Dual-Hop Relaying Networks over Nakagami-m Fading Channels

Christos K. Datsikas*, Nikos C. Sagias†, Fotis I. Lazarakis†, George C. Alexandropoulos†‡, Antonis A. Alexandridis†, Kostas P. Dangakis†, and George S. Tombras*

*Laboratory of Electronics, Physics Department, University of Athens, Panepistimiopolis, Zografou, GR-15784 Athens, Greece (e-mail: cdatsikas@gmail.com, gtombras@cc.uoa.gr)
†Laboratory of Mobile Communications, Institute of Informatics and Telecommunications, National Centre for Scientific Research–"Demokritos," Patriarhou Grigoriou & Neapoleos, Agia Paraskevi, GR-15310 Athens, Greece (e-mail: nsagias@ieee.org, flaz;aalex;kdang@iit.demokritos.gr)
‡Department of Computer Engineering & Informatics, University of Patras, Rion, GR-26500 Patras, Greece, (e-mail: alexandg@ieee.org)

***Abstract*— In this paper, for an L-relays dual-hop plus a direct link wireless network in which the decode-and-forward relaying protocol is employed, closed-form expressions for the end-end outage probability are presented. Our analysis generalizes previous results on Rayleigh fading, considering a Nakagami-m fading environment with either equal or distinct second hops fading parameter to average power ratios. Various numerical examples illustrate the proposed analysis.**

I. Introduction

Multihop networks have several advantages over traditional communication networks in terms of deployment, connectivity, and capacity, while minimizing the need for fixed infrastructure [1]–[3]. A special class of multihop communication networks are the cooperative networks, where the destination terminal combines the signals received from both source and relays terminals [4], [5]. The destination terminal can employ a variety of diversity techniques to benefit from the multiple signal replicas available, such as maximal-ratio, equal-gain, and selection diversity [6]. Besides, there are various protocols that achieve the benefits of user cooperation. One of them is the decode-and-forward (DF) relaying protocol, which uses relays that encode and retransmit the signal towards the destination after demodulate and decode the received signal from the source.

An important performance criterion which is useful for evaluating the transmission protocol under consideration is the outage probability (OP). In [7], a closed-form solution for the OP of DF relaying has been presented, assuming independent and identically distributed (id) relay channels among source, relays, and destination terminals. In addition, lower and asymptotic bounds have been proposed in [7] and [8], respectively, assuming independent but not id channels. In a recent contribution [9], Beaulieu and Hu have presented closed-form expression for the OP in dissimilar Rayleigh fading channels. Real wireless channels are more accurately modeled by more generic than Rayleigh distributions, such as the well-known Nakagami-m [10]. However, to the best of the authors knowledge, an OP analysis for DF relaying networks, with exact closed-form expressions has not been performed.

Motivated by all above, in this paper, we extend previously published results by analyzing the performance of DF dual-hop networks over Nakagami-m fading channels. More specifically, exact closed-form expressions for the OP of DF relaying networks operating over either equal or distinct second hops Nakagami-m fading parameters to average signal-to-noise ratios (SNRs) ratios are presented.

The paper is organized as follows: The next section presents the system and channel model of the dual-hop network. In Section III, OP expressions are presented for equal or distinct parameters, while in Section IV, a few numerical examples are demonstrated. Some useful conclusions are finally presented in Section V.

II. System and Channel Model

We consider a dual-hop system with a source node, $\mathbf{S}$, communicating with a destination node, $\mathbf{D}$, via a direct link as well as through L relay nodes, $\mathbf{R}_\ell$, as in Fig. 1 ($\ell = 1, 2, \ldots, L$). We assume that the receivers at the relays and destination have perfect channel state information (CSI) so that maximum-likelihood combining can be employed; also, no transmitter CSI is available at the source or relays. For having orthogonal transmission, a time-division channel allocation scheme with $L+1$ time slots is used [7], [8]. During the first time slot, $\mathbf{S}$ broadcasts its signal to the set of L-relay nodes and also directly to $\mathbf{D}$. We define the decoding set $\mathcal{C}$ as those relays with the ability to fully decode the source message. If the channel between the source and the relay node is strong enough to allow for successful decoding, the relay node is said to belong to $\mathcal{C}$. During the rest L time slots, the members of the decoding set first decode and then forward the source information to the destination node in a predetermined order.

