The Third International Workshop on Signal Design and Its Applications in Communications
September 23 -27, 2007, Chengdu, China (IWSDA'07)

Triple-branch MRC Diversity in Weibull Fading Channels

Zafiro G. PAPADIMITRIOU†, Petros S. BITHAS†, P.Takis MATHIOPOULOS†,
Nikos C. SAGIAS††, Lazaros MERAKOS†††

† Institute for Space Applications and Remote Sensing, National Observatory of Athens, Metaxa & Vas. Pavlou Street
†† Institute of Informatics and Telecommunications, National Centre for Scientific Research Demokritos, Agia Paraskevi
††† Department of Informatics and Telecommunications, University of Athens, Athens 15784, Greece
E-mail: †{zpapadim,pbithas,mathio}@space.noa.gr, ††nsagias@ieee.org, †††merakos@di.uoa.gr

Abstract In this paper a performance analysis of triple-branch maximal ratio combining (MRC) diversity receivers operating over an arbitrarily correlated Weibull fading environment is presented. For the trivariate Weibull distribution infinite series representations for the joint probability density function (PDF) and the cumulative distribution function (CDF) are derived. Moreover, a novel analytical expression for the joint moment-generating function (MGF) is presented. It is assumed that the arbitrarily correlated variates do not necessarily have identical fading parameters and average powers. These series representations are readily applicable to the performance analysis of a triple-branch MRC receiver. Furthermore, the average bit error probability (ABEP) is then derived and analytically studied. The proposed mathematical analysis is accompanied by various numerical results, with parameters of interest the fading severity, the correlation and the power decay factor.

key words: *Triple-branch diversity, arbitrary correlation, maximal-ratio combining (MRC), Weibull fading.*

1. Introduction

Diversity reception techniques have received a great deal of research interest because of their possible use in mobile phones or base stations. In diversity reception it is frequently assumed that the antennae are sufficiently separated so that the combined signals are independent of each other [1]. However, this assumption is not always valid, e.g., for applications employing small size terminals. As a result the received signals are correlated with each other, which leads to a reduced diversity gain. Performance evaluation of correlated diversity systems requires the use of multivariate statistics [1].

Several spatial correlation models have been proposed for the performance analysis of various wireless systems. The most widely used are the constant and the exponential correlation models [1]. For the first one, the correlation depends on the distance among the combining antennas and therefore it is more suitable for equidistant antennas. For the second one, the correlation among the pairs of combined signals decays as the spacing among the antennas increases [1]. Thus, the exponential correlation model corresponds to the scenario of multichannel reception from equispaced diversity antennas. This model has been widely used for performance analysis of space diversity techniques [2], [3], [4] or multiple-input multiple-output (MIMO) systems [5]. However, in this paper, the arbitrary correlation model is used and analyzed. This is due to the fact that it is more generic and it doesn't involve antenna limitations, thus it includes the two previously mentioned models as special cases.

The Weibull distribution, although it was originally used in reliability and failure data analysis, it has been recently considered for wireless digital communication systems. The main motivation for this is the fact that it exhibits a very good fit to experimental fading channel measurements for both indoor and outdoor terrestrial radio propagation environments [6], [7]. Additionally, in [8], it was argued that the Weibull distribution could also been considered as an alternative channel model for land-mobile satellite systems.

