Bit-Error-Rate Performance of Semiconductor Optical Amplifiers in Negative Exponential Fading

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***Abstract*—We present analytical relations for estimating the bit-error-rate performance of semiconductor optical amplifier assisted outdoor optical wireless systems in the presence of negative-exponential fading. The relations are derived by associating the optical signal and noise powers with the channel state and are utilized to assess the system performance in terms of first and order statistics. We demonstrate that both first order (average bit-error-rate and outage probability) and second order (average fade duration) statistics are improved by a significant factor, since the deployment of the amplifier leads to a considerably reduced receiver sensitivity.**

***Index Terms*—Bit-error-rate, outage probability, level crossing rate, average fade duration, negative-exponential fading, outdoor optical wireless, semiconductor optical amplifiers.**

I. Introduction

Outdoor optical wireless (OW) systems have been extensively studied with a goal to facilitate high capacity links in networking scenarios that can not be implemented or are simply too expensive to implement by conventional fiber optics, including broadband airborne/space communications [1] and semi-permanent terrestrial links [2]. The propagation of light through the atmosphere, however, is hindered by a number of factors, including rain, fog, snow, air pollution and temperature variations along the transmission path. Due to the volatile nature of the atmosphere, the optical power that is received at the OW receiver is not constant but it exhibits random time-varying fluctuations (scintillations) around its mean value, and whenever scintillations are intense enough to lower the received optical power under a certain threshold then the link is lost and a fade event occurs. The impact of scintillations is particularly detrimental on the OW link operation and typically a fade margin is included in the link budget with a goal to reduce the probability of a fade. Alternatively, a number of techniques can also be utilized to partially compensate for fades and beam focusing [3], aperture averaging [4], [5], spatial and temporal diversity [6], [7], coding [8], relaying [9] and amplification [10]–[12] are candidate techniques that have been proposed to effectively reduce the impact of fading and contribute to more reliable outdoor OW links.

In the current work we focus on assessing the fade compensation potential of semiconductor-optical-amplifiers (SOAs) in outdoor OW systems that operate in the saturated (negative-exponential) fading regime. This work is instigated by the observation that the optical amplifier improves the required receiver sensitivity due to the signal and noise beating process which generates electrical noise variances that add to the already existing thermal and shot noise [13], [14]. It is therefore of interest to establish whether this sensitivity improvement can be fully exploited in an outdoor OW system and determine in a quantitative manner the degree of the expected benefit. With his goal in mind, we derive a mathematical framework that evaluates the system performance in terms of the bit-error-rate (BER) that is attainable at each channel state, as defined by the negative-exponential statistics. We then utilize the proposed framework to analytically calculate metrics such as the average BER, the outage probability and the average fade duration of the SOA-assisted system and demonstrate that the aforementioned sensitivity improvement does have a significant impact on the system performance even in this challenging fading regime.

The rest of this paper is organized as follows: in Section II we present the mathematical model that describes the BER performance of the system in the presence of negative-exponential fading. The model takes into account all noise variances that arise from the signal and noise beating process and associates them with the channel state. This enables the treatment of the system BER as a random variable whose values are determined by the negative-exponential statistics. In the following section, we analytically present the first order statistics performance of the system, focusing on the average BER and outage probability. The presented results demonstrate that the SOA improves the average BER by one order of magnitude, while the outage probability is also reduced by over 94% in practical systems. Finally, in Section IV we present analytical results on the second order statistics of the system emphasizing on the average fade duration. We show that the predicted sensitivity improvement also positively affects the duration of fades, which is reduced by over 78%. Finally, Section V main points of the presented work and concludes the paper.

II. Noise Model and Bit-Error-Rate Calculation

In the current section we derive the mathematical model that will enable the BER assessment the SOA-assisted OW system. To this end, we first briefly detail the signal and noise properties of at the SOA output and then correlate the system BER with the OW channel state. We begin our analysis by considering that the optical power $P_{\rm in}$ of each pulse that is received by the OW system is a random variable and in the saturated fading regime this variable follows negative-

