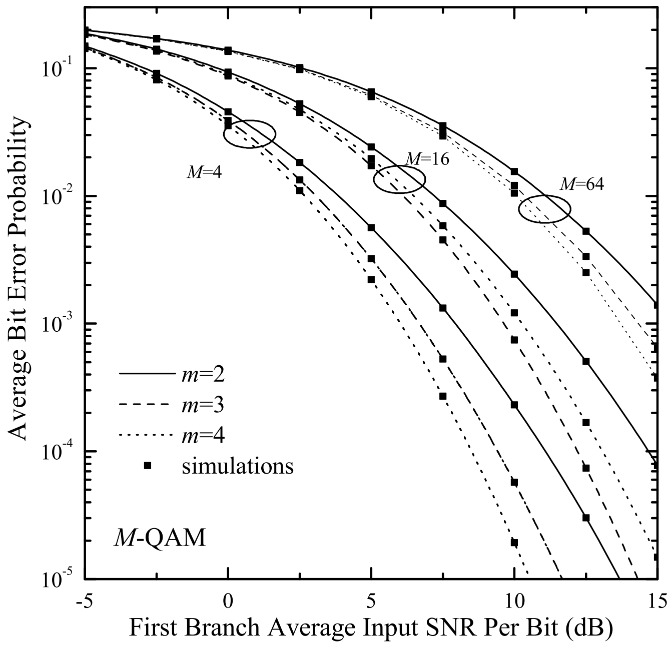

Fig. 1. ABEP of Gray-encoded M-QAM with GSC(2,3) receivers for a linearly arbitrary correlation model as a function of the first branch average input SNR per bit.

IV. NUMERICAL AND COMPUTER SIMULATION RESULTS

Setting equal summation limits for the truncation of (18) to all sums, Table II summarizes the number of terms needed so as the ABEP of BDPSK to converge with relative error $e_r \leq 5\%$ comparing to accurate computer simulations. The constant correlation model is considered with id channels ($\overline{\gamma}_\ell = \overline{\gamma}$) various values of ρ and $m = 2$, 4. Interestingly enough, only a few terms are required in order the series in (18) to converge. An increase on $\overline{\gamma}$, results to a decrease of the required number of terms, while for a fixed $\overline{\gamma}$, the required number of terms for convergence increases with increasing m and/or ρ.

Next, an exponential power delay profile $\overline{\gamma}_\ell = \exp[-\delta\,(\ell - 1)]\,\overline{\gamma}_1$, with power decaying factor $\delta = 0.1$ is considered. In Fig. 1, the ABEP for Gray-encoded M-ary square-QAM schemes is plotted as a function of the first branch average input SNR per bit $\overline{\gamma}_b = \overline{\gamma}_1/\log_2(M)$. The linearly arbitrary correlation model has been adopted with $\rho_{12} = \rho_{23} = 0.795$

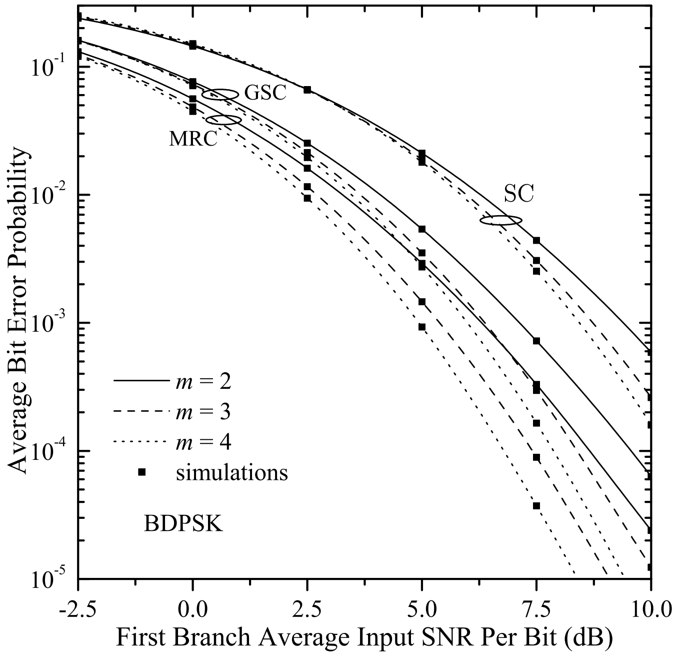

Fig. 2. ABEP of BDPSK with GSC(2,3), MRC, and SC for a constant correlation model as a function of the first branch average input SNR per bit.