Most of the published papers concerning multivariate fading statistics deal with Rayleigh and Nakagami-m distributions. For example in [9], closed-form expressions for the multivariate, exponentially correlated Nakagami-m joint probability density function (PDF) and the cumulative distribution function (CDF) were derived. Past works concerning multivariate distributions with arbitrary correlation can be found in [10], [11], [12], [13]. In [10] new infinite series representations for the PDF and the CDF of three and four arbitrarily correlated Rayleigh random variables were presented. In [11] closed-form expressions for the joint Nakagami-m multivariate PDF and CDF with arbitrary correlation were derived and the correlation matrix was approximated by a Green's matrix. Similarly in [12], the Green's matrix was used to approximate the correlation matrix of L-branch selection combining (SC) receivers with arbitrary correlation and the outage probability for lognormal fading channels has been obtained. In [13] expressions for multivariate Rayleigh and exponential PDFs generated from correlated Gaussian random variables were presented. Furthermore, in the same reference a general expression for the multivariate exponential characteristic function (CF), in terms of determinants, was also derived. More recently, the joint PDF, CDF and the moment-generating function (MGF) for the bivariate Weibull distribution have been analytically derived [14]. The multivariate Weibull distribution has also been studied for the exponential and the constant correlation case with equal average fading powers. However, to the best of the authors' knowledge, an analytical performance study of triple-branch MRC receivers operating over an arbitrarily correlated Weibull environment, has not yet been presented in the open technical literature. Thus, this is the subject of our paper, whereby we present the statistical characteristics of the trivariate Weibull distribution and then we apply the developed theoretical results to the performance analysis of a 3-branch MRC receiver.

2. Statistical Characteristics

Let h_ℓ, ($\ell = 1, 2, 3$) be the complex envelope of the Weibull fading model, written as

$$h_\ell = (X_\ell + jY_\ell)^{2/\beta_\ell} = G_\ell^{2/\beta_\ell} \tag{1}$$

where X_ℓ and Y_ℓ are the Gaussian in-phase and quadrature elements of the multipath components [14]. The magnitude of h_ℓ, i.e., $Z_\ell = |h_\ell|$, where $|\cdot|$ denotes absolute value, it can be expressed as a power transformation of a Rayleigh distributed random variable (RV), $R_\ell = |X_\ell + jY_\ell|$ as [15]

$$Z_\ell = R_\ell^{2/\beta_\ell}. \tag{2}$$

Let $G = \{G_1, G_2, G_3\}$ be joint complex Gaussian RVs with zero mean and positive definite covariance matrix $\mathbf{\Psi}$, having elements $\psi_{i\kappa} = E\langle G_i G_\kappa^*\rangle$, where $E\langle\cdot\rangle$ denotes expectation with $i,\kappa \in \{1,2,3\}$. $\mathbf{\Psi}$ can be expressed as

$$\mathbf{\Psi} = \begin{bmatrix} \Omega_1 & \rho_{12}\sqrt{\Omega_1\Omega_2} & \rho_{13}\sqrt{\Omega_1\Omega_3} \\ \rho_{12}\sqrt{\Omega_1\Omega_2} & \Omega_2 & \rho_{23}\sqrt{\Omega_2\Omega_3} \\ \rho_{13}\sqrt{\Omega_1\Omega_3} & \rho_{23}\sqrt{\Omega_2\Omega_3} & \Omega_3 \end{bmatrix} \tag{3}$$

where $\Omega_\ell = E\left\langle Z_\ell^{\beta_\ell}\right\rangle$ and ρ_{ij} is the correlation coefficient between the ith and the jth branch. The PDF of the trivariate Weibull distribution with arbitrary correlation is expressed as

$$\begin{aligned} &f_{Z_1,Z_2,Z_3}(z_1,z_2,z_3) = \frac{\beta_1\beta_2\beta_3\det(\mathbf{\Phi})}{z_1^{(2-\beta_1)/2} z_2^{(2-\beta_2)/2} z_3^{(2-\beta_3)/2}} \\ &\times \exp\left[-\left(z_1^{\beta_1}\phi_{11} + z_2^{\beta_2}\phi_{22} + z_3^{\beta_3}\phi_{33}\right)\right] \\ &\times \sum_{k=0}^{\infty} \epsilon_k (-1)^k \cos(k\chi) \\ &\times \sum_{\ell,m,n=0}^{\infty} \frac{|\phi_{12}|^{2\ell+k}}{\ell!(\ell+k)!} \frac{|\phi_{23}|^{2m+k}}{m!(m+k)!} \frac{|\phi_{31}|^{2n+k}}{n!(n+k)!} \\ &\times z_1^{\beta_1(\ell+n+k)+\beta_1/2} z_2^{\beta_2(\ell+m+k)+\beta_2/2} z_3^{\beta_3(m+n+k)+\beta_3/2} \end{aligned} \tag{4}$$

