Selection diversity receivers in Weibull fading: outage probability and average signal-to-noise ratio

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Novel expressions for the probability and the cumulative density function of the signal-to-noise ratio (SNR) at the output of an L-branch selection combining receiver, operating in Weibull fading, are derived. Capitalising on these expressions, the outage probability and the average output SNR are obtained in closed-form.

Introduction: The performance analysis of digital communications diversity receivers with selection combining (SC), has been studied extensively in the past for several well-known fading channel models, such as Rayleigh and Nakagami-m, assuming independent or correlative fading [1]. However, another well-known fading channel model, namely the Weibull model, has not yet received as much attention, despite the fact that it is a flexible model providing a very good fit to experimental fading channel measurements for both indoor [2] and outdoor environments [3]. Sagias *et al.* in [4], studied the performance of switched diversity receivers in Weibull fading. Alouini and Simon in [5] have presented an analysis for the evaluation of generalised selection combining (GSC) receivers over Weibull fading channels. In this Letter, closed-form expressions for the probability density function (PDF) and the cumulative density function (CDF) of the signal-to-noise ratio (SNR), measured at the output of a SC receiver, operating over independent Weibull fading channels, are derived. Furthermore, closed-form expressions for important performance measures such as the outage probability and the average output SNR, are obtained.

Statistics of output SNR: We consider an L-branch SC diversity receiver, operating in a Weibull fading environment. Assuming statistical independent input paths, the CDF of the envelopes r, $(i=1, 2, \ldots, L)$ is given by [6]

$$F_{r_i}(r_i) = 1 - \exp\left[-\left(\frac{r_i}{\omega_i}\right)^{\beta_i}\right] \tag{1}$$

with $\omega_i = \sqrt{(\overline{r_i^2}/\Gamma(d_{2,i}))}$, $d_{k,i} = 1 + k/\beta_i$ where k is a positive integer and $\Gamma(\cdot)$ is the Gamma function [[7], equation (8.310/1)]. $\overline{r_i^2}$ and $\beta_i > 0$ are the average signal power and the Weibull fading parameterof the ith channel, respectively. As β_i decreases, the fading severity increases and, for $\beta_i = 2$, (1) reduces to the well-known Rayleigh CDF. Since we are ultimately interested in the PDF of the instantaneous input SNR per symbol, $\gamma_i = r_i^2\, E_s/N_o$, where E_s is the symbol energy, N_o is the noise power spectral density, with the corresponding average SNR, $\bar{\gamma}_i = \overline{r_i^2}\, E_s/N_o = \omega_i^2 \Gamma(d_{2,i}) E_s/N_o$, it is convenient to rearrange (1) expressing the CDF of the Weibull distributed input SNR as

$$F_{\gamma_i}(\gamma_i) = 1 - \exp\left[-\left(\frac{\gamma_i}{a_i \bar{\gamma}_i}\right)^{\beta_i/2}\right] \tag{2}$$

where $a_i = 1/\Gamma(d_{2,i})$. The instantaneous SNR per symbol at the SC combiner's output is the probability, P_r, that the signal levels of all branches fall below a certain level γ_{sc}, i.e. $P_r = \prod_{i=1}^{L} F_{\gamma_i}(\gamma_i \le \gamma_{sc})$. Using (2), the CDF of the signal at the output of the combiner, γ_{sc}, can be expressed as

$$F_{\gamma_{sc}}(\gamma_{sc}) = \prod_{i=1}^{L}\left\{1 - \exp\left[-\left(\frac{\gamma_{sc}}{a_i \bar{\gamma}_i}\right)^{\beta_i/2}\right]\right\} \tag{3}$$

and by differentiating (3), the corresponding PDF is obtained as

$$p_{\gamma_{sc}}(\gamma_{sc}) = \frac{1}{2}\sum_{j=1}^{L} \frac{\beta_j \gamma_{sc}^{(\beta_j/2)-1}}{(a_j \bar{\gamma}_j)^{\beta_j/2}} \exp\left[-\left(\frac{\gamma_{sc}}{a_j \bar{\gamma}_j}\right)^{\beta_j/2}\right] \times \prod_{\substack{i=1\\ i\neq j}}^{L}\left\{1 - \exp\left[-\left(\frac{\gamma_{sc}}{a_i \bar{\gamma}_i}\right)^{\beta_i/2}\right]\right\} \tag{4}$$

For independent and identical distributed ($\bar{\gamma}_i = \bar{\gamma}_0$, $\beta_i = \beta$ and $a = a_i$, for every i) input paths, using the binomial theorem [[7], equation (1.111)], (4) reduces to

$$p_{\gamma_{sc}}(\gamma_{sc}) = \frac{\beta L \gamma_{sc}^{(\beta/2)-1}}{2(a\bar{\gamma}_0)^{\beta/2}} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \exp\left[-(k+1)\left(\frac{\gamma_{sc}}{a\bar{\gamma}_0}\right)^{\beta/2}\right] \tag{5}$$

