New Results for the Shannon Channel Capacity in Generalized Fading Channels

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***Abstract*— Novel, closed-form expressions for the average Shannon capacity of single-branch receivers, operating over generalized fading channels (Nakagami-m, Rice and Weibull), are derived. As an application, the optimum switching threshold for maximizing the data transmission rate of switched and stay combining receivers is obtained and several numerical results are presented.**

***Index Terms*— Shannon channel capacity, generalized fading channels, switched diversity, switching threshold.**

I. INTRODUCTION

THE growing demand on wireless multimedia services and products leads to increasing needs for radio channel spectrum and information data rates. Therefore, channel capacity is an important performance metric of primary concern in the design of future digital telecommunications systems. Shannon channel capacity provides an upper bound of maximum transmission rate in a given Gaussian environment [1-3].

Diversity is an effective and widely used technique for mitigating the effects of multipath fading. Among the well-known diversity techniques, switched and stay combining (SSC) requires reduced complexity, avoiding the need for knowledge of the channel's state information for all the diversity branches and the need of a dedicated receiver for each one of them [4]. In a dual-branch SSC receiver, if the instantaneous signal-to-noise ratio (SNR) of the first branch falls below a predefined switching threshold, the second branch is immediately selected, regardless of whether or not the SNR of that branch is above or below the predetermined threshold. Previously published works concerning the performance of SSC receivers are included in [5-11]. In most of these papers the optimum switching threshold for maximum average output SNR or / and error rate performance has been studied. However, the needs for higher data rates requires the maximization of the channel's capacity for given bandwidth and transmission power.

In this letter, we present novel closed-form and analytical expressions (in terms of Meijer's G-function [12, eq. (9.301)]) for the average Shannon channel capacity for single-branch receivers operating in Nakagami/Rayleigh, Rice and Weibull fading environments with arbitrary values for the fading severity parameters. As an application, the derived expressions are used to evaluate the Shannon capacity of SSC receivers. Moreover, the optimum common switching threshold for maximum capacity is obtained in a useful closed-form. Finally, several numerical results are presented to outline the effect of the fading severity on the maximum achieved data rate.

Manuscript submitted April 27, 2004. The associate editor coordinating the review of this paper and approving it for publication was Prof. Suhas Diggavi.

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Digital Object Identifier 10.1109/LCOMM.2005.02031.

II. CAPACITY OF SINGLE-BRANCH RECEIVERS

Considering a signal's transmission of bandwidth BW over the additive white Gaussian noise (AWGN) channel, the Shannon capacity is defined as $C_\gamma \triangleq BW \log_2(1+\gamma)$, where γ is received SNR. When the same signal is transmitted over a fading channel, the capacity can be considered as a random variable. The average channel capacity can be obtained averaging C_γ over the probability density function (pdf) of γ, $p_\gamma(\cdot)$ [1], i.e.,

$$\overline{C}_\gamma \triangleq BW \int_0^\infty \log_2(1+\gamma)\, p_\gamma(\gamma)\, d\gamma. \quad (1)$$

Next, using well-known expressions for the $p_\gamma(\cdot)$, the average capacity is obtained in closed-forms for the Nakagami/Rayleigh, Rice and Weibull fading channel models.

A. Nakagami Fading

In Nakagami-m fading, the pdf and the cumulative density function (cdf) of the received SNR, are [4, Table 9.5]

$$p_\gamma(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\overline{\gamma}}\right)^m \gamma^{m-1}\, e^{-m\frac{\gamma}{\overline{\gamma}}} \quad (2)$$

and $P_\gamma(\gamma) = 1 - \Gamma(m, m\,\gamma/\overline{\gamma})/\Gamma(m)$, respectively, where $\overline{\gamma}$ is the corresponding average SNR per, $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function [12, eq. (8.350/2)], $\Gamma(\cdot)$ is the Gamma function [12 eq. (8.310/1)] $(\Gamma(\cdot,0)=\Gamma(\cdot))$ and m is the fading severity parameter which ranges from 0.5 to ∞. Note, that for $m=1$, (2) reduces to the pdf (exponential) of the well-known Rayleigh fading channel. By replacing (2) into (1), an integral of the form $\int_0^\infty \gamma^{m-1}\ln(1+\gamma)\, e^{-m\,\gamma/\overline{\gamma}}\, d\gamma$, appears. This kind of integral has been solved when m is an integer [3]. To the best of the authors' knowledge, this integral can not be analytically solved for arbitrary values of m, using tables included in classical reference books, such as in [12]. However, this type of integral can be efficiently expressed in closed-form using [13], for arbitrary values of m, as follows. By expressing the logarithmic and exponential integrands as

