New Results on SC and MRC over Nakagami-m Fading Channels with Arbitrary Correlation Matrix

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***Abstract*— New results for the multichannel Nakagami-m fading model with an arbitrary correlation matrix are presented. By using an efficient tridiagonalization method based on Householder matrices, a union upper bound for the joint Nakagami-m probability density function and an infinite series representation for the moment generating function of the sum of gamma random variables are derived. Based on the proposed mathematical analysis, a tight union upper bound for the outage probability of multibranch selection diversity, as well as, exact analytical expressions for the outage and average error probability of multibranch maximal-ratio diversity receivers operating over identically distributed and arbitrarily correlated Nakagami-m fading channels are obtained. Our analysis is verified by comparisons of numerically evaluated results with extensive computer simulation ones.**

I. INTRODUCTION

Multivariate statistics can be used as an essential mathematical tool for modeling and analyzing realistic wireless channels with correlated fading. Such fading channels are usually met in digital contemporary communications systems employing diversity receivers with not sufficiently separated antennas, where space or polarization diversity is applied [1] (e.g. antenna arrays, handheld mobile terminals, and indoor base-stations).

In past, numerous papers have been published in the open technical literature dealing with multivariate fading channel models and/or systems performance analysis (see [1]–[15] and references therein). A very generic expression for the multivariate gamma-type distribution with an arbitrary covariance matrix being in the form of a multiple series of generalized Laguerre polynomials was presented back in 1951 [6]. That expression has been used for the derivation of the outage probability (OP) of selection combining (SC) receivers over arbitrarily correlated Nakagami-m [7] and generalized gamma [8] fading channels. However, the associated multivariate probability density function (PDF) used in [7], [8] for deriving the OP becomes fairly complicated with poor convergence properties [9], when the statistics of more than three random variables (RVs) needs to be considered. Hence, simpler formulas have been introduced considering specific structures of the correlation matrix. For example in [10], Mallik has presented exact closed-form PDF expressions for the multivariate Rayleigh distribution with exponential and constant correlation matrices. In a parallel and independent work, Karagiannidis *et al.* [11] have introduced the multivariate Nakagami-m PDF with exponential correlation and identically distributed (id) fading channels. By approximating the correlation matrix with a Green's matrix, the same authors have also generalized [11], presenting approximate expressions for an arbitrarily correlated Nakagami-m distribution [12].

In this paper, we provide a new statistical approach for the multivariate Nakagami-m fading channel model with arbitrary correlation structures. By using the Householder tridiagonalization method and an orthogonal transformation of Gaussian RVs, a union upper bound for the joint Nakagami-m PDF, as well as, an infinite series representation for the moment generating function (MGF) of the sum of arbitrarily correlated and id gamma RVs are derived. As indicative applications, a tight union upper bound for the OP of multibranch SC receivers is extracted, while simple exact analytical expressions for the outage and average bit error probability (ABEP) of multibranch MRC receivers are obtained. Our analysis is also verified by extensive computer simulations.

II. MULTIVARIATE NAKAGAMI-m FADE STATISTICS

Let $\mathbf{Y}_k = [Y_{k,1}\, Y_{k,2}\, \cdots\, Y_{k,L}]^{\mathrm{T}}$ ($k = 1, 2, \ldots, 2m$) be $2m$ L-dimensional real column vectors[1] ($[]^{\mathrm{T}}$ denotes the transpose matrix), which are independent and id zero mean $\mathbb{E}\langle Y_{k,l}\rangle = 0$ with variance $\mathbb{E}\langle Y_{k,l}^2\rangle = \sigma^2$ ($l = 1, 2, \ldots, L$ and $\mathbb{E}\langle\cdot\rangle$ denotes expectation) Gaussian RVs having a symmetric and positive-definite correlation matrix $\mathbf{\Sigma_G} \in \Re^{L\times L}$. Also, let $R_l = \|\mathbf{X}_l\| = \sqrt{\sum_{k=1}^{2m} Y_{k,l}^2}$ be the Euclidean norm of the $2m$-dimensional column vector $\mathbf{X}_l = [Y_{1,l}\, Y_{2,l}\, \cdots\, Y_{2m,l}]^{\mathrm{T}}$ composed of the lth components of $\mathbf{Y}_k$'s. Clearly, R_l's are correlated Nakagami-m RVs with marginal PDFs described by

