Simulation of Generalized Gamma Fading Channels with Finite State Markov Chains

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Abstract–An efficient and fast simulator is developed for generating slow generalized Gamma fading channels. The level crossing rates and the cumulative distribution function of the simulation results are shown to be very close to their theoretical values. The Weibull fading channel model is examined as a special case.

I. INTRODUCTION

Several fading simulators which have been proposed are complex and time consuming, e.g. filtered Gaussian type, while others are restricted to particular sets of parameters, e.g. integer-order. Also, it is well known that realistic fading channels are characterised by the Doppler phenomenon. The theory of first-order Markov stochastic processes to model fading channels has been efficiently used in several works including [1]—[3]. It is based on the partitioning of the received signal-to-noise ratio (SNR) in a finite number of states.

A versatile envelope distribution generalizing several fading channel models is the so-called generalized Gamma (GG) distribution, which was introduced by Stacy [4] more than four decades ago. Recently it has been also considered for wireless digital communication systems [5]—[8]. The main motivation for this is the fact that it exhibits a very good fit to experimental fading channel measurements for both indoor and outdoor terrestrial radio propagation environments.

In this paper, a first-order finite state Markov channel (FSMC) is developed for simulating the GG fading channel model, while special attention is also paid to the Weibull fading simulator [9], which is a special case if the GG model.

The rest of the paper is organized as follows: In Section II, the GG fading channel model is introduced. In Section III, a first-order finite state Markov channel (FSMC) is developed, while in Section IV, numerical and computer simulation results are presented to study the accuracy of the proposed simulator. Finally, Section V concludes the paper.

II. THE GG FADING CHANNEL MODEL

Let Z be the instantaneous SNR of the GG fading channel. Then, the probability density function (pdf) and the cumulative distribution function (cdf) of Z are [[8], eqs. (1) and (2)]

$$f_Z(z) = \frac{\beta/\Gamma(m)}{2\Xi\bar{Z}}\left(\frac{z}{\Xi\bar{Z}}\right)^{m\beta/2-1} \times \exp\left[-\left(\frac{z}{\Xi\bar{Z}}\right)^{\beta/2}\right] \quad (1)$$

and

$$F_Z(z) = 1 - \frac{1}{\Gamma(m)}\Gamma\left[m, \left(\frac{z}{\Xi\bar{Z}}\right)^{\beta/2}\right] \quad (2)$$

respectively, where $\bar{Z}$ is the corresponding average SNR, $\Xi = \Gamma(m)/\Gamma(m+2/\beta)$, $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function [[10], eq. (8.350/2)], $\Gamma(\cdot)$ is the Gamma function [[10], eq. (8.310/1)], and $\beta > 0$ and $m \geq 1/2$ are the GG's fading parameters. As β and/or m increase the severity of the fading decreases. For $\beta = 2$ and $m = 1$, (1) reduces to the well-known Nakagami-*m* and Weibull pdfs, respectively, while when $\beta \to 0$ and $m \to \infty$, (1) becomes the well-known lognormal pdf (as a limiting case).

The average level crossing rate (lcr) of Z is defined as the average number of times that Z 'falls' bellow a specified threshold z to the negative direction and for the GG channel model it is given by [5], eq. (25)]

$$L_Z(z) = \sqrt{2\pi}\, f_D \frac{m^{m-1/2}}{\Gamma(m)} \left(\frac{z}{\Xi \bar{Z}}\right)^{(m-1/2)\beta/2} \times \exp\left[-m\left(\frac{z}{\Xi \bar{Z}}\right)^{\beta/2}\right] \quad (3)$$

where f_D is the maximum Doppler frequency shift, while for the Weibull model reduces to ([[9]], eq. (12))

$$L_Z(z) = \sqrt{2\pi}\, f_D \left(\frac{z}{a\bar{Z}}\right)^{\beta/4} \exp\left[-\left(\frac{z}{a\bar{Z}}\right)^{\beta/2}\right] \quad (4)$$

with $a = 1/\Gamma(1 + 2/\beta)$.