where ϵ_k is the Neumann factor ($\epsilon_0 = 1, \epsilon_k = 2$ for $k = 1,2,\cdots$) [10], $\chi = \chi_{12} + \chi_{23} + \chi_{31}$. Moreover, $\mathbf{\Phi}$ is the inverse covariance matrix in the case of the trivariate Weibull distribution, given by

$$\mathbf{\Phi} = \mathbf{\Psi^{-1}} = \begin{bmatrix} \phi_{11}, & \phi_{12}, & \phi_{13} \\ \phi_{12}^*, & \phi_{22}, & \phi_{23} \\ \phi_{13}^*, & \phi_{23}^*, & \phi_{33} \end{bmatrix} \tag{5}$$

for $\phi_{i\kappa} = |\phi_{i\kappa}|\exp(j\chi_{i\kappa})$. The equivalent infinite series representation for the CDF can be obtained as

$$\begin{aligned} &F_{Z_1,Z_2,Z_3}(z_1,z_2,z_3) \frac{\det(\mathbf{\Phi})}{\phi_{11}\phi_{22}\phi_{33}} \sum_{k=0}^{\infty} \epsilon_k (-1)^k \cos(k\chi) \\ &\times \sum_{\ell,m,n=0}^{\infty} C_{\ell,m,k}\, \nu_{12}^{\ell+k/2} \nu_{23}^{m+k/2} \nu_{31}^{n+k/2} \\ &\times \gamma\left(\delta_1, z_1^{\beta_1}\phi_{11}\right) \gamma\left(\delta_2, z_2^{\beta_2}\phi_{22}\right) \gamma\left(\delta_3, z_3^{\beta_3}\phi_{33}\right) \end{aligned} \tag{6}$$

where

$$C_{\ell,m,k} = \frac{1}{\ell!(\ell+k)!m!(m+k)!n!(n+k)!},$$

$$\nu_{i\kappa} = \frac{|\phi_{i\kappa}|^2}{\phi_{ii}\phi_{\kappa\kappa}},$$

and $\delta_1 = \ell+n+k+1$, $\delta_2 = m+\ell+k+1$, $\delta_3 = n+m+k+1$ with $\gamma(\cdot,\cdot)$ denoting the incomplete lower Gamma function [16, eq. (3.381/1)].

Furthermore, using (4), the moments of the $(\kappa_1+\kappa_2+\kappa_3)th$ order of Z_1, Z_2 and Z_3 can been derived as

$$\begin{aligned} &E\langle Z_1^{\kappa_1}, Z_2^{\kappa_2}, Z_3^{\kappa_3}\rangle = \frac{\det(\mathbf{\Phi})}{\phi_{11}^{1+\kappa_1/\beta_1}\phi_{22}^{1+\kappa_2/\beta_2}\phi_{33}^{1+\kappa_3/\beta_3}} \\ &\times \sum_{k=0}^{\infty} \epsilon_k (-1)^k \cos(k\chi) \sum_{\ell,m,n=0}^{\infty} C_{\ell,m,k} \\ &\times \nu_{12}^{\ell+k/2} \nu_{23}^{m+k/2} \nu_{31}^{n+k/2} \\ &\times \Gamma\left(\delta_1 + \frac{\kappa_1}{\beta_1}\right) \Gamma\left(\delta_2 + \frac{\kappa_2}{\beta_2}\right) \Gamma\left(\delta_3 + \frac{\kappa_3}{\beta_3}\right) \end{aligned} \tag{7}$$

where $\Gamma(\cdot)$ is the gamma function [16, eq. (8.310/1)].