$$f_{R_l}(r) = \frac{2r^{2m-1}}{\Gamma(m)\,\Omega^m}\exp\left(-\frac{r^2}{\Omega}\right) \tag{1}$$

with $\Gamma(\cdot)$ being the Gamma function [16, eq. (8.310/1)], $\Omega = 2\sigma^2 = \mathbb{E}\langle R_l^2\rangle/m$ being the average fading power, and $m \geq 1/2$ being the fading parameter. Their power correlation matrix $\mathbf{\Sigma} \in \Re^{L\times L}$ is given by $\mathbf{\Sigma}_{i,j} \equiv 1$ for $i = j$ and $\mathbf{\Sigma}_{i,j} = \mathbf{\Sigma}_{j,i} \equiv \rho_{i,j}$ for $i \neq j$, with $0 \leq \rho_{i,j} < 1$ ($i, j = 1, 2, \ldots, L$) being the power correlation coefficient (i.e. between R_i^2 and R_j^2) [1, eq.

[1]Positive integer or half-integer values for $m \geq 1/2$ are here assumed.

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(9.195)]. It can be easily proved that the correlation matrix of the underlying Gaussian processes, $\mathbf{\Sigma_G}$, is related to the power correlation matrix, $\mathbf{\Sigma}$, as $\mathbf{\Sigma_G} = \sqrt{\mathbf{\Sigma}}$ ($\sqrt{\mathbf{\Sigma}}$ stands for a matrix with elements the square root ones of $\mathbf{\Sigma}$).

A. Joint PDF

In the proposed mathematical analysis, a class of orthogonal and symmetric matrices, known as Householder matrices [17], are used for the tridiagonal decomposition [18] of the inverse of the Gaussian correlation matrix, $\mathbf{W} = \mathbf{\Sigma_G}^{-1}$. By applying a similarity transformation

$$\mathbf{W}' = \mathbf{Q}^{\mathrm{T}}\,\mathbf{W}\,\mathbf{Q} \tag{2}$$

$\mathbf{W}'$ becomes real, symmetric, and tridiagonal, where $\mathbf{Q} = [q_{i,j}] \in \Re^{L\times L}$ is an orthogonal matrix given by a product of $L-2$ properly chosen Householder matrices. A computationally efficient method for the tridiagonal decomposition of (2), as well as, some properties concerning matrix $\mathbf{Q}$ are described in [13].

By using (2) and the previous results of [5], a union upper bound for the joint Nakagami-m PDF can be derived by means of the following theorem.

Theorem 1 (Upper bound for the PDF): The joint PDF of $\mathbf{R} = [R_1\, R_2 \cdots R_L]$ with an arbitrary power correlation matrix is upper bounded as[2]

$$\begin{aligned} f_{\mathbf{R}}(\mathbf{r}) \leq & \frac{\det(\mathbf{A})\,\det(\mathbf{W})^m\,(2/\Omega)^L}{\Omega^{m-1}\Gamma(m)}\, r_1^{m-1}\left(\sum_{i=1}^{L}|q_{L,i}|\; r_i\right)^m \\ & \times \exp\left[-\frac{p_{L,L}}{\Omega}\left(\sum_{i=1}^{L}|q_{L,i}|\; r_i\right)^2\right] \\ & \times \prod_{k=1}^{L-1}|p_{k,k+1}|^{-(m-1)}\sum_{i=1}^{L}|q_{k,i}|\; r_i \\ & \times \exp\left[-\frac{p_{k,k}}{\Omega}\left(\sum_{i=1}^{L}|q_{k,i}|\; r_i\right)^2\right] \\ & \times I_{m-1}\left(\frac{2\,|p_{k,k+1}|}{\Omega}\sum_{l_1=1}^{L}\sum_{l_2=1}^{L}|q_{k,l_2}\,q_{k+1,l_1}|\; r_{l_1}\, r_{l_2}\right) \end{aligned} \tag{3}$$