Meijer's G-functions, i.e., $\ln(1+\gamma) = G_{2,2}^{1,2}\left[\gamma\middle|\,{}^{1,1}_{1,0}\right]$ and $e^{-m\gamma/\overline{\gamma}} = G_{0,1}^{1,0}\left[m\gamma/\overline{\gamma}\middle|\,{}^{-}_{0}\right]$ [13, eq. (11)] and using [13, eq. (21)], the average channel capacity can be obtained in a simple closed-form as

$$\overline{C}_\gamma = \frac{BW}{\ln(2)}\frac{1}{\Gamma(m)}\left(\frac{m}{\overline{\gamma}}\right)^m G_{2,3}^{3,1}\left[\frac{m}{\overline{\gamma}}\middle|\begin{matrix}-m\ ,\ 1-m\\ 0\ ,\ -m\ ,\ -m\end{matrix}\right]. \quad (3)$$

Note, that using [14, /07.34.03.0987.01], $G_{2,3}^{3,1}[\cdot]$ can be written in terms of the more familiar generalized hypergeometric function ${}_2F_2(\cdot,\cdot;\cdot,\cdot;\cdot)$ [12, eq. (9.14/1)]. Moreover, for $m = 1$, using [14, /06.35.26.0001.01], (3) reduces to the average capacity of the well-known Rayleigh model [1, eq. (5)].

B. Rice Fading

In Rice fading, the pdf and the cdf of the received SNR, are [4, Table 9.5]

$$p_\gamma(\gamma) = \frac{1}{\overline{\gamma}}(1+K)\,e^{-K}\,e^{-\gamma(1+K)/\overline{\gamma}}\,I_0\left[\sqrt{4K(1+K)\frac{\gamma}{\overline{\gamma}}}\right] \quad (4)$$

and $P_\gamma(\gamma) = 1 - Q_1\left[\sqrt{2K}, \sqrt{2(1+K)\gamma/\overline{\gamma}}\right]$, respectively, where $\overline{\gamma}$ is the corresponding average SNR, $Q_1(\cdot)$ is the first order Marcum Q-function [4, eq. (4.10)], $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind [12, ch. (8.40)] and K is the Rician factor. Using (1), (4) and an infinite series representation for $I_0(\cdot)$ [12, eq. (8.447/1)], an integral of the same type as that in Nakagami-m fading case appears. Solving this integral, following the same method as in Section II-A, the Rice average channel capacity can be obtained as

$$\overline{C}_\gamma = \frac{BW}{\ln(2)}\frac{(1+K)\,e^{-K}}{\overline{\gamma}}\sum_{n=0}^{\infty}\frac{1}{(n!)^2}\left[\frac{K(1+K)}{\overline{\gamma}}\right]^n \times G_{2,3}^{3,1}\left[\frac{K+1}{\overline{\gamma}}\middle|\begin{matrix}-1-n\ ,\ -n\\ 0\ ,\ -1-n\ ,\ -1-n\end{matrix}\right]. \quad (5)$$

C. Weibull Fading

The Weibull distribution is a flexible model which exhibits an excellent fit to experimental fading channel measurements, for indoor and outdoor environments [15]. The pdf and the cdf of the received SNR, are [11] $p_\gamma(\gamma) = \beta\left[\gamma/(a\overline{\gamma})\right]^{\beta/2-1} e^{-[\gamma/(a\overline{\gamma})]^{\beta/2}}/(2a\overline{\gamma})$ and $P_\gamma(\gamma) = 1 - e^{-[\gamma/(a\overline{\gamma})]^{\beta/2}}$, respectively, where $\overline{\gamma}$ is the corresponding average SNR, $a = 1/\Gamma(1+2/\beta)$ and $\beta > 0$ is the Weibull fading severity parameter. The average channel capacity is [16]

$$\overline{C}_\gamma = \frac{BW}{\ln(2)}\frac{\beta}{2\,(a\overline{\gamma})^{\frac{\beta}{2}}}\frac{\sqrt{k}\,l^{-1}}{(2\pi)^{\frac{k+2l-3}{2}}} \times G_{2l,k+2l}^{k+2l,l}\left[\frac{(a\overline{\gamma})^{-\frac{\beta k}{2}}}{k^k}\middle|\begin{matrix}\Upsilon(l,-\frac{\beta}{2})\ ,\ \Upsilon(l,1-\frac{\beta}{2})\\ \Upsilon(k,0)\ ,\ \Upsilon(l,-\frac{\beta}{2})\ ,\ \Upsilon(l,-\frac{\beta}{2})\end{matrix}\right] \quad (6)$$

where $\Upsilon(n,\xi) \triangleq \xi/n, (\xi+1)/n, \ldots, (\xi+n-1)/n$, with ξ an arbitrary real value and n positive integer. Moreover, $l/k = \beta/2$, where k and l are positive integers. Depending upon the value of β, a set with minimum values of k and l can be properly chosen.

