Performance Analysis of a Class of GSC Receivers Over Nonidentical Weibull Fading Channels

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***Abstract*—The performance of a class of generalized-selection combining (GSC) receivers operating over independent but nonidentically distributed Weibull fading channels is studied. We consider the case where the two branches with the largest instantaneous signal-to-noise ratio (SNR), from a total of L available, GSC(2, L) are selected. By introducing a novel property for the product of moments of ordered Weibull random variables, convenient closed form expressions for the moments of the GSC(2,L) output SNR are derived. Using these expressions, important performance criteria, such as average output SNR and amount of fading, are obtained in closed form. Furthermore, employing the Padé approximants theory and the moment-generating function approach, outage and bit-error rate performance are studied. An attempt is also made to identify the equivalency between the Weibull and the Rice fading channel, which is typically used to model the mobile satellite channel. We present various numerical performance evaluation results for different modulation formats and channel conditions. These results are complemented by equivalent computer simulated results which validate the accuracy of the proposed analysis.**

***Index Terms*—Amount of fading (A_F), bit-error rate (BER), generalized-selection combining (GSC), land-mobile satellite, moment-generating function (MGF), ordered statistics, outage probability, Padé approximants, Weibull fading channels.**

I. Introduction

DIVERSITY is an efficient communication receiver technique providing wireless link performance improvement at relatively low cost [1]. Among a wide range of diversity combining implementations, the most well-known techniques are maximal-ratio combining (MRC), selection combining (SC), and a combination of MRC and SC, identified as generalized-selection combining (GSC). The latter type has been proposed to bridge the gap between the two extreme cases (MRC and SC) in terms of maximizing the performance and minimizing the overall complexity. While MRC receivers provide optimum performance at the expense of high implementation complexity, GSC receivers are simpler and provide comparable performance. In GSC (N, L), the N strongest branches having the highest instantaneous signal-to-noise ratio (SNR) are selected among the L available and adaptively combined [2]. The GSC reception is equivalent to MRC reception if all L branches are combined (i.e., $N = L$), while it is equivalent to SC reception if only one out of the L branches is selected (i.e., $N = 1$).

The technical literature concerning GSC (N, L) receivers is quite extensive. For example, in [3], a simple closed-form expression for the moment-generating function (MGF) of the GSC output SNR has been derived for an independent but not identically distributed (i.n.d.) Rayleigh fading environment. In [4], closed-form expressions for the MGFs of the GSC output SNR over independent and identically distributed (i.i.d.) Nakagami-m channels, as well as i.n.d. Rayleigh channels, have been presented. In [5], a unified performance analysis for GSC has been proposed which has provided a general MGF expression of the GSC (N, L) output SNR, including i.n.d. input statistics and the Nakagami-m and Rice fading models. Considering different families of distributions and i.n.d. input channels, in [6], a general asymptotic MGF expression for large average input SNRs has been derived. Furthermore, in [7], special attention has been given to the important class of GSC receivers GSC(2,3) and GSC(2,4), in which the two strongest branches are selected $(N = 2)$ among the total available $L = 3$ or 4. In the same work, closed-form expressions for the average bit-error probability (ABEP) over i.i.d. Nakagami-m fading channels, for several modulation schemes, have been derived.

The Weibull fading model has recently received renewed interest (e.g., [8]–[10]) mainly due to the fact that it fits well with experimental fading channel measurements, for both indoor [11], [12] and outdoor [13]–[15] terrestrial radio propagation environments. Furthermore, due to the well-known fact that the land-mobile satellite channel has some similarities with the terrestrial radio propagation environment [16], the Weibull distribution could be also considered as an alternative channel model for land-mobile satellite systems. For example, the Weibull distribution is more general than the Rice distribution, which is the most commonly used channel model for mobile satellite systems, as it is able to model worse fading environments than Rayleigh. Furthermore, for rainfall-induced signal attenuation caused by rain drop energy absorption, extensive experimental measurements for satellite communications systems

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operating at frequency above 10 GHz have shown varying levels (i.e., fading) of the received signal [17]. The survivorship function of this attenuation signal, i.e., the ratio between the number of crossings lasting longer than a given duration and the total number of crossings of that attenuation level was found to be well represented by the Weibull distribution [18]. Moreover, the Weibull distribution can be used as a single state channel model, and its combination with a log-normal distribution to include shadowing effects is an open research problem. The composite Weibull-lognormal distribution could also become part of a novel multistate mobile satellite channel model [19].

On the subject of GSC(2,L) receivers for Weibull fading channels, the only paper published in the open technical literature the authors are aware of is [20]. In that paper, by assuming i.i.d. diversity input channels, analytical expressions for the first- and second-order moments of the GSC output SNR and the amount of fading (A_F) have been derived. However, this assumption is not always valid for real radio propagation environments, such as wideband code division multiple access (CDMA) and various IEEE-standardized channels, where the power delay profile (PDP) is considered to be an exponentially decaying distribution [1].

Motivated by the preceding, in this paper, we investigate the case of i.n.d. Weibull frequency flat-fading statistics for the diversity branches by deriving exact closed-form expressions for several important performance quality indicators for a GSC(2,L) combiner. The remainder of this paper is organized as follows. In Section II, closed-form expressions for the moments of GSC $(2, L)$ output SNR are obtained. Based on these expressions, in Section III, several important performance criteria are studied. Numerical and computer simulations results are presented in Section IV, and Section V contains the conclusions of the paper.

II. Statistical Properties of the GSC Output SNR

In this section, we first discuss the system and channel models under consideration. Then, a novel closed-form expression for the moments of the GSC output SNR, which allows the evaluation of the corresponding MGF with the aid of the Padé approximants, is derived.