where $\mathbf{r} = [r_1\, r_2 \cdots r_L]$, $I_{m-1}(\cdot)$ is the $(m-1)$th-order modified Bessel function of the first kind [16, eq. (8.406/1)], $\det(\mathbf{W})$ stands for the determinant of $\mathbf{W}$, $p_{i,j} \in \Re$ are the elements of $\mathbf{W}'$, and $\mathbf{A} = [|q_{i,j}|]$, with $|q_{i,j}|$ denoting the absolute value of $q_{i,j}$.

Proof: Let us consider an orthogonal transformation of RVs

$$\mathbf{Y}'_k = \mathbf{Q}^{\mathrm{T}}\mathbf{Y}_k\,. \tag{4}$$

Then, $\mathbf{Y}'_k$'s form another set of zero mean real Gaussian RVs with correlation matrix $\mathbf{\Sigma_G}' = \mathbf{Q}^{\mathrm{T}}\,\mathbf{\Sigma_G}\,\mathbf{Q}$ [19]. Also, let $R'_l = \|\mathbf{X}'_l\| = \sqrt{\sum_{k=1}^{2m} Y'^{\,2}_{k,l}}$ be the Euclidean norm of the $2m$-dimensional column vector $\mathbf{X}'_l = \left[Y'_{1,l}\, Y'_{2,l} \cdots Y'_{2m,l}\right]^{\mathrm{T}}$ composed of the lth components of $\mathbf{Y}'_k$'s. Since $\mathbf{Y}'_k$'s are independent, their joint PDF can be expressed as a product of marginal PDFs, i.e.,

$$\begin{aligned} f_{\mathbf{Y}'}(\mathbf{y}) &= \prod_{k=1}^{2m} f_{\mathbf{Y}'_k}(\mathbf{y}_k) \\ &= \frac{\det(\mathbf{W}')^m}{(2\pi)^{m\,L}}\prod_{k=1}^{2m}\exp\left(-\frac{1}{2}\,\mathbf{y}_k{}^{\mathrm{T}}\,\mathbf{W}'\,\mathbf{y}_k\right) \end{aligned} \tag{5}$$

with $\mathbf{Y}' = [\mathbf{Y}'_1\, \mathbf{Y}'_2 \cdots \mathbf{Y}'_{2m}]$, $\mathbf{y} = [\mathbf{y}_1\, \mathbf{y}_2 \cdots \mathbf{y}_{2m}]$, and $\mathbf{W}' = \left(\mathbf{\Sigma_G}'\right)^{-1}$ given by (2). Equation (5) has a similar form as [5, eq. (2.3)] with $\mathbf{W}'$ being tridiagonal, and hence, following a similar procedure such that in [5, Theorem I], the joint PDF of $\mathbf{R}' = [R'_1\, R'_2 \cdots R'_L]$ can be easily obtained [11, eq (3)].