III. The Markov Fading Channel Model

We define N partitions for Z so that if $Z_n<Z<Z_{n+1}$ ($1 \leq n \leq N$), then the FSMC is said to be in the state s_n. The Z_n's are the thresholds of the partition, with $Z_1 = 0$ and $Z_{N+1} = +\infty$. A simple way to define these thresholds consists in specifying that the steady-state probabilities π_n of each state be all equal

$$\pi_n = F_Z(Z_n) - F_Z(Z_{n-1}) = \frac{1}{N}. \quad (5)$$

In order to solve the last equation, for the GG fading channel, a standard numerical method may be used (e.g. the bisection method), while for the Weibull channel a closed-form solution yields

$$Z_n = a\bar{Z} \ln^{2/\beta}\left\{\exp\left[-\left(\frac{Z_{n-1}}{a\bar{Z}}\right)^{\beta/2}\right] - \frac{1}{N}\right\} \quad (6)$$

In a slow fading environment, the variations of the SNR during a symbol period are slow enough that only stays in the current state or adjacent state transitions may be considered. Let $p+(n)$ and $p-(n)$ denote the transition probability between states $s_n \to s_{n+1}$ and $s_n \to s_{n-1}$, then they can be approximated as

$$p_+(n) = \frac{\pi_n L_{n+1}}{R_S} \quad (7)$$

and

$$p_-(n) = \frac{\pi_n L_n}{R_S} \quad (8)$$

respectively, with $L_n = L(Z_n)$ and R_S being the symbol rate. The probability that remains at the same state is given by $p(n)=1-p_+(n)-p_-(n)$, with $p_-(1)=0$ and $p_+(N)=0$.

IV. Numerical and Simulation Results

For the simulation results N=64 states have been considered, $\bar{Z} = 1$, and about 17000 samples have been generated. In Figs. 1 and 2, L_Z / f_D and $F_{\sqrt{Z}}$ are plotted as a function of z or m = 1.3, $f_D/R_S = 10^{-3}$, and several values of β. Also in Figs. 3 and 4, N_Z / f_D and $F_{\sqrt{Z}}$ are plotted as a function of z for the Weibull fading channel (m = 1), $f_D/R_S = 5 \cdot 10^{-3}$, and several values of β. In all figures it is shown that the average lcr and the cdf of the simulated envelopes are very close to their theoretical values, pointing the accuracy and usefulness of the proposed fading simulator. Note that for higher values of f_D/R_S, the simulator also provides accurate performance, but such results are not presented due to space limitations.

V. Conclusions

By developing a generic simulator for GG fading channels, previously published simulators for the Rayleigh and Nakagami-*m* fading models were extended. The simulation results were shown to be very close to their theoretical values. Special attention was also given to the Weibull fading channel model.

ACKNOWLEDGMENT

This paper is part of the 03ED910 research project implemented within the framework of the "reinforcement program of human research manpower" (PENED) and cofinanced by national and community funds (25% from the Greek ministry of development—General Secretariat of Research and Development and 75% from EU—European Social Funds)

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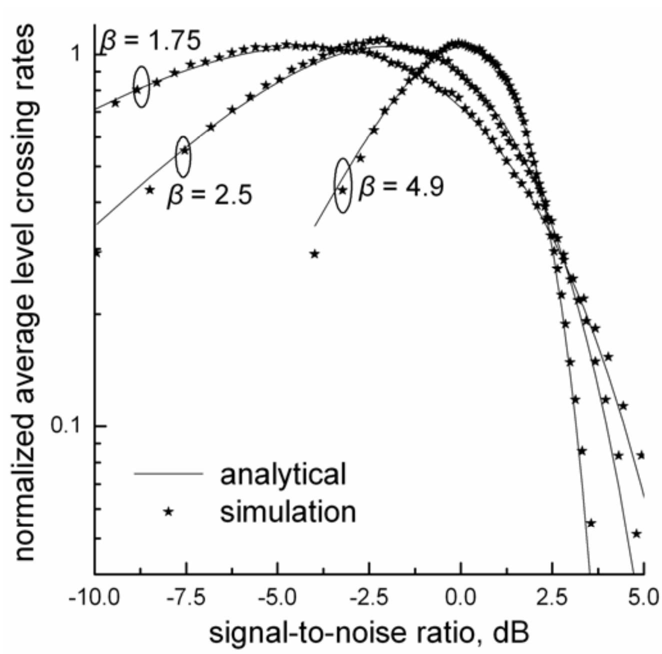

Fig. 1: N_Z / f_D as a function of z for m=1.3 and $f_D/R_S = 10^{-3}$.