TABLE I
SYSTEM PARAMETERS

Parameter	Symbol	Value
Wavelength	λ	1550 nm
Line Rate	$T_b{}^{-1}$	10 Gb/s
SOA small-signal gain	G	20 or 30 dB
Saturation parameter	P_{sat}	1 mW
Population inversion factor	n_{sp}	4.0
Photodiode responsivity	R	1.25 A/W
Receiver temperature	T	300^o K
Resistor load	R_L	$100\,\Omega$
Electrical noise figure	F_n	3 dB
Electrical bandwidth	B_e	7 GHz
Optical bandwidth	B_o	50 GHz

exponential statistics with a probability density function given by

$$f_{P_{\rm in}}(z) = \frac{1}{\overline{P}_{\rm in}} \exp\left(-\frac{z}{\overline{P}_{\rm in}}\right), \tag{1}$$

where $\overline{P}_{\rm in}$ is the average received optical power. Due to the nonlinear amplification in the SOA, its output power $P_{\rm out}$ can be calculated by [15], [16]

$$P_{\rm out}(z) = P_{\rm sat}\, \log\left[1 + G\left(\exp\left(\frac{z}{P_{\rm sat}}\right) - 1\right)\right], \tag{2}$$

where G is the SOA small-signal gain, $P_{\rm sat}$ is the rate-dependent saturation parameter of the SOA

$$P_{\rm sat} = \frac{U_{\rm sat}}{T_b}, \tag{3}$$

$U_{\rm sat}$ is the SOA saturation energy and T_b is the bit duration. The SOA, however, also generates an optical noise component due to amplified spontaneous emission (ASE), which is described by the ASE spectral density

$$P_n = n_{sp}\, h\, \frac{c}{\lambda}, \tag{4}$$

where c is the vacuum light speed, h is the Planck constant, n_{sp} is the population inversion factor and λ denotes the wavelength. The optical signal and the ASE beat on the square-law detector of the receiver (photodiode) and as a result several electrical noise components appear at the photodiode output. The associated noise variances are denoted as thermal, shot, signal-spontaneous beating and spontaneous-spontaneous beating, and are calculated as

$$\sigma_{th}^2 = \frac{4\, k_B\, T\, F_n\, B_e}{R_L}, \tag{5a}$$

$$\sigma_{shot}^2(z) = 2\, q\, R\, \left(P_{\rm out}(z) + (G-1)\, P_n\, B_o\right)\, B_e, \tag{5b}$$

$$\sigma_{sig-sp}^2(z) = 4\, R^2\, P_{\rm out}(z)\, (G-1)\, P_n\, B_e, \tag{5c}$$

and

$$\sigma_{sp-sp}^2 = 2\, R^2\, \left((G-1)\, P_n\right)^2\, (2\, B_o - B_e)\, B_e, \tag{5d}$$

respectively. In (5), B_e and B_o are the electrical and optical bandwidths, respectively, R is the photodiode responsivity, T is the receiver temperature, k_B denotes the Boltzmann constant, F_n is the electric noise figure and R_L is the resistor load. All respective parameters and the values that are used for the presentation are summarized in Table I.

Given (2) and (5), the noise variances during the signal and noise powers during '1' and '0' symbols are directly calculated as

$$I_1(z) = R\, P_{\rm out}(z), \tag{6a}$$

$$\sigma_1^2(z) = \sigma_{th}^2 + \sigma_{shot}^2(z) + \sigma_{sig-sp}^2(z) + \sigma_{sp-sp}^2, \tag{6b}$$

and

$$I_0 = 0, \tag{7a}$$

$$\sigma_0^2 = \sigma_{th}^2 + \sigma_{shot}^2(0) + \sigma_{sp-sp}^2, \tag{7b}$$

respectively. Following the above, the BER performance of the system is evaluated by

$$BER(z) = \frac{1}{2}\, \text{erfc}\left(\frac{Q(z)}{\sqrt{2}}\right), \tag{8}$$

with

$$Q(z) = \frac{I_1(z)}{\sigma_0 + \sigma_1(z)}. \tag{9}$$

In (8) it is assumed that the receiver is capable of estimating the channel state (CSI-capable) and setting its decision threshold on a symbol-by-symbol fashion to [17]

$$I_{th}(z) = \frac{\sigma_0\, I_1(z)}{\sigma_0 + \sigma_1(z)}. \tag{10}$$

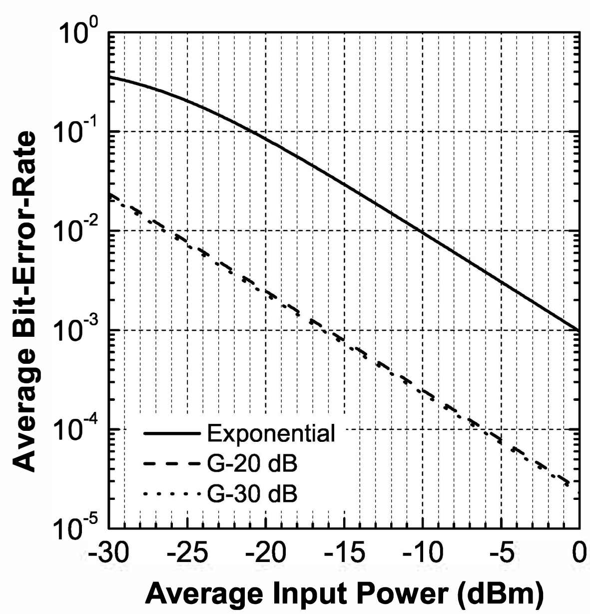

Fig. 1. Average bit-error-rate versus the average input power.