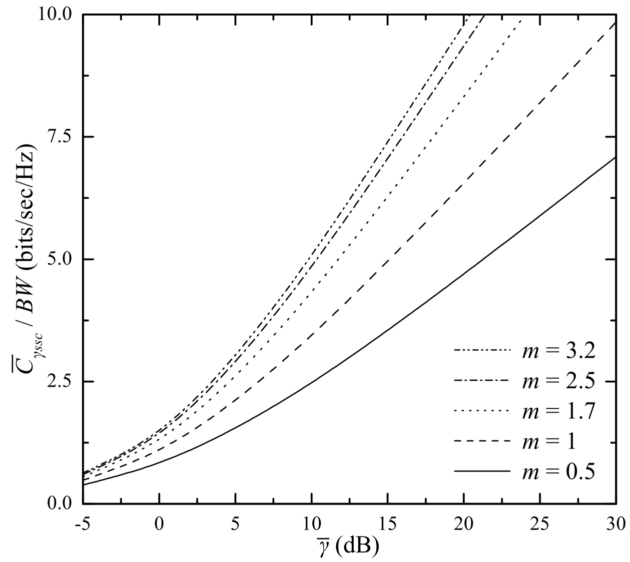

Fig. 1. Normalized average channel capacity of SSC versus average SNR in Nakagami-m fading at optimum common switching threshold.

III. Capacity of Dual-Branch SSC Receivers

We consider a dual-branch SSC receiver with common switching threshold γ_τ operating in a flat fading environment. Here, it is convenient to replace γ with γ_ℓ, $\ell = 1$ and 2, in the pdfs and cdfs of the aforementioned fading channel models, i.e., denoting them as $p_{\gamma_\ell}(\cdot)$ and $P_{\gamma_\ell}(\cdot)$, respectively, as well as the fading severity parameter of each model m, K and β, with m_ℓ, K_ℓ and β_ℓ, respectively. The pdf of the SSC instantaneous output SNR γ_{ssc} is [4, eq. (9.272)]

$$p_{\gamma_{ssc}}(\gamma_{ssc}) = \begin{cases} \frac{P_1P_2}{P_1+P_2}\left[p_{\gamma_1}(\gamma_{ssc}) + p_{\gamma_2}(\gamma_{ssc})\right] & ,\gamma_{ssc} \le \gamma_\tau \\ \frac{(1+P_1)P_2}{P_1+P_2}\,p_{\gamma_1}(\gamma_{ssc}) + \frac{(1+P_2)P_1}{P_1+P_2}\,p_{\gamma_2}(\gamma_{ssc}) & ,\gamma_{ssc} > \gamma_\tau \end{cases} \quad (7)$$

where $P_\ell = P_{\gamma_\ell}(\gamma_\ell = \gamma_\tau)$. Using (1) and (7), the average channel capacity at the output of the SSC can be expressed as

$$\overline{C}_{\gamma_{ssc}} = \frac{(1+P_2)\,P_1}{P_1+P_2}\,\overline{C}_{\gamma_1} + \frac{(1+P_1)\,P_2}{P_1+P_2}\,\overline{C}_{\gamma_2} - \frac{BW\,P_2}{P_1+P_2}\int_0^{\gamma_\tau}\log_2(1+\gamma)p_{\gamma_1}(\gamma)d\gamma - \frac{BW\,P_1}{P_1+P_2}\int_0^{\gamma_\tau}\log_2(1+\gamma)p_{\gamma_2}(\gamma)d\gamma. \quad (8)$$

The above equation includes finite integrals, which can be easily evaluated via numerical integration. Using (3), (5) and (6) and the corresponding expressions for the pdfs, the average channel capacity of SSC can be expressed in closed-form for Nakagami/Rayleigh, Rice and Weibull fading, respectively. For independent and identically distributed (i.i.d.) input branch SNRs, (8) reduces to

$$\overline{C}_{\gamma_{ssc}} = (1+P)\,\overline{C}_\gamma - BW\int_0^{\gamma_\tau}\log_2(1+\gamma)p_\gamma(\gamma)d\gamma \quad (9)$$

where $\overline{\gamma} = \overline{\gamma}_\ell$, $P = P_\ell$, $p_{\gamma_\ell}(\cdot) = p_\gamma(\cdot)$ and for Nakagami $m = m_\ell$, Rice $K = K_\ell$ and Weibull $\beta = \beta_\ell$. In this case, the common optimum switching threshold γ_τ^* for maximum average channel capacity can be obtained as