It is useful to obtain the MGF of Z_ℓ in order to evaluate the average bit error probability (ABEP). It can be derived as

$$M_{Z_\ell}(s) = E\langle\exp(-sZ_\ell)\rangle. \tag{8}$$

By substituting (4) in (8), integrals of the following form need to be solved

$$\Upsilon(\xi,u) = \int_0^\infty x^{u-1}\exp(-x - \xi x^{\beta_\ell})dx. \tag{9}$$

Note here that u and ξ take arbitrary positive values. Following [17] the integrals of this kind can be obtained, under the constraint that β_ℓ is a rational number, as

$$\begin{aligned} \Upsilon(\xi,u) &= \frac{\lambda^u\sqrt{\frac{\kappa}{\lambda}}}{(\sqrt{2\pi})^{\kappa+\lambda-2}} \\ &\times G_{\lambda,\kappa}^{\kappa,\lambda}[\xi^\kappa \frac{\lambda^\lambda}{\kappa^\kappa}|_{0/\kappa,1/\kappa,\ldots,(\kappa-1)/\kappa}^{(1-u)/\lambda,(2-u)/\lambda,\ldots,(\lambda-u)/\lambda}] \end{aligned} \tag{10}$$

where $G[.]$ is the Meijer's G-function [16, eq. (9.301)], and κ and λ are positive integers so that

$$\frac{\lambda}{\kappa} = \beta_\ell. \tag{11}$$

A set of minimum values of κ and λ can be chosen according to the specific value of β_ℓ and the MGF can be expressed as

$$
\begin{aligned}
&M_{Z_1,Z_2,Z_3}(s_1,s_2,s_3)=\beta_1\beta_2\beta_3\det(\boldsymbol{\Phi})\sum_{\mathbf{k}=\mathbf{0}}^{\infty}\epsilon_{\mathbf{k}}(-\mathbf{1})^{\mathbf{k}}\\
&\times\cos(k\chi)\sum_{\ell,m,n=0}^{\infty}\frac{|\phi_{12}|^{2\ell+k}}{\ell!(\ell+k)!}\frac{|\phi_{23}|^{2m+k}}{m!(m+k)!}\frac{|\phi_{31}|^{2n+k}}{n!(n+k)!}\\
&\times\frac{1}{s_1^{\beta_1(\ell+n+k+1)}s_2^{\beta_2(\ell+m+k+1)}s_3^{\beta_3(m+n+k+1)}}\\
&\times\Upsilon\left(\frac{\phi_{11}}{s_1^{\beta_1}},\beta_1(\ell+n+k+1)\right)\\
&\times\Upsilon\left(\frac{\phi_{22}}{s_2^{\beta_2}},\beta_2(\ell+m+k+1)\right)\\
&\times\Upsilon\left(\frac{\phi_{33}}{s_3^{\beta_3}},\beta_3(m+n+k+1)\right).
\end{aligned}
\tag{12}
$$

3. Performance Analysis of MRC Receivers

Let us now consider a MRC diversity receiver with three branches operating over correlated Weibull fading channels. Let $\zeta_\ell = wh_\ell + n_\ell$ be the received baseband signal at the ℓth input branch, where w is the transmitted complex symbol, $E_s = E\langle|w|^2\rangle$ is the transmitted average symbol energy, with $|\cdot|$ denoting absolute value, and n_ℓ is the additive white Gaussian noise (AWGN) having a single-sided power spectral density N_0.

The instantaneous SNR per symbol at the ℓth diversity branch is given by

$$\gamma_\ell = Z_\ell^2\frac{E_s}{N_0}. \tag{13}$$

Moreover, the corresponding average SNR can be expressed as

$$\overline{\gamma}_\ell = E\langle Z_\ell^2\rangle\frac{E_s}{N_0} = \Gamma(d_{2,\ell})\Omega_\ell^{2/\beta_\ell}\frac{E_s}{N_0} \tag{14}$$

where $d_{\tau,\ell} = 1 + \tau/\beta_\ell$ with τ taking non-negative values. Expressions for the statistics of γ_ℓ can be easily derived by replacing β_ℓ with $\beta_\ell/2$ and Ω_ℓ with $(\alpha_\ell\overline{\gamma}_\ell)^{\beta_\ell/2}$, in the corresponding expressions for the fading envelope Z_ℓ, where $\alpha_\ell = 1/\Gamma(d_{2,\ell})$.