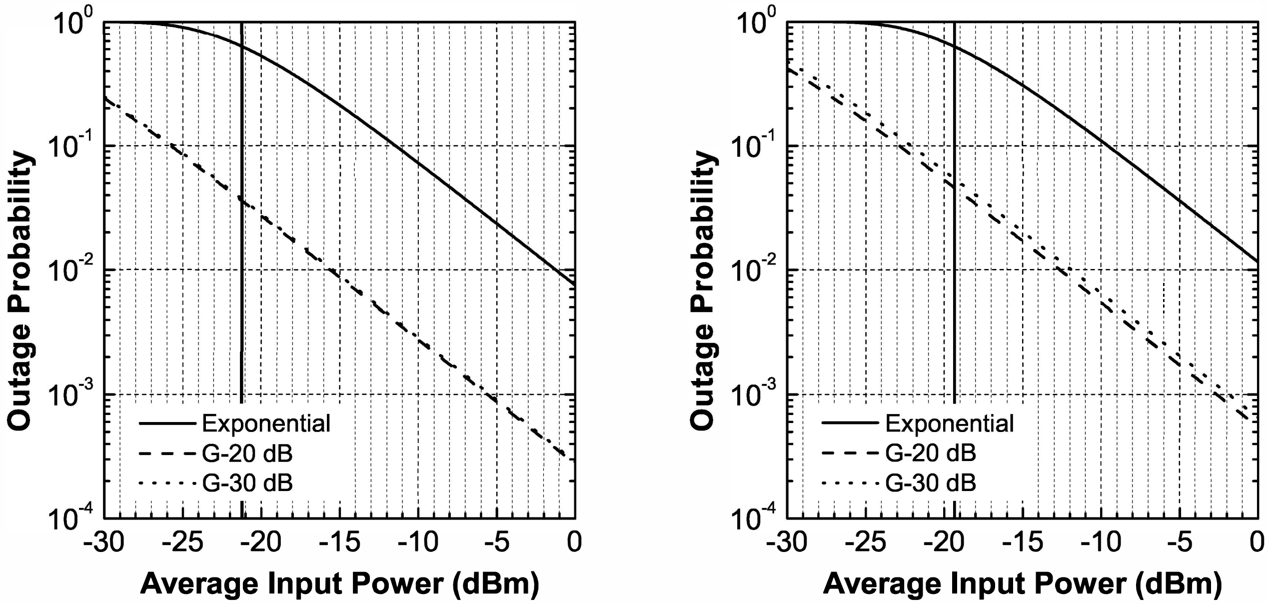

Fig. 2. Outage probability versus the average input power for $BER_0 = 10^{-3}$ (left) and $BER_0 = 10^{-6}$ (right). The solid vertical line corresponds to the receiver sensitivity without the SOA.

III. Performance Analysis of First Order Statistics

The current section discusses the performance improvement that is introduced by the SOA in the first order statistics of the OW system, and in particular the average BER and the outage probability. The average BER is calculated after integrating the BER variable over all possible negative-exponential channel states following

$$\overline{BER} = \frac{1}{2} \int_0^\infty \operatorname{erfc}\left(\frac{Q(z)}{\sqrt{2}}\right) f_{P_{\mathrm{in}}}(z)\, dz\,. \tag{11}$$

The integral is evaluated in a numerical fashion from the values of Table I and the results are plotted in Fig. 1 for two SOAs with 20 and 30 dB small signal gains. The figure clearly demonstrates a very significant improvement of the average BER of the system with the SOA and a 16 dB gain in the link budget is observed at any fixed BER level. This is expected from the sensitivity increase that is brought about by the SOA, which equals 14.3 dB at a BER of 10^{-3} and 12.5-13.8 dB at a BER of 10^{-6} given the numerical values under consideration. Moreover, the deployment of the SOA improves the average BER at least one order of magnitude for the presented input powers, thus the SOA adds to the reliability of the system and can contribute towards lowering the fade margin that is required in a practical system.