Starting from the definition of R'_l, R_l and after some algebraic manipulations with (4), the two groups of Nakagami-m RVs, $\mathbf{R}'$ and $\mathbf{R}$, are related as

$$R'_1 = R_1 \tag{6a}$$

$$R'_n = \left\|\sum_{i=1}^{L} q_{n,i}\,\mathbf{X}_i\right\|\,,\; n = 2,3,\ldots,L. \tag{6b}$$

The generalization of the triangle inequality can be applied in (6b), extracting union bounds between the two groups of Nakagami-m RVs, as

$$R'_n \leq \sum_{i=1}^{L}|q_{n,i}|\; R_i. \tag{7}$$

By using a standard method for RVs transformation, an upper bound for the joint PDF of $\mathbf{R}$ can be easily obtained as in (3), with $\mathbf{A}$ being the Jacobian matrix of the RVs transformations described by (6a) and (7). ■

Although the exact joint PDF [6, eq. (3.7)] is very generic, it is fairly complicated with poor convergence properties, since is in a form of infinite series of generalized Laguerre polynomials. On the other hand, the proposed union bound for the joint PDF given by (3) is also for arbitrary correlation structures, less complicated, and in closed form.

In order to demonstrate the simplicity of (3), based on the similarity transformation in (2), we consider a typical example of a 4×4 power correlation matrix, which is arbitrary, symmetric, and positive definite

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & 0.618 & 0.384 & 0.203 \\ 0.618 & 1 & 0.563 & 0.348 \\ 0.384 & 0.563 & 1 & 0.640 \\ 0.203 & 0.348 & 0.640 & 1 \end{bmatrix}. \tag{8}$$

The inverse matrix of $\mathbf{\Sigma_G}$ is $\mathbf{W}=\left(\sqrt{\mathbf{\Sigma}}\right)^{-1}$, yielding

$$\mathbf{W} = \begin{bmatrix} 4.796 & -3.909 & -0.996 & 0.676 \\ -3.909 & 7.189 & -2.772 & -0.421 \\ -0.996 & -2.772 & 8.414 & -4.729 \\ 0.676 & -0.421 & -4.729 & 5.099 \end{bmatrix}. \tag{9}$$

[2] Since (3) holds for any $m \geq 1/2$, from now on this paper with except to the proof of the Theorem 1, $m \geq 1/2$.

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$$F_{\mathbf{R}}(\mathbf{r}) \leq \frac{\det(\mathbf{W})^m}{\Gamma(m)} \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \frac{\prod_{i=1}^{L-1} p_{i,i+1}^{2k_i} / [k_i!\,\Gamma(k_i+m)]}{p_{1,1}^{k_1+m} \left(\prod_{i=2}^{L-1} p_{i,i}^{k_{i-1}+k_i+m}\right) p_{L,L}^{k_{L-1}+m}} \gamma\left(k_1+m, \frac{p_{1,1}}{\Omega} r_1^2\right) \times \left\{\prod_{j=2}^{L-1} \gamma\left[k_{j-1}+k_j+m, \frac{p_{j,j}}{\Omega}\left(\sum_{i=1}^{L} |q_{j,i}|\, r_i\right)^2\right]\right\} \gamma\left[k_{L-1}+m, \frac{p_{L,L}}{\Omega}\left(\sum_{i=1}^{L} |q_{L,i}|\, r_i\right)^2\right] \quad (11)$$

$$P_{\text{out}}(\gamma_{\text{th}}) \leq \frac{\det(\mathbf{W})^m}{\Gamma(m)} \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \frac{\prod_{i=1}^{L-1} p_{i,i+1}^{2k_i} / [k_i!\,\Gamma(k_i+m)]}{p_{1,1}^{k_1+m} \left(\prod_{i=2}^{L-1} p_{i,i}^{k_{i-1}+k_i+m}\right) p_{L,L}^{k_{L-1}+m}} \gamma\left(k_1+m, \frac{p_{1,1}\, m}{\overline{\gamma}_s} \gamma_{\text{th}}\right) \times \left\{\prod_{j=2}^{L-1} \gamma\left[k_{j-1}+k_j+m, \frac{p_{j,j}\, m}{\overline{\gamma}_s}\left(\sum_{i=1}^{L} |q_{j,i}|\right)^2 \gamma_{\text{th}}\right]\right\} \gamma\left[k_{L-1}+m, \frac{p_{L,L}\, m}{\overline{\gamma}_s}\left(\sum_{i=1}^{L} |q_{L,i}|\right)^2 \gamma_{\text{th}}\right] \quad (12)$$