Performance analysis: A well-accepted performance criterion for diversity systems operating in fading environments is the outage probability, P_{out}, defined as the probability that the output SNR of the SC falls below a given outage threshold γ_{th}. The outage probability for independent, but not necessarily identical, Weibull input paths can be obtained by replacing γ_{sc} with γ_{th} in (3) resulting in

$$P_{\text{out}}(\gamma_{th}) = F_{\gamma_{sc}}(\gamma_{th}) \tag{6}$$

Another important and probably the most common performance criterion, measured at the output of the SC diversity receiver, is the average SNR. The average output SNR of the SC, $\bar{\gamma}_{sc}$ can be obtained by averaging γ_{sc} over the PDF of the combined signal, i.e.

$$\bar{\gamma}_{sc} = \int_0^{\infty} f_{\gamma_{sc}}(\gamma_{sc}) \gamma_{sc}\, d\gamma_{sc} \tag{7}$$

Using (5) and [[7], equation (3.326/2)], the average output SNR of the SC can be expressed in simple closed-form as

$$\bar{\gamma}_{sc} = \bar{\gamma}_0 \sum_{k=0}^{L-1} (-1)^{L-1-k} \binom{L}{k} (L-k)^{-2/\beta} \tag{8}$$

Numerical results: We have numerically evaluated (6) and (8) and the results are depicted in Figs. 1 and 2, respectively. In Fig. 1, the outage probability, P_{out}, is plotted against the first branch normalised outage threshold, $\gamma_{th}/\bar{\gamma}_1$, for $L=3$ branches with an exponentially decaying power delay profile (PDP) ($\bar{\gamma}_k = \bar{\gamma}_1 e^{-\delta(k-1)}$), decaying factor $\delta = 0.5$ and for several identical values of β. For a given value of $\gamma_{th}/\bar{\gamma}_1$ an increase of β leads to a decrease of P_{out}. In Fig. 2, the normalised average output SNR, $\bar{\gamma}_{sc}/\bar{\gamma}_0$, is plotted against β for several values of L. As expected, the higher the diversity order is, the higher SNR gain becomes. This is also the case when the severity of fading increases (i.e. β decreases). However, as β increases, the gain of the SC becomes less sensitive to β.

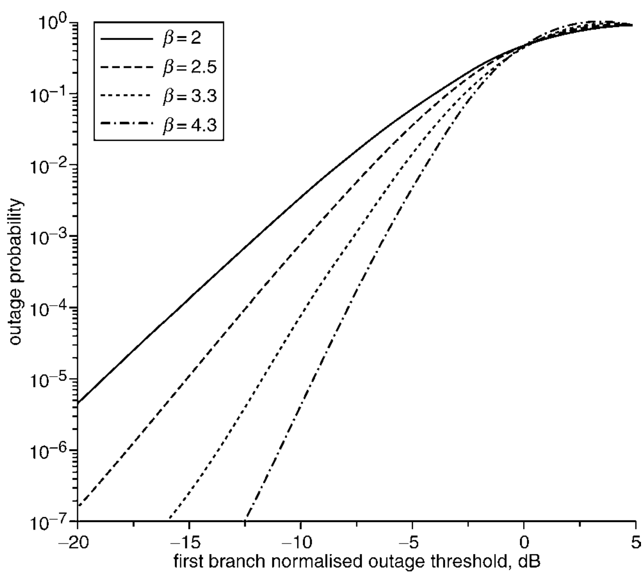

Fig. 1 *Outage probability against first branch normalised outage threshold, for $L=3$ branches with exponentially decaying PDP, $\delta=0.5$ and several values of β*

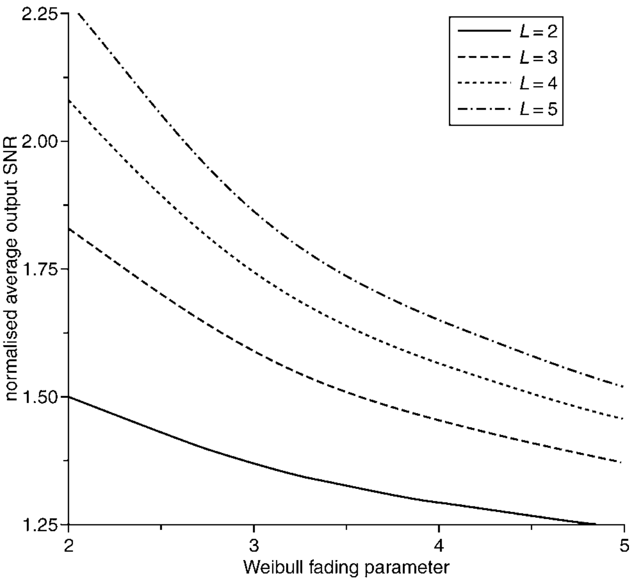

Fig. 2 *Normalised average output SNR against Weibull fading parameter for several values of L*

Conclusions: New closed-form expressions were derived for the PDF and the CDF of the average output SNR for an L-branch SC diversity receiver operating in Weibull fading. Using these expressions, analytical results for the probability of outage and the average output SNR were also derived in closed-form.

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