The mutually independent channel envelopes among $\mathbf{S} \longrightarrow \mathbf{D}$, $\mathbf{S} \longrightarrow \mathbf{R}_\ell$, and $\mathbf{R}_\ell \longrightarrow \mathbf{D}$, represented by $h_{\rm SD}, h_{{\rm SR}_\ell}$, and $h_{{\rm R}_\ell{\rm D}}$, respectively, are modeled as Nakagami-m random variables (RVs), while additive white Gaussian noise with N_0 single side power spectral density is also assumed. The corresponding powers $X = h_{\rm SD}^2$, $Z_\ell = h_{{\rm SR}_\ell}^2$, and $Y_\ell = h_{{\rm R}_\ell{\rm D}}^2$ are gamma distributed RVs. Let us define the instantaneous

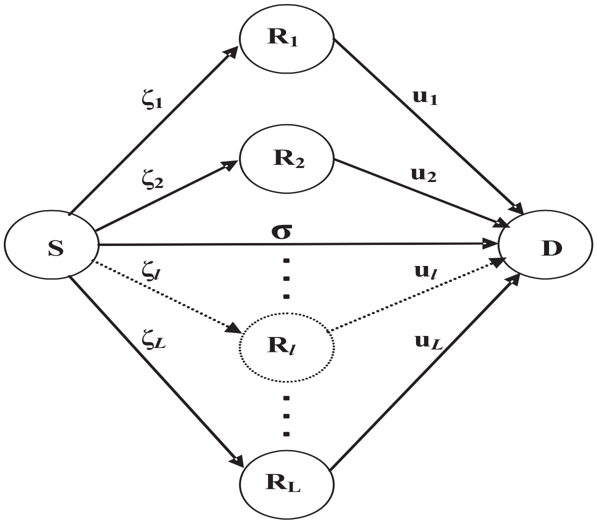

Fig. 1. Block diagram of an L-relay dual-hop plus a direct link wireless network.

SNR of the second hop of the ℓ-th link as

$$\upsilon_\ell = Y_\ell^2\,\frac{E_s}{N_0}$$

with E_s being the transmitted by **S** symbol energy. This SNR also follows the gamma distribution with probability density function (PDF) given by[1]

$$f_{\upsilon_\ell}(\gamma) = \frac{B_\ell^{m_\ell}}{\Gamma\left(m_\ell\right)}\,\gamma^{m_\ell - 1}\,\exp\left(-\gamma\, B_\ell\right) \tag{1}$$

where $m_\ell \geq 1/2$ denotes the Nakagami-m fading parameter, $\Gamma\left(\cdot\right)$ is the gamma function [11, eq. (8.310/1)], and $B_\ell = m_\ell/\overline{\upsilon}_\ell$, with

$$\overline{\upsilon}_\ell = \mathbb{E}\langle Y_\ell^2\rangle\,\frac{E_s}{N_0}$$

($\mathbb{E}\langle\cdot\rangle$ denotes expectation) being the corresponding average SNR. In our research, we assume that B_ℓ's are either equal, i.e., $B_\ell = B\ \forall\ell$, or distinct, i.e., $B_\ell \neq B_k\ \forall\ell \neq k$. Also, the cumulative distribution function (CDF) of υ_ℓ is

$$F_{\upsilon_\ell}(\gamma) = 1 - \frac{1}{\Gamma\left(m_\ell\right)}\Gamma\left(m_\ell, B_\ell\,\gamma\right) \tag{2}$$

where $\Gamma\left(\cdot,\cdot\right)$ is the upper incomplete gamma function [11, eq. (8.350/2)]. We also define the instantaneous SNR of the direct link and the first hop of the ℓ-th link as $\sigma = X^2\,E_s/N_0$ and $\zeta_\ell = Z_\ell^2\,E_s/N_0$, which similarly to υ_ℓ are gamma distributed with parameters $(m_0, \overline{\sigma})$ and $(n_\ell, \overline{\zeta}_\ell)$, respectively.