and $\rho_{13} = 0.605$. As expected, the ABEP improves as M decreases and/or m and $\overline{\gamma}_b$ increase. In Fig. 2, the constant correlation model [2] with $\rho_{\ell\ell'} = 0.1 \ \forall \ell \neq \ell'$ has been considered. More specifically, a few curves are illustrated for the ABEP of BDPSK modulation for GSC(K,3) receivers as a function of $\overline{\gamma}_b$, several values of m, and $K = 1, 2, 3$. For comparison purposes, corresponding curves for SC and MRC are also included. As expected, it is clear that the MRC scheme outperforms both GSC(2,3) and SC ones. In both figures, the numerically evaluated results are compared to equivalent simulation ones. This comparisons clearly show that the curves for the ABEP coincide with square pattern signs obtained via simulations, verifying the correctness of the proposed analysis.

V. CONCLUSIONS

A rapidly convergent infinite series of a trivariate Nakagami-m PDF with arbitrary covariance matrix was derived from the diagonal elements of the Wishart matrix. Following the MGF-based approach and extracting the MGF of the GSC output SNR, the error rate performance of GSC receivers was analyzed and compared to conventional ones such as MRC and SC. Finally, extensive numerical and computer simulation results were presented and compared, and a perfect match was observed.

REFERENCES

[1] M. Nakagami, "The m-distribution–A general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*. Oxford, U.K.: Pergamon Press, 1960, pp. 3–36.

[2] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[3] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoulos, "On the multivariate Nakagami-m distribution with exponential correlation," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1240–1244, Aug. 2003.

[4] R. K. Mallik and M. Z. Win, "Analysis of hybrid selection/maximal-ratio combining in correlated Nakagami fading," *IEEE Trans. Commun.*, vol. 50, no. 8, pp. 1372–1383, Aug. 2002.

[5] X. Zhang and N. C. Beaulieu, "SER of threshold-based hybrid selection/maximal-ratio combining in correlated Nakagami fading," *IEEE Trans. Commun.*, vol. 53, no. 9, pp. 1423–1426, Sept. 2005.

[6] ——, "Performance analysis of generalized selection combining in generalized correlated Nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, no. 11, pp. 2103–2112, Nov. 2006.

[7] M. Hagedorn, P. J. Smith, P. J. Bones, R. P. Millane, and D. Pairman, "A trivariate chi-square distribution derived from the complex Wishart distribution," *J. Multivariate Anal.*, vol. 97, no. 3, pp. 655–674, Mar. 2006.

[8] I. Gradshteyn and I. M. Ryzhic, *Tables of Integrals, Series, and Products*, 6th ed. New York: Academic Press, 2000.

[9] Y. Chen and C. Tellambura, "Infinite series representations of the trivariate and quadrivariate Rayleigh distribution and their applications," *IEEE Trans. Commun.*, vol. 53, no. 12, pp. 2092–2101, Dec. 2005.

[10] K. D. P. Dharmawansa, R. M. A. P. Rajatheva, and C. Tellambura, "Infinite series representations of the trivariate and quadrivariate Nakagami-m distributions," in *Proc. IEEE Inter. Conf. Commun.*, Glasgow, Scotland, UK, June 2007, pp. 1114–1118.

[11] X. Zhang and N. C. Beaulieu, "Approximate SER of H-S/MRC in Nakagami fading with arbitrary branch power correlation," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 56–60, Jan. 2006.

[12] Q. T. Zhang, "Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1141–1150, July 1999.