3.1 Average Bit Error Probability (ABEP)

Using (12), the MGF of the SNR for the trivariate Weibull distribution with arbitrary correlation can be expressed as

$$
\begin{aligned}
&M_{\gamma_1,\gamma_2,\gamma_3}(s_1,s_2,s_3)=\frac{\beta_1\beta_2\beta_3\det(\boldsymbol{\Phi'})}{8}\sum_{k=0}^{\infty}\epsilon_k(-1)^k\\
&\times\cos(k\chi')\sum_{\ell,m,n=0}^{\infty}\frac{|\phi'_{12}|^{2\ell+k}}{\ell!(\ell+k)!}\frac{|\phi'_{23}|^{2m+k}}{m!(m+k)!}\frac{|\phi'_{31}|^{2n+k}}{n!(n+k)!}\\
&\times\frac{1}{s_1^{\beta_1(\ell+n+k+1)/2}s_2^{\beta_2(\ell+m+k+1)/2}s_3^{\beta_3(m+n+k+1)/2}}\\
&\times\Upsilon\left(\frac{\phi'_{11}}{s_1^{\beta_1/2}},\frac{\beta_1}{2}(\ell+n+k+1)\right)\\
&\times\Upsilon\left(\frac{\phi'_{22}}{s_2^{\beta_2/2}},\frac{\beta_2}{2}(\ell+m+k+1)\right)\\
&\times\Upsilon\left(\frac{\phi'_{33}}{s_3^{\beta_3/2}},\frac{\beta_3}{2}(m+n+k+1)\right)
\end{aligned}
\tag{15}
$$

where $\boldsymbol{\Phi'} = \boldsymbol{\Psi'^{-1}}$ and $\boldsymbol{\Psi'}$ is the covariance matrix given in (3) after the substitutions described previously. The instantaneous SNR per symbol at the output of a triple branch MRC receiver is

$$\gamma_{mrc} = \sum_{i=1}^{3}\gamma_i = \gamma_1 + \gamma_2 + \gamma_3. \tag{16}$$

Using (15), the MGF of the triple branch MRC output with arbitrary correlation can be obtained as

$$M_{\gamma_{mrc}}(s_1,s_2,s_3) = M_{\gamma_1,\gamma_2,\gamma_3}(s,s,s) \tag{17}$$

The ABEP can be obtained using the MGF-based approach [1]. Specifically, the above expression is very useful in the direct calculation of the ABEP of noncoherent binary frequency-shift keying (NBFSK) and binary differential phase-shift keying (BDPSK) modulation signaling. For other types of modulation formats, including binary phase-shift keying (BPSK), M-ary phase-shift keying (M-PSK), M-ary quadrature amplitude modulation (M-QAM), M-ary amplitude modulation (M-AM) and M-ary differential phase-shift keying (M-DPSK), numerical integration is needed in order to evaluate single integrals with finite limits and integrands composed of elementary functions [18].

3.2 Output SNR Moments

By using the multinomial identity [16, eq. (1.111)], the corresponding wth order moment of γ_{mrc}, $\mu_w = E\langle\gamma_{mrc}^w\rangle$, can be obtained as

$$\mu_w = w!\sum_{\kappa_1,\kappa_2,\kappa_3=0}^{w}\frac{E\langle\gamma_1^{\kappa_1}\gamma_2^{\kappa_2}\gamma_3^{\kappa_3}\rangle}{\kappa_1!\kappa_2!\kappa_3!} \tag{18}$$

with $\kappa_1+\kappa_2+\kappa_3 = w$. Substituting (7), (13) and (14) in (18), it yields

$$\mu_w = w! \sum_{\kappa_1,\kappa_2,\kappa_3=0}^{w} \frac{1}{\kappa_1!\kappa_2!\kappa_3!}$$
$$\times \frac{\det(\mathbf{\Phi}')}{{\phi'}_{11}^{1+2\kappa_1/\beta_1}{\phi'}_{22}^{1+2\kappa_2/\beta_2}{\phi'}_{33}^{1+2\kappa_3/\beta_3}} \sum_{k=0}^{\infty} \epsilon_k (-1)^k \cos(k\chi')$$
$$\times \sum_{\ell,m,n=0}^{\infty} C_{\ell,m,k}\, {\nu'}_{12}^{\ell+k/2} {\nu'}_{23}^{m+k/2} {\nu'}_{31}^{n+k/2}$$
$$\times \Gamma\left(\delta_1 + \frac{2\kappa_1}{\beta_1}\right) \Gamma\left(\delta_2 + \frac{2\kappa_2}{\beta_2}\right) \Gamma\left(\delta_3 + \frac{2\kappa_3}{\beta_3}\right). \tag{19}$$