By applying (2), the matrix $\mathbf{W}$ is decomposed to a tridiagonal form after two Householder transformations, resulting to

$$\mathbf{W}' = \begin{bmatrix} 4.796 & 4.090 & 0 & 0 \\ 4.090 & 6.428 & -1.926 & 0 \\ 0 & -1.926 & 2.403 & 2.596 \\ 0 & 0 & 2.596 & 11.871 \end{bmatrix}. \quad (10)$$

Note that for demonstration purposes the elements of matrices given by (9) and (10) have been rounded to the third decimal digit.

B. Joint CDF

By using an infinite series representation for the Bessel function [16, eq. (8.445)] in (3), and after L integrations, the joint Nakagami-m CDF of $\mathbf{R}$ can be upper bounded as in (11) (top of this page), where $\gamma(x, y)$ is the lower incomplete Gamma function [16, eq. (8.350)]. Note that, when x is integer, the incomplete Gamma functions in (11) can be further simplified to standard functions using [16, eq. (8.352/1)].

III. Performance Analysis of Multibranch Receivers

We consider an L-branch diversity receiver operating over id and arbitrarily correlated Nakagami-m fading channels. Let a signal's transmission over the lth flat Nakagami-m fading channel ($l = 1, 2, \ldots, L$) corrupted by additive white Gaussian noise (AWGN), with E_s being the transmitted symbols' energy and N_0 the single-sided noise power spectral density of the AWGN. The instantaneous signal-to-noise ratio (SNR) per symbol of the lth diversity channel can be expressed by $\gamma_l = R_l^2\, E_s/N_0$, with its corresponding average value being $\overline{\gamma}_l = \mathbb{E}\langle R_l^2\rangle\, E_s/N_0 = m\,\Omega\, E_s/N_0 = \overline{\gamma}_s\ \forall\, l$.

The proposed mathematical analysis of Section II is helpful in the study of several performance criteria of multibranch diversity receivers. Specifically, a tight union upper bound for the OP of SC, as well as, exact analytical expressions for the OP and ABEP of MRC are derived.

A. Multibranch SC Receivers

The instantaneous SNR per symbol at the output of an L-branch SC receiver will be one with the highest instantaneous value among the L branches, i.e., $\gamma_{\text{sc}} = \max\{\gamma_1, \gamma_2, \ldots, \gamma_L\}$. The OP, P_{out}, is defined as the probability that the SC output SNR falls below a given outage threshold, γ_{th}. This probability can be easily obtained as $P_{\text{out}}(\gamma_{\text{th}}) = F_{\boldsymbol{\gamma}}(\gamma_{\text{th}}, \gamma_{\text{th}}, \ldots, \gamma_{\text{th}})$, $\boldsymbol{\gamma} = [\gamma_1\, \gamma_2\, \cdots\, \gamma_L]$, where using (11) and a standard method for RVs transformation yields as in (12) (top of the this page).

B. Multibranch MRC Receivers

An exact performance analysis of multibranch MRC receivers operating over id and arbitrarily correlated Nakagami-m fading channels can be carried out using the generic expressions derived for $\mathbf{R}'$, due to the following theorem.

Theorem 2 (Equal norms): Both groups of Nakagami-m RVs, $\mathbf{R}'$ and $\mathbf{R}$, have the same norm, i.e.,

$$\|\mathbf{R}'\| = \|\mathbf{R}\|. \quad (13)$$

Proof: Since $\mathbf{Y}'_k$ is an orthogonal transformation of $\mathbf{Y}_k$, as shown in (4), they both have the same norm, i.e.,

$$\|\mathbf{Y}'_k\| = \|\mathbf{Y}_k\|. \quad (14)$$

By squaring both sides of (14), adding by parts the associated $2m$ equations, and using the definition of R'_l and R_l, (13) is extracted. ■