III. Outage Probability of DF Relaying

The mutual information between the source and ℓ-th relay node is

$$I_\ell = \frac{1}{L+1}\,\log\left[1 + \left(\frac{\mathcal{S}}{\mathcal{N}}\right)\,Z_\ell\right] \tag{3}$$

[1]The parameters set of a gamma distributed RV of the form of (1) is denoted as $(m_\ell, \overline{\upsilon}_\ell)$.

where $(\mathcal{S}/\mathcal{N})$ is the transmit SNR. Thus, if I_ℓ is higher than a fixed spectral efficiency, $\mathcal{R}$, $\mathbf{R}_\ell$ is able to successfully decode the transmitted signal from **S** and belongs to the decoding set $\mathcal{C}$. The end-to-end mutual information, when DF relaying is used, can then be expressed as

$$I_{\text{DF}} = \frac{1}{L+1}\,\log\left[1 + \left(\frac{\mathcal{S}}{\mathcal{N}}\right)\left(X + \sum_{c\in\mathcal{C}} X_c\right)\right]. \tag{4}$$

By the total probability law, the end-to-end OP is given by

$$P_{\text{out}} = \Pr\left[I_{\text{DF}} < \mathcal{R}\right] = \sum_{c} \Pr\left[I_{\text{DF}} < \mathcal{R}|\mathcal{C}\right]\,\Pr\left[\mathcal{C}\right]. \tag{5}$$

Similarly to [9], we view the wireless L-relay plus the direct channel as effectively having $L+1$ paths between the source and destination. Let the zeroth path represents the $\mathbf{S}\longrightarrow\mathbf{D}$ direct link and the ℓ-th path represents the $\mathbf{S} \longrightarrow \mathbf{R}_\ell \longrightarrow \mathbf{D}$ cascaded link. Also, let γ_ℓ denotes the received SNR of both the fading on $\mathbf{S} \longrightarrow \mathbf{R}_\ell$ and $\mathbf{R}_\ell \longrightarrow \mathbf{D}$. Then, γ_ℓ has PDF (link active: la, link not active: lna)

$$f_{\gamma_\ell}(\gamma) = f_{\gamma_\ell|\text{lna}}(\gamma)\,\Pr\left[\text{lna}\right] + f_{\gamma_\ell|\text{la}}(\gamma)\,\Pr\left[\text{la}\right] \tag{6}$$

with $f_{\gamma_\ell|\text{la}}(\gamma)$ and $f_{\gamma_\ell|\text{lna}}(\gamma)$ being the conditional PDFs, and $\Pr[\text{la}]$ and $\Pr[\text{lna}]$ the associated probabilities the ℓ-th link being active and not active, respectively. When the ℓ-th link is not active, $f_{\gamma_\ell|\text{lna}}(\gamma) = \delta(0)$, with $\delta\left(\cdot\right)$ being the Kronecker delta function, and the associated probability is

$$A_\ell = \Pr\left[\zeta_\ell < \frac{2^{(L+1)\mathcal{R}} - 1}{(\mathcal{S}/\mathcal{N})}\right]$$

which can be expressed as

$$A_\ell = F_{\zeta_\ell}\left[\frac{2^{(L+1)\,\mathcal{R}} - 1}{(\mathcal{S}/\mathcal{N})}\right] \tag{7}$$

with the $F_{\zeta_\ell}\left(\cdot\right)$ denoting the CDF of ζ_ℓ. The above equation represents the probability that the ℓ-th relay node does not belong to $\mathcal{C}$. Note that since the direct $\mathbf{S}\longrightarrow\mathbf{D}$ path is not linked via a relay, $A_0 = 0$. The probability that the ℓ-th link is up is $1 - A_\ell$, and the associated conditional PDF is $f_{\gamma_\ell|\text{la}}(\gamma) = f_{\upsilon_\ell}(\gamma)$. Hence, by substituting (1) in (6), yields[2]

$$f_{\gamma_\ell}(\gamma) = A_\ell\,\delta(0) + (1 - A_\ell)\,B_\ell^{m_\ell}\,\frac{\gamma^{m_\ell - 1}}{\Gamma\left(m_\ell\right)}\,\exp\left(-\gamma\,B_\ell\right). \tag{8}$$

The above PDF represents the ℓ-th cascaded link from source to destination.