where

$$\nu'_{i\kappa} = \frac{|\phi'_{i\kappa}|^2}{\phi'_{ii}\phi'_{\kappa\kappa}}.$$

3.3 Outage Probability Evaluation

The outage probability is defined as the probability that γ_{mrc} falls below a predetermined threshold γ_{th} and is given by

$$P_{out}(\gamma_{th}) = F_{\gamma_{mrc}}(\gamma_{th}) = L^{-1}\left[\frac{M_{\gamma_{mrc}}(s)}{s}; \gamma_{mrc}\right] |_{\gamma_{mrc}=\gamma_{th}} \tag{20}$$

where $F_{\gamma_{mrc}}(.)$ is the CDF of γ_{mrc} and $L^{-1}[\cdot;\cdot]$ denotes the inverse Laplace transform. Because of the complicated form of $M_{\gamma_{mrc}}(s)/s$ in (20), the so-called Euler summation-based algorithm for the inversion of CDFs may be applied [1, Appendix 9B.1].

4. Performance Evaluation Results

Using the previous mathematical analysis various performance evaluation results of triple-branch MRC receivers operating over an arbitrarily correlated Weibull fading environment have been obtained. Without loss of generality it is assumed that $\beta_\ell = \beta \,\forall\, \ell$ and BDPSK signaling is considered. Furthermore, non-identical distributed Weibull channels, i.e., $\overline{\gamma}_\ell = \overline{\gamma}_1 \exp(-(\ell-1)\delta)$ where δ is the power decay factor are assumed [8]. The presented performance evaluation results have been obtained by numerically evaluating (17).

In Fig. 1, the ABEP is plotted as a function of the average input SNR for the special case of exponential correlation with $\beta = 2.5$ and $\rho = 0.5$ and for different values of the power decay factor δ. In Fig. 2, the ABEP is also plotted as a function of the average input SNR assuming $\rho_{12} = 0.6, \rho_{13} = 0.4$ and $\rho_{23} = 0.3$ and again for different values of δ. The obtained results in both figures show clearly that the system's performance degrades with increasing δ.

In Fig. 3, the ABEP is presented for $\rho_{12} = 0.5, \rho_{13} = 0.1$ and $\rho_{23} = 0.7$ and for different values of β and δ. It is evident that an increase of β improves the ABEP.

5. Conclusions

A performance analysis for the trivariate Weibull distribution with arbitrary correlation was presented. Initially, infinite series representations for the joint PDF, CDF and MGF were derived. These theoretical results were applied to analyze the

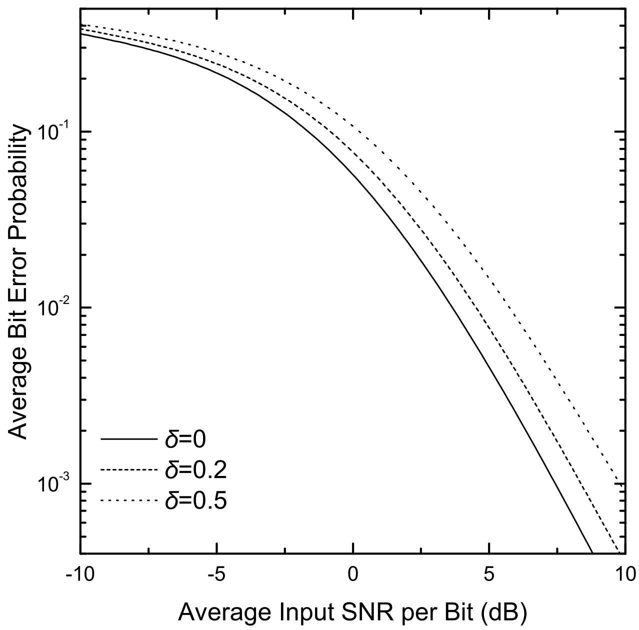

Fig. 1 ABEP versus the average input SNR for the exponential correlation fading model, $\beta = 2.5$ and $\rho = 0.5$.