Consequently, it directly follows from (13) that $\sum_{l=1}^{L} R_l'^2 = \sum_{l=1}^{L} R_l^2$, which implies that the instantaneous MRC output SNR per symbol can be expressed as

$$\gamma_{\text{mrc}} = \frac{E_s}{N_0} \sum_{l=1}^{L} R_l'^2. \quad (15)$$

Due to (15), one benefit of our approach is that for the performance evaluation of MRC over id and arbitrarily correlated Nakagami-m fading channels, the joint PDF of $\mathbf{R}'$ [11, eq (3)] in conjunction with the tridiagonalization method can be utilized instead of using the complicated PDF of $\mathbf{R}$ [6, eq. (3.7)]. Also, for id fading parameters, our approach seems to be less complicated than the PDF-based one presented in [14].

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$$\mathcal{M}_{\gamma_{\rm mrc}}(s)=\frac{\det(\mathbf{W})^m}{\Gamma(m)}\left(\frac{m}{\overline{\gamma}_s}\right)^{Lm}\sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty}\left[\prod_{i=1}^{L-1}\frac{(m\,p_{i,i+1}/\overline{\gamma}_s)^{2k_i}}{(k_i!)\,\Gamma(k_i+m)}\right]\left(s+\frac{mp_{1,1}}{\overline{\gamma}_s}\right)^{-(k_1+m)}\Gamma(k_1+m)$$
$$\times\left(s+\frac{m\,p_{L,L}}{\overline{\gamma}_s}\right)^{-(k_{L-1}+m)}\Gamma(k_{L-1}+m)\prod_{i=2}^{L-1}\left(s+\frac{m\,p_{i,i}}{\overline{\gamma}_s}\right)^{-(k_{i-1}+k_i+m)}\Gamma(k_{i-1}+k_i+m) \quad (16)$$

TABLE I
NUMBER OF REQUIRED TERMS FOR CONVERGENCE TO THE SIXTH SIGNIFICANT DIGIT OF THE UNION BOUND OF THE OP OF SC WITH A LINEARLY ARBITRARY MODEL ($L=3$).

$\gamma_{\rm th}/\overline{\gamma}_s$ (dB)	$m=1$	$m=2$	$m=4$
5	14	21	37
0	11	19	30
-5	7	14	21
-10	5	7	12
-15	2	2	5
-20	1	1	1

1) Error Probability: By using [11, eq (3)], Theorem 2, and the definition of the MGF of the output SNR per symbol of an L-branch MRC, i.e., $\mathcal{M}_{\gamma_{\rm mrc}}(s)=\mathbb{E}\langle\exp(-s\,\gamma_{\rm mrc})\rangle$, $\mathcal{M}_{\gamma_{\rm mrc}}(s)$ can be obtained as in (16) (top of the this page). Based on this MGF expression, the average symbol error probability (ASEP) at the output of an L-branch MRC receiver, for differential binary phase shift keying (DBPSK) modulation schemes can be directly calculated (e.g. ABEP for DBPSK is given by $\overline{P}_{be}=0.5\,\mathcal{M}_{\gamma_{\rm mrc}}(1)$). For other schemes, including binary phase shift keying (BPSK), single integrals with finite limits and integrands composed of elementary (exponential and trigonometric) functions, have to be readily evaluated via numerical integration [1].

2) Outage Probability: The OP can be obtained as [1]

$$\begin{aligned}P_{\rm out}(\gamma_{\rm th})&=F_{\gamma_{\rm mrc}}(\gamma_{\rm th})\\&=\mathbb{L}^{-1}\left[\frac{\mathcal{M}_{\gamma_{\rm mrc}}(s)}{s};\gamma_{\rm mrc}\right]\Big|_{\gamma_{\rm mrc}=\gamma_{\rm th}}\end{aligned} \quad (17)$$

where $F_{\gamma_{\rm mrc}}(\cdot)$ is the CDF of $\gamma_{\rm mrc}$ and $\mathbb{L}^{-1}[\cdot;\cdot]$ denotes the inverse Laplace transform.