The exact contribution of the SOA to the link margin, however, can only be explored via the outage probability, since real-world systems are designed with a specific maximum acceptable BER level in mind, and additional techniques (mainly forward error correction) are introduced to recover from link errors. In these systems, an outage occurs whenever the BER at the receiver remains below the pre-defined target level BER_0, and following the analysis presented in the previous section the outage probability is calculated by

$$\Pr\left\{BER\,(z) > BER_0\right\}\,, \tag{12}$$

or equivalently by

$$\Pr\left\{z \le P_{\mathrm{s}}\right\} = \int_0^{P_{\mathrm{s}}} f_{P_{\mathrm{in}}}(z)\, dz \, = 1 \, - \, \exp\left(-\frac{P_{\mathrm{s}}}{\overline{P}_{\mathrm{in}}}\right)\,, \tag{13}$$

where P_{s} is the receiver sensitivity that is required to achieve BER_0. The equivalence of (12) and (13) is justified from the fact that BER_0 is exceeded whenever the input power remains below the corresponding sensitivity P_{s}. Furthermore, the corresponding receiver sensitivity is obtained after numerically solving

$$BER\,(P_{\mathrm{s}}) = \frac{1}{2} \operatorname{erfc}\left(\frac{Q(P_{\mathrm{s}})}{\sqrt{2}}\right) = BER_0\,. \tag{14}$$

The outage probability is plotted in Fig. 2, which illustrates the performance of a SOA-assisted receiver assuming acceptable BER levels of 10^{-3} and 10^{-6}. In agreement with the average BER results, the figure shows that a significant gain in the link budget is obtained by using the SOA, still the gain is a couple of dBs lower than this predicted by the average BER analysis. In fact, the link budget gain is exactly equal to the receiver sensitivity improvement that the SOA

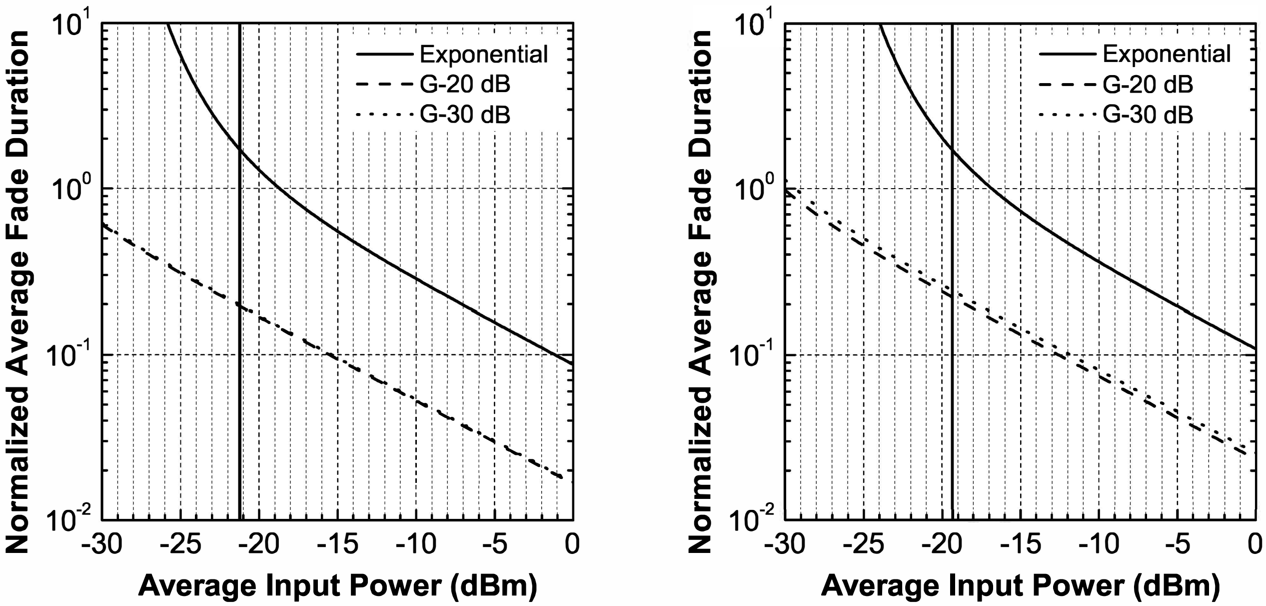

Fig. 3. Average fade duration versus the average input power for $BER_0 = 10^{-3}$ (left) and $BER_0 = 10^{-6}$ (right). The solid vertical line corresponds to the receiver sensitivity without the SOA.

introduces and equals 14.3 dB for a maximum BER of 10^{-3} and 12.5-13.8 dB for a maximum BER of 10^{-6}. The sensitivity increase is almost identical for both SOA gains at a BER of 10^{-3}, and as a result the plots in Fig. 2a practically coincide, while a slightly worse performance is observed for the 30 dB gain SOA at a BER of 10^{-6} due to the higher levels of ASE that is generated according to (5). With respect to the outage probability itself, the SOA-assisted system exhibits a reduced outage probability for all practically anticipated power levels. The reduction amounts to 96% for the 10^{-3} and over 94% for the 10^{-6} BER level, and exceeds the 90% reduction that is required to increase the system percentile availability by one additional '9'.