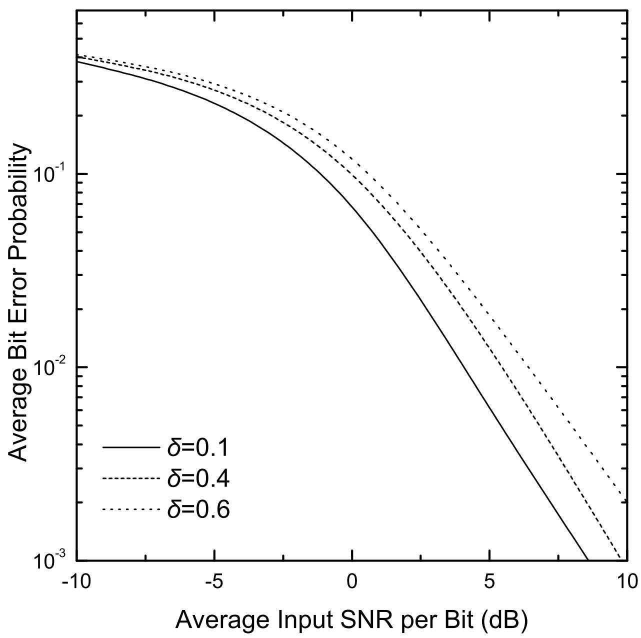

Fig. 2 ABEP versus the average input SNR for $\beta = 2.5$, $\rho_{12} = 0.6, \rho_{13} = 0.4$ and $\rho_{23} = 0.3$.

performance of triple-branch MRC receivers, operating in an arbitrarily correlated fading environment. The ABEP, one of the most useful performance criteria, was studied. Various performance evaluation results were presented showing the effects of fading severity, correlation coefficient and the power decay factor on the system's performance.

Acknowledgments

This paper is part of the 03ED910 research project, implemented within the framework of the "Reinforcement Programme of Human Research Manpower" (PENED) and co-

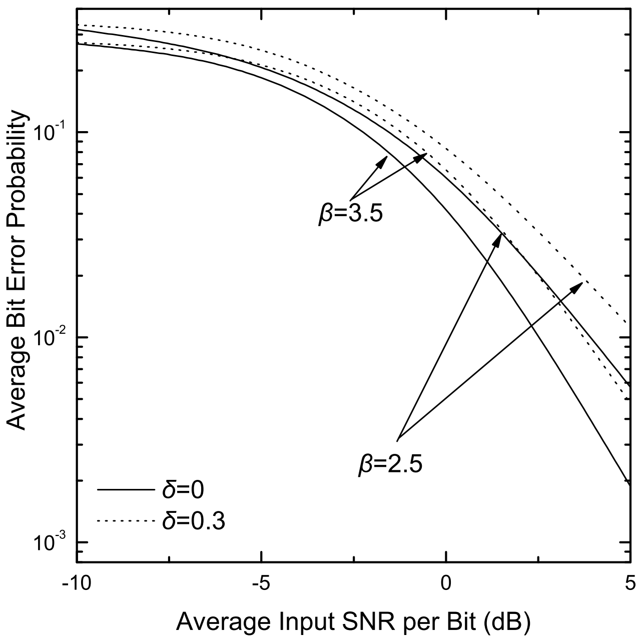

Fig. 3 ABEP versus the average input SNR for $\rho_{12} = 0.5, \rho_{13} = 0.1$ and $\rho_{23} = 0.7$.

financed by National and Community Funds (25% from the Greek Ministry of Development-General Secretariat of Research and Technology and 75% from E.U.- European Social Fund).

References

[1] M. K. Simon and M.-S. Alouini, Digital Communication over Fading Channels, Wiley, New York, 2005.