The end-to-end OP can be written as

$$P_{\text{out}} = \Pr\left(\sum_{\ell=0}^{L} \gamma_\ell < \gamma_{\text{th}}\right) \tag{9}$$

with

$$\gamma_{\text{th}} = \frac{2^{(L+1)\,\mathcal{R}} - 1}{(\mathcal{S}/\mathcal{N})}$$

being the outage threshold. Equation (9) shows that the OP in this model is just the CDF of the end-to-end SNR $\gamma_{\text{end}} =$

[2]Hereafter, $\ell = 0, 1, \ldots, L$.

$\sum_{\ell=0}^{L}\gamma_\ell$ evaluated at $\gamma_{\rm th}$ (by design). In order to extract this CDF, we first obtain the moment-generating function (MGF) of γ_ℓ, i.e., $M_{\gamma_\ell}(s)=\mathbb{E}\langle\exp(-s\,\gamma_\ell)\rangle$. Using (8), this MGF can be obtained as

$$M_{\gamma_\ell}(s)=A_\ell+\frac{1-A_\ell}{(1+s/B_\ell)^{m_\ell}}. \tag{10}$$

Moreover, since γ_ℓ's are independent, the MGF of $\gamma_{\rm end}$ can be expressed as

$$M_{\gamma_{\rm end}}(s)=\prod_{\ell=0}^{L}M_{\gamma_\ell}(s) \tag{11}$$

while the end-to-end OP can be extracted as

$$P_{\rm out}=\mathbb{L}^{-1}\left\{\frac{1}{s}\,M_{\gamma_{\rm end}}(s);t\right\}\Big|_{t=\gamma_{\rm th}} \tag{12}$$

with $\mathbb{L}^{-1}\{\cdot;\cdot\}$ denoting inverse Laplace transformation.

A. Distinct B_ℓ's

We consider the case of distinct B_ℓ's, i.e., $B_\ell\neq B_k\ \forall\ \ell\neq k$, and integer-order fading parameters. By substituting (10) and (11) in (12), a product of L+1 terms $\prod_{k=0}^{L}(1+B_k)$ is formed. This product can be expanded using the following useful formula that we have developed

$$\prod_{k=0}^{L}(1+B_k)=1+\sum_{k=0}^{L}\sum_{\lambda_0=0}^{L-k}\sum_{\lambda_1=\lambda_0+1}^{L-k+1}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{L}\prod_{n=0}^{k}B_{\lambda_n}$$

helping us to simplify the integration appears in (12). Taking into account that the fading parameters are considered as integers, the integration theory of rational functions [11, Section 2.102] can be applied. Moreover, inverse laplace transformations of the form $\mathbb{L}^{-1}\{(s+B_\ell)^{-q}/s;t\}$, with q integer, need to be performed, which using [11, Section 17.1] can be solved as

$$\mathbb{L}^{-1}\left\{\frac{1}{s}\,(s+B_\ell)^{-q};t\right\}=\frac{\gamma\left(q,B_\ell\,t\right)}{\Gamma(q)\,B_\ell^q}$$

with $\gamma(q,x)$ being the lower incomplete gamma function, $\gamma(q,x)=\Gamma(q)-\Gamma(q,x)$, which can be further simplified to standard functions [11, eq. (8.352/2)]. Hence, after a lot of algebraic manipulations, a closed-form expression for the end-to-end OP of DF relaying with L-relay nodes over Nakagami-m with integer-order values for m_ℓ's can be obtained as

$$P_{\rm out}=\left(\prod_{i=0}^{L}A_i\right)\left\{1+\sum_{k=0}^{L}\sum_{\lambda_0=0}^{L-k}\sum_{\lambda_1=\lambda_0+1}^{L-k+1}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{L}\left(\prod_{n=0}^{k}\frac{1-A_{\lambda_n}}{A_{\lambda_n}}\,B_{\lambda_n}^{m_{\lambda_n}}\right)\sum_{p=0}^{k}\sum_{q=1}^{m_{\lambda_p}}\frac{\xi_q}{B_{\lambda_p}^q}\times\left[1-\exp\left(-\gamma_{\rm th}\,B_{\lambda_p}\right)\sum_{l=0}^{q-1}\frac{1}{l!}\left(\gamma_{\rm th}\,B_{\lambda_p}\right)^l\right]\right\} \tag{13a}$$

with

$$\xi_q=\frac{\Psi_p(s)^{(m_{\lambda_p}-q)}|_{s=-B_{\lambda_p}}}{\left(m_{\lambda_p}-q\right)!} \tag{13b}$$

and

$$\Psi_p(s)=\left(s+B_{\lambda_p}\right)^{m_{\lambda_p}}\prod_{n=0}^{k}(B_{\lambda_n}+s)^{-m_{\lambda_n}}. \tag{13c}$$

Note that for Rayleigh fading, i.e., $m_\ell=1\ \forall\ell$, (13) agrees with [9, eq. (10a)].