The analysis of the first order statistics establishes that the sensitivity improvement which is achieved by the SOA deployment translates directly into a link budget gain. As a result the SOA can provide a significant link margin that, depending on the design parameters, may balance out or even exceed the fade margin that is required in a high availability OW system. This potentially leads to an energy friendlier or more reliable system OW system, or a combination of both depending on the requirements. As an alternative, the link margin could be further utilized to extend the reach of the OW system even in this intense fading environment.

IV. Performance Analysis of Second Order Statistics

Apart from the average BER and the outage probability, which provide a measure of the OW link reliability, it is equally important to evaluate the second order statistics of the SOA-assisted system. In this work we give emphasis to the average fade duration (AFD) which is defined as the average duration that the system remains below the desired BER level BER_0 after a fade event. The AFD affects the OW system latency and its evaluation is particularly useful when considering temporal diversity techniques, since it provide an estimate of the required buffering, as well as when designing link layer automatic request protocols in which the frame and window sizes should be tailored so that a full-window transmission exceeds the fade duration.

The AFD is calculated from the outage probability (Eq. (12)) and the level crossing rate (LCR) of the link. For negative-exponential fading the LCR (normalized over the maximum Doppler shift) is given by

$$LCR\left(P_{\mathrm{s}}\right) = \exp\left(-\frac{P_{\mathrm{s}}}{\overline{P}_{\mathrm{in}}}\right)\sqrt{\left(\frac{P_{\mathrm{s}}}{\overline{P}_{\mathrm{in}}}\right)}. \qquad (15)$$

As we detail in [18], it is sufficient to evaluate the LCR at the receiver sensitivity P_{s} that is the required to attain the desired BER level BER_0 following (14). This results simplifies the calculation of the AFD, which is obtained in a straight-forward manner as

$$AFD\left(P_{\mathrm{s}}\right) = \frac{\Pr\left\{z \le P_{\mathrm{s}}\right\}}{LCR\left(P_{\mathrm{s}}\right)} = \frac{\exp\left(\frac{P_{\mathrm{s}}}{\overline{P}_{\mathrm{in}}}\right) - 1}{\sqrt{\left(\frac{P_{\mathrm{s}}}{\overline{P}_{\mathrm{in}}}\right)}}. \qquad (16)$$

Eq. (16) is plotted in Fig. 3 for a system with maximum BER level of 10^{-3} and 10^{-6}. It can be verified from the figure that the SOA has a significant impact on the OW system AFD, owing to the expected sensitivity improvement. As a result, the link margin of 14.3 dB for the BER of 10^{-3} and 12.5-13.8 dB for the BER of 10^{-6} that was calculated in the outage probability analysis is also valid for the AFD analysis. With respect to the AFD itself, an AFD improvement of over 80% is predicted for the 10^{-3} BER level, while the corresponding improvement is marginally reduced to 78% for the 10^{-6} BER level.

Following the above, the SOA is well capable of drastically reducing both the probability of a fade and its average duration thus enabling a more reliable and lower latency outdoor OW system. In addition, the same link margin that is predicted for the outage probability in the first order statistics analysis is also observed for the AFD and the LCR, although the LCR-related results are not included in the presentation for brevity purposes. As a result the SOA-assisted receiver performance can be thoroughly described in a systematic fashion by the expected sensitivity improvement and the statistics that govern fading.

V. Conclusions and Future Work

We have evaluated in an analytical fashion the performance of a SOA-assisted outdoor OW link in the negative-exponential fading regime. To this end, we provided a mathematical framework that fully describes that signal and noise components at the SOA output and enables the treatment of the system BER as a stochastic variable that depends of the channel state. Given the presented mathematical framework, we expressed key first and second order statistics as functions of the BER and demonstrated that the SOA is well capable of improving both categories. Regarding first order statistics, our results show that the average BER is lowered by at least one order of magnitude and that the outage probability is also reduced by 94%-96%. With respect to second order statistics, it has been demonstrated that the AFD is also improved by 78%-80%. Even though the presentation is limited within the negative-exponential (saturated) fading regime, the proposed framework can also be utilized to predict the SOA-assisted system performance in less challenging fading scenarios, by properly modifying the negative-exponential statistics with log-normal or gamma-gamma statistics.

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