IV. NUMERICAL AND COMPUTER SIMULATION RESULTS

The numerical evaluation of several expressions presented in Section III requires the summation of an infinite number of terms. As an indicative example, Table I summarizes the number of terms in each sum needed, so as the bound expression for the OP using (12) to converge after the truncation of the infinite series. The findings are not very different, concerning the convergence, if (16) or (17) are used. A linearly arbitrary correlation model has been considered with $L=3$ [12, p. 886]. As Table I indicates, the number of required terms depends strongly on the normalized outage threshold, $\gamma_{\rm th}/\overline{\gamma}_s$. As $\gamma_{\rm th}/\overline{\gamma}_s$ decreases, fewer terms are required to be summed. Moreover, for a fixed $\gamma_{\rm th}/\overline{\gamma}_s$, an increase on m results to an increase on the required number of terms that are essential to be summed in order the OP to converge. It is interesting to be mentioned that additional convergence experiments were conducted, the following findings were obtained. *i*) The convergence rate depends slightly on the diversity order and *ii*) an increase on the correlation coefficients results to an increase of the required number of terms needed for convergence.

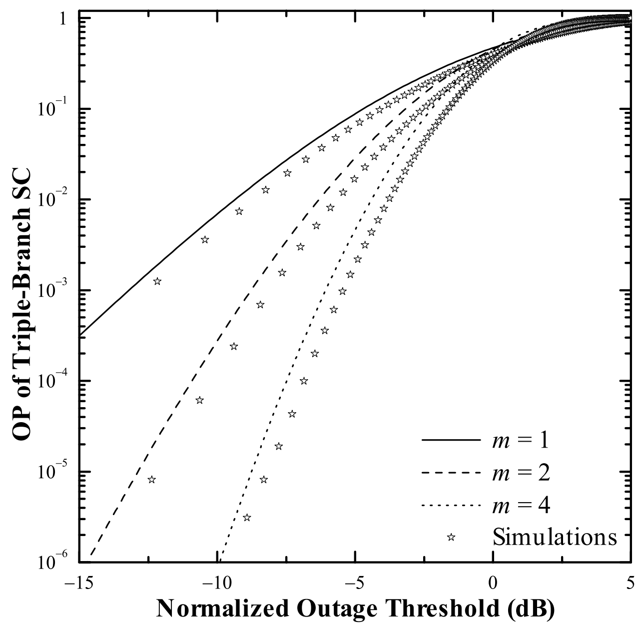

Fig. 1. Upper bound for the outage probability of a triple-branch SC as a function of the normalized outage threshold.

Having numerically evaluated (12), in Fig. 1, upper bounds for $P_{\rm out}$ are plotted as a function of the $\gamma_{\rm th}/\overline{\gamma}_s$, for a triple-branch SC receiver, different values of m, and a linearly arbitrary correlation matrix given in [12, p. 886]. It can be easily verified that $P_{\rm out}$ degrades with a decrease of m and/or an increase of $\gamma_{\rm th}/\overline{\gamma}_s$. More importantly, the obtained results clearly show, that the derived upper bounds for $P_{\rm out}$ are tight bounds compared with equivalent computer simulation results for different values of m.

Based on (17), Fig. 2 demonstrates a few numerically evaluated results for $P_{\rm out}$ as a function of the $\gamma_{\rm th}/\overline{\gamma}_s$, for a quadruple-branch MRC receiver, various m, and an arbitrary correlation matrix given by (8). As expected, $P_{\rm out}$ degrades with a decrease of m and/or an increase of $\gamma_{\rm th}/\overline{\gamma}_s$. It is obvious that all numerically evaluated curves for $P_{\rm out}$ perfectly match with equivalent computer simulation results.