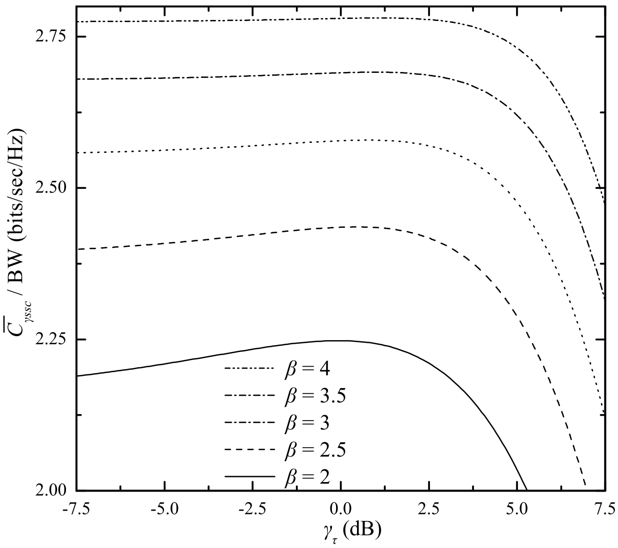

Fig. 2. Normalized average channel capacity of SSC versus common switching threshold in Weibull fading for $\overline{\gamma} = 10\,\text{dB}$.

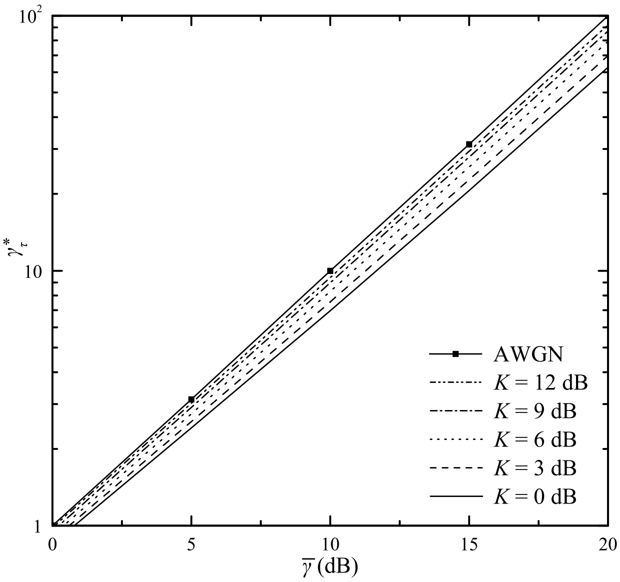

Fig. 3. Optimum common switching threshold of SSC versus average SNR in Rice fading.

$\partial\overline{C}_{\gamma_{ssc}}/\partial\gamma_\tau\big|_{\gamma_\tau^*=\gamma_\tau} = 0$, where after some straightforward mathematical manipulations, yields

$$\gamma_\tau^* = 2^{\overline{C}_\gamma/BW} - 1. \tag{10}$$

For the non-identically distributed case, the optimum switching threshold for maximum average channel capacity can be derived using numerical methods, available in most of the well-known mathematical software packages, such as Mathematica and Maple.

IV. Numerical Results

Using (9), in Fig. 1, the normalized to BW average channel capacity (spectral efficiency) of SSC, operating at the optimum switching threshold, is plotted as a function of the average SNR in Nakagami-m fading with i.i.d. input branches and for several values of m. As it was expected, $\overline{C}_{\gamma_{ssc}}$ improves with an increase of m. In Fig. 2, the normalized to BW average channel capacity of SSC is plotted as a function of the switching threshold in Weibull fading with i.i.d. input branches for several values of β and for $\overline{\gamma} = 10\,\text{dB}$. As shown, $\overline{C}_{\gamma_{ssc}}$ also improves with an increase of β and a maximum value of $\overline{C}_{\gamma_{ssc}}$ is observed, related to (10), which is more obvious as the severity of fading increases (e.g. β decreases). In both Figs. 1 and 2 and for a given value of $\overline{\gamma}$, the relative capacity advantage is more pronounced in a poorer channel condition. In Fig. 3, using (10), the optimum common switching threshold is plotted as a function of the average SNR in Rice fading with i.i.d. input branches for several values of K. It is evident that as $\overline{\gamma}$ increases, γ_τ^* also increases. In the same figure, the common switching threshold is also plotted for comparison reasons for the AWGN channel $(\gamma_\tau = \gamma)$, representing its upper bound.

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