[2] V. V. Veeravalli, ”On the Performance Analysis for Signaling on Correlated Fading Channels”, IEEE Trans. on Communications, Vol.49, No.11, pp.1879-1883, November, 2001.

[3] Y.-K. Ko and M.-S. Alouini and M. K. Simon, ”Outage Probability of Diversity Systems over Generalized Fading Channels”, IEEE Trans. on Communications, Vol.48, No.11, pp.1783-1787, November, 2000.

[4] P. Lombardo and G. Fedele and M. M. Rao, ”MRC Performance for Binary Signals in Nakagami Fading with General Branch Correlation”, IEEE Trans. on Communications, Vol.47, No.1, pp.44-52, January, 1999.

[5] S. L. Loyka, ”Channel Capacity of MIMO Architecture Using the Exponential Correlation Matrix”, IEEE Communication Letters, Vol.5, No.9, pp.369-371, September, 2001.

[6] F. Babich and G. Lombardi, ”Statistical Analysis and Characterization of the Indoor Propagation Channel”, IEEE Trans. on Communications, Vol.48, No.3, pp.455-464, March, 2000.

[7] N. H. Shepherd, ”Radio Wave Loss Deviation and Shadow Loss at 900 MHz”, IEEE Trans. on Vehicular Technology, Vol.VT-26, pp.309-313, 1977.

[8] P. S. Bithas and G. K. Karagiannidis and N. C. Sagias and P. T. Mathiopoulos and S. A. Kotsopoulos and G.E. Corazza, ”Performance Analysis of a Class of GSC Receivers Over Nonidentical Weibull Fading Channels”, IEEE Trans. on Vehicular Technology, Vol.54, No.6, pp.1963-1970, November, 2005.

[9] G. K. Karagiannidis and D. A. Zogas and S. A. Kotsopoloulos, ”On the Multivariate Nakagami-m Distribution With Exponential Correlation”, IEEE Trans. on Communications, Vol.51, No.8, pp.1

[10] Y. Chen and C. Tellambura, ”Infinite Series Representation of the Trivariate and Quadrivariate Rayleigh Distribution and their Applications”, IEEE Trans. on Communications, Vol.53, No.12, pp.2092-2101, December, 2005.

[11] G. K. Karagiannidis and D. A. Zogas and S. A. Kotsopoloulos, ”An Efficient Approach to Multivariate Nakagami-m Distribution Using Green’s matrix approximation”, IEEE Trans. on Wireless Communications, Vol.2, No.5, pp.883-889, September, 2003.

[12] T. Piboongungon and V. A. Aalo, ”Outage Probability of L-Branch Selection Combining in Correlated Lognormal Fading Channels”, IEEE Electronic Letters, Vol.40, No.14, pp.886-888, July, 2004.

[13] R. K. Mallik, ”On the Multivariate Rayleigh and Exponential Distributions”, IEEE Trans. on Information Theory, Vol.49, No.6, pp.1499-1515, June, 2003.

[14] N. C. Sagias and G. K. Karagiannidis, ”Gaussian Class Multivariate Weibull Distributions: Theory and Applications in Fading Channels”, IEEE Trans. on Information Theory, Vol.51, No.10, pp.3608-3619, October, 2005.

[15] M. D. Jacoub, ”The $\alpha-\mu$ distribution: A general fading distribution ”, Proc. of the IEEE International Symposium on Personal and Mobile Radio Communications, Lisbon, Portugal, Sept., 2003.

[16] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, Academic Press, 6th Ed., New York, 2000.

[17] N. C. Sagias and G. K. Karagiannidis and D. A. Zogas and P. T. Mathiopoulos and G. S. Tombras, ”Performance Analysis of Dual Selection Diversity in Correlated Weibull Fading Channels”, IEEE Trans. on Communications, Vol.52, No.7, pp.1063-1067, July, 2004.

[18] G. K. Karagiannidis, ”Moments-Based Approach to the Performance Analysis of Equal Gain Diversity in Nakagami-m Fading”, IEEE Trans. on Communications, Vol.52, No.5, pp.685-690, May, 2004. 240-1244, August, 2003.