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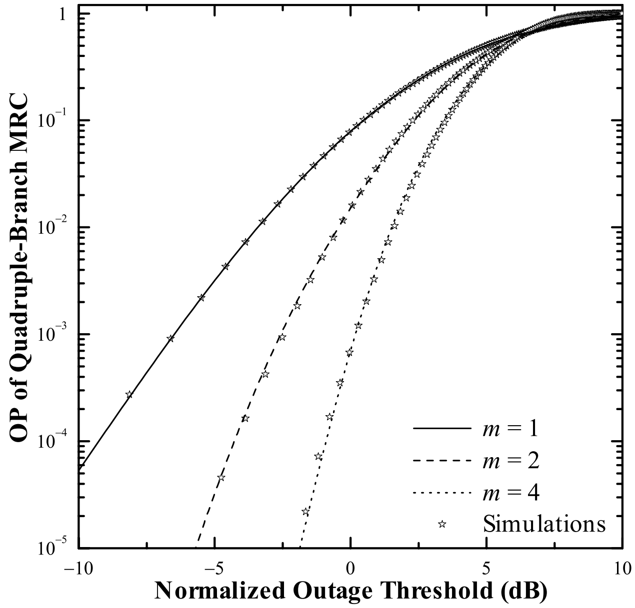

Fig. 2. Outage probability of a quadruple-branch MRC as a function of the normalized outage threshold.

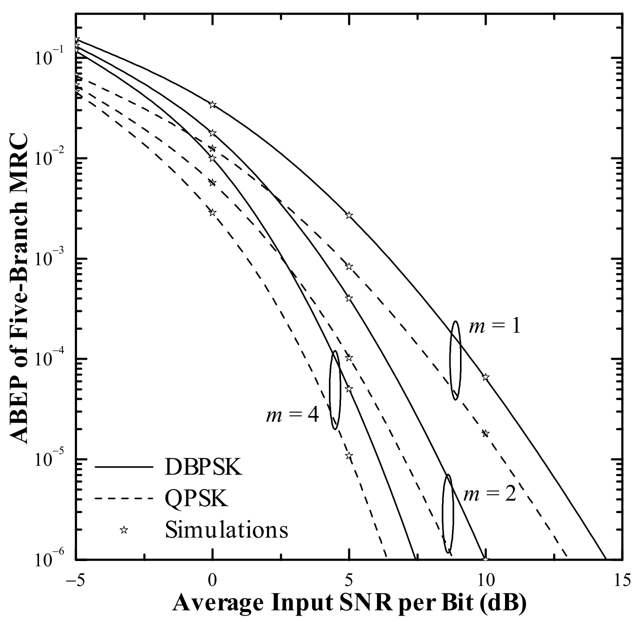

Fig. 3. ABEP of DBPSK and QPSK for a five-branch MRC as a function of the average input SNR per bit.

In Fig. 3, the ABEP performance of DBPSK and Gray-encoded quadrature phase shift keying (QPSK) signalling is plotted as a function of $\overline{\gamma}_b = \overline{\gamma}_s / \log_2(M)$ for a five-branch MRC receiver with a linearly arbitrary correlation matrix given by [15, eq. (40)]. It is clearly shown that numerically evaluated curves for the ABEP coincides to the equivalent computer simulation results.

V. Conclusions

In this paper, new results for the multivariate Nakagami-m fading channel model with arbitrary correlation structures were presented. By using the Householder tridiagonalization method, a union upper bound for the joint Nakagami-m PDF, as well as, an infinite series representation for the MGF of the sum of gamma RVs were derived. As for applications, a tight union upper bound for the OP of multibranch SC, as well as, exact analytical expressions for the OP and the ABEP of multibranch MRC receivers operating over id and arbitrarily correlated Nakagami-m fading channels were obtained. Comparisons between numerically evaluated results and extensive computer simulation ones verified the validity of our approach.

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