B. Equal B_ℓ's

For equal B_ℓ's, i.e., $B_\ell=B\ \forall\ell$, following a similar procedure as that in Section III-A, a closed-form expression for the end-to-end OP of DF relaying over Nakagami-m with arbitrary fading parameters can be obtained as

$$P_{\rm out}=\left(\prod_{i=0}^{L}A_i\right)\Bigg[1+\sum_{k=0}^{L}\sum_{\lambda_0=0}^{L-k}\sum_{\lambda_1=\lambda_0+1}^{L-k+1}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{L}\left(\prod_{n=0}^{k}\frac{1-A_{\lambda_n}}{A_{\lambda_n}}\right)\times\gamma\left(\sum_{n=0}^{k}m_{\lambda_n},B\,\gamma_{\rm th}\right)/\Gamma\left(\sum_{n=0}^{k}m_{\lambda_n}\right)\Bigg]. \tag{14}$$

Moreover, using [11, eq. (8.352/2)] for integer values of m_ℓ, (14) simplifies to

$$P_{\rm out}=\left(\prod_{i=0}^{L}A_i\right)\Bigg\{1+\sum_{k=0}^{L}\sum_{\lambda_0=0}^{L-k}\sum_{\lambda_1=\lambda_0+1}^{L-k+1}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{L}\left(\prod_{n=0}^{k}\frac{1-A_{\lambda_n}}{A_{\lambda_n}}\right)\times\left[1-\exp\left(-\gamma_{\rm th}\,B\right)\sum_{l=0}^{\sum_{n=0}^{k}m_{\lambda_n}-1}\frac{\left(\gamma_{\rm th}\,B\right)^l}{l!}\right]\Bigg\}. \tag{15}$$

IV. Numerical Results and Discussion

Without loss of generality, we assume equal fading parameters for both first and second hop for each one of the L links, i.e., $m_\ell=n_\ell$ and $\overline{\upsilon}_\ell=\overline{\zeta}_\ell\ \forall\ell$. Figs. 2 and 3 show the end-to-end OP with distinct and id Nakagami-m channel statistics, respectively, as a function of the transmit SNR of the **S**$\longrightarrow$**D** direct link,$(\mathcal{S}/\mathcal{N})$. In Fig. 2, a few curves are plotted for $L=2$ nodes and $\overline{\zeta}_1=0.75\,\overline{\sigma}$, $\overline{\zeta}_2=0.5\,\overline{\sigma}$. Moreover, in Fig. 3, $L=2$ and 4 nodes with $\overline{\zeta}_\ell=\overline{\sigma}$ and $m_\ell=m_0=1$, 1.8, 3.2 are considered. As expected, the OP decreases as the transmit SNR increases. Interestingly enough, the presence of many nodes does not necessarily mean improved OP as also observed in [9]. For example, from Fig. 3, a certain SNR value can be found, for lower values of which, the OP is improved for two nodes compared to four, e.g., for $m_0=1$, this value is at 11.5dB. As m_0 increases, this SNR value, for which the curves for L=2 and 4 are crossed, moves towards lower OP values. Hence, we may conclude that depending on