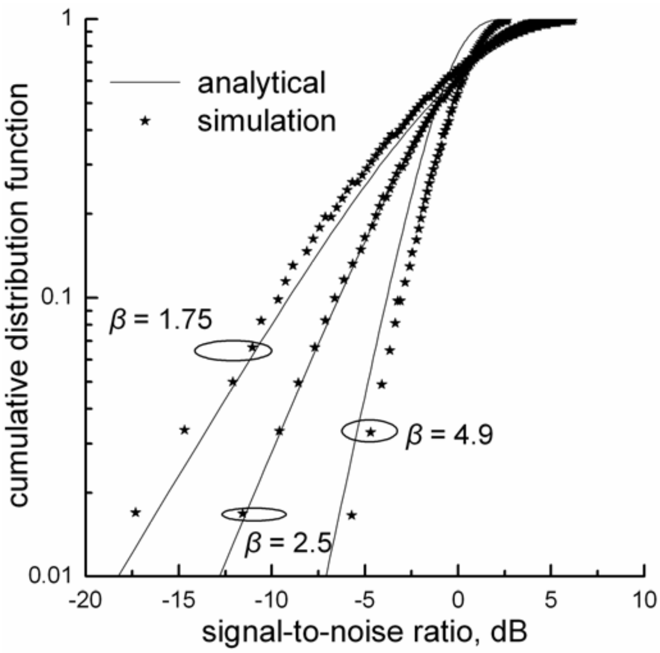

Fig. 2: CDF of envelopes as a function of z for m=1.3 and $f_D/R_S = 10^{-3}$.

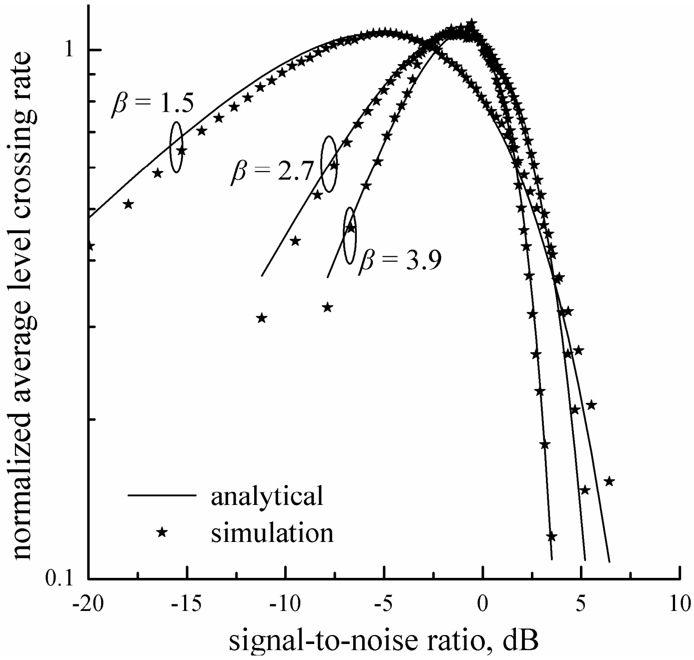

Fig. 3: N_Z / f_D as a function of z for m=1 and $f_D/R_S = 5\cdot10^{-3}$.

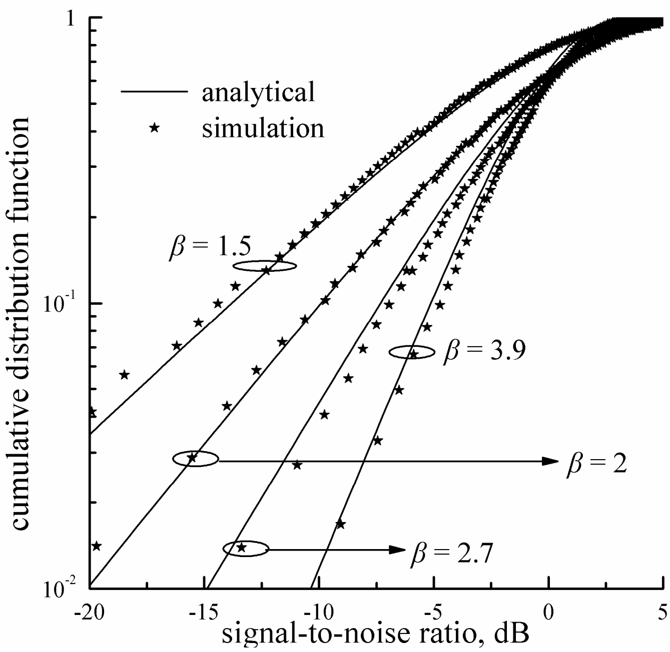

Fig. 4: CDF of envelopes as a function of z for m=1 and $f_D/R_S = 5\cdot10^{-3}$.