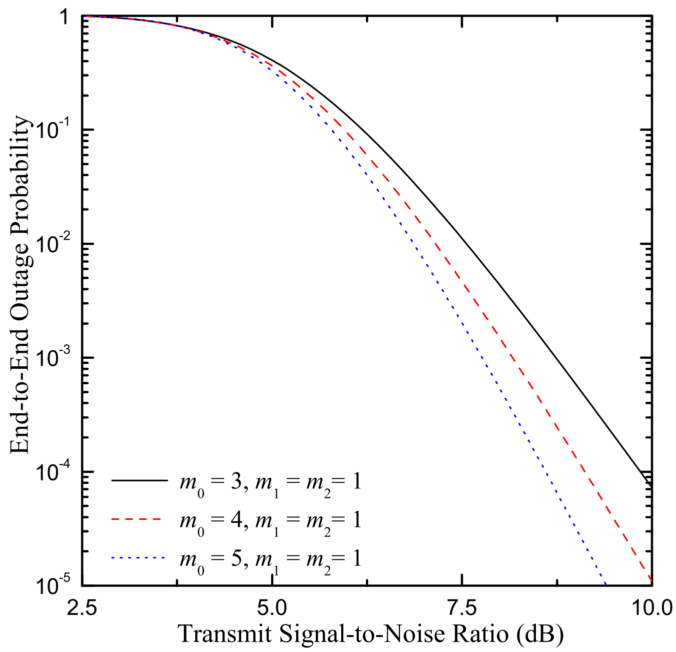

Fig. 2. End-to-end OP with two nodes operating in a Nakagami-m fading environment with distinct B_ℓ's ($\mathcal{R} = 1$ bit/sec/Hz).

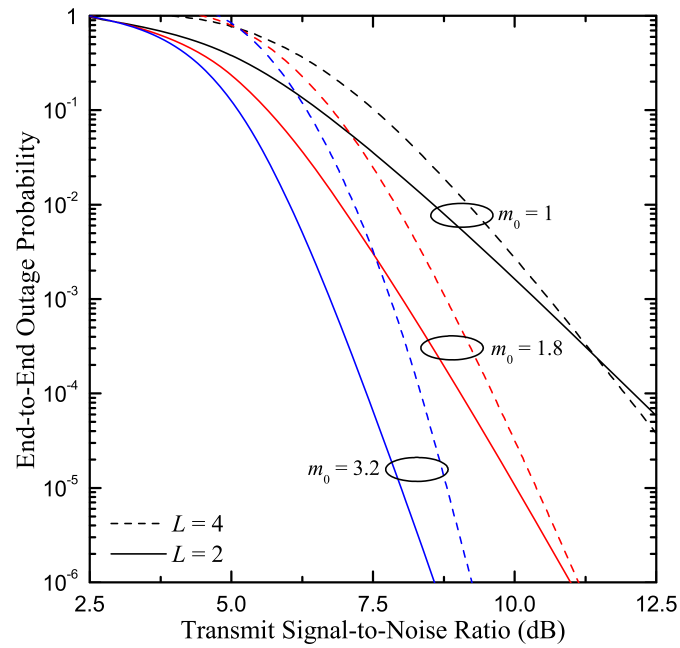

Fig. 3. End-to-end OP with two and four nodes operating in a Nakagami-m fading environment with equal B_ℓ's ($\mathcal{R} = 1$ bit/sec/Hz).

the channels conditions, an optimal number of relaying nodes should be utilized, when the DF relaying protocol is employed.

V. Conclusions

Closed-form outage probability expressions were presented, for an L-relays dual-hop plus a direct link wireless network, in which the DF relaying protocol is employed. Our analysis significantly extended previous results, considering a Nakagami-m fading environment with either equal or distinct second hops fading parameter to average power ratios. Various numerical examples illustrated the proposed analysis, where an interesting finding was that depending on the channels conditions, an optimal number of relaying nodes should be utilized, when the DF relaying protocol is employed.

References

[1] M. O. Hasna and M.-S. Alouini, "Outage probability of multihop transmission over Nakagami fading channels," *IEEE Commun. Lett.*, vol. 7, no. 5, pp. 216–218, May 2003.

[2] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1820–1830, Oct. 2004.

[3] G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, "Bounds for multihop relayed communications in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, no. 1, pp. 18–22, Jan. 2006.

[4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[5] G. K. Karagiannidis, "Performance bounds of multihop wireless communications with blind relays over generalized fading channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 498–503, Mar. 2006.

[6] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2004.

[7] Y. Zhao, R. Adve, and T. J. Lim, "Outage probability at arbitrary SNR with cooperative diversity," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 700–702, Aug. 2005.

[8] Y. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[9] N. C. Beaulieu and J. Hu, "A closed-form expression for the outage probability of decode-and-forward relaying in dissimilar Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 813–815, Dec. 2006.

[10] H. A. Suraweera, P. J. Smith, and J. Armstrong, "Outage probability of cooperative relay networks in Nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 10, no. 12, pp. 834–836, Dec. 2006.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.