A. System and Channel Models

Let us consider a transmitted complex symbol s with average energy $E_s = \langle |s|^2 \rangle$ in a multipath fading environment ($E\langle\cdot\rangle$ denotes averaging). The baseband received signal in the ℓth $(\ell = 1, 2, \ldots, L)$ antenna of the GSC (N, L) receiver is

$$r_\ell = s Z_\ell + n_\ell \tag{1}$$

where n_ℓ is the complex sample of the additive white Gaussian noise (AWGN) having one-sided power spectral density N_0 identical to all branches, and Z_ℓ is the complex channel path gain. Under the assumption of ideal phase estimation, only the distributed fading envelope affects the received signal. Hence, let Y_ℓ be the magnitude of Z_ℓ, i.e., $Y_\ell = |Z_\ell|$, modeled as a statistically independent Weibull random variable (RV) with probability density function (PDF) given by [9]

$$f_{Y_\ell}(y) = \beta \frac{y^{\beta-1}}{\omega_\ell} \exp\left(-\frac{y^\beta}{\omega_\ell}\right) \tag{2}$$

where ω_ℓ is a positive scaling parameter, $\omega_\ell = \sqrt[\beta/2]{E\langle Y_\ell^2\rangle / \Gamma(1+2/\beta)}$, $\Gamma(\cdot)$ is the Gamma function [21, Eq. (8.310/1)], and β is the Weibull fading parameter identical to all input channels $(\beta > 0)$. As the value of β increases the severity of fading decreases, while for $\beta = 2$, (2) reduces to the well-known Rayleigh PDF.

Defining $d_x \triangleq 1 + x/\beta$, with $x \in \Re$, the cumulative distribution function (CDF) and the moments of Y_ℓ can be expressed as

$$F_{Y_\ell}(y) = 1 - \exp\left(-\frac{y^\beta}{\omega_\ell}\right) \tag{3}$$

and

$$E\left\langle Y_\ell^n \right\rangle = \omega_\ell^{n/\beta} \Gamma(d_n) \tag{4}$$

respectively. Rearranging $\{Y_\ell\}$ as $Y_{(1)} \geq Y_{(2)} \geq Y_{(i)} \geq 0 \quad \forall i = 3, 4, \ldots, L$, the joint PDF of this ordered set is given by [22]

$$f_{Y_{(1)}Y_{(2)}}(y_1, y_2) = \sum_{n_1=1}^{L} \sum_{\substack{n_2=1 \\ n_2 \neq n_1}}^{L} f_{Y_{n_1}}(y_1) f_{Y_{n_2}}(y_2) \prod_{l'=3}^{L} F_{Y_{l'}}(y_2) \tag{5}$$

where the index with the prime refers to the $L-2$ unselected channel outputs, thus excluding all unprimed indexes occurring in any outer summations.

B. Moments of the Output SNR

Theorem 1: Let $\{Y_{(1)}, Y_{(2)}\}$ be a sample of an ordered statistical set consisting of L i.n.d. Weibull RVs, satisfying $Y_{(1)} \geq Y_{(2)} \geq Y_{(i)} \geq 0, \forall i = 3, 4, \ldots, L$. The moments of the product of $Y_{(1)}$ and $Y_{(2)}$ are given by

$$\begin{aligned} E\left\langle Y_{(1)}^m Y_{(2)}^n \right\rangle = & \sum_{n_1=1}^{L} \sum_{\substack{n_2=1 \\ n_2 \neq n_1}}^{L} \frac{\Gamma(d_m + d_n)}{\omega_{n_1}\omega_{n_2} d_n} \\ & \times \left[\left(\frac{\omega_{n_1}\omega_{n_2}}{\omega_{n_1} + \omega_{n_2}} \right)^{d_m + d_n} g_1\left(m, n, \{\omega_i\}_{i=1}^L\right) \right. \\ & + \sum_{k=3}^{L} (-1)^k \sum_{\lambda_3=3}^{L-k+3} \sum_{\lambda_4=\lambda_3+1}^{L-k+4} \cdots \sum_{\lambda_k=\lambda_{k-1}+1}^{L} \\ & \left. \times \frac{g_2\left(m, n, \{\omega_i\}_{i=1}^L\right)}{\left(\omega_{n_1}^{-1} + \omega_{n_2}^{-1} + \sum_{t=3}^{k} \omega_{\lambda_t}^{-1}\right)^{d_m + d_n}} \right] \end{aligned} \tag{6}$$

where

$$g_1\left(m, n, \{z_i\}_{i=1}^L\right) = {}_2F_1\left(1, d_m + d_n; d_m + 1; \frac{z_{n_1}}{z_{n_2} + z_{n_1}}\right)$$

and

$$g_2\left(m,n,\{z_i\}_{i=1}^{L}\right) = {}_2F_1\left(1, d_m+d_n; d_n+1; \frac{z_{n_2}^{-1}+\sum_{t=3}^{k} z_{\lambda_t}^{-1}}{z_{n_1}^{-1}+z_{n_2}^{-1}+\sum_{t=3}^{k} z_{\lambda_t}^{-1}}\right)$$

with ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ being the Gauss hypergeometric function [21, Eq. (9.100)], m, n positive integers, and $\{z_i\}_{i=1}^{L}$ positive values.

Proof: See the Appendix.

Using the property of the Weibull distribution that the nth power of a Weibull RV with parameters (β,ω) is another Weibull distributed RV with parameters $(\beta/n,\omega)$, it can be easily derived that the SNR per symbol of the channel is also a Weibull RV with parameters $(\beta/2,\omega)$ [8]. The instantaneous output SNR of a GSC(2,L) receiver is

$$\gamma_{\rm gsc} = \gamma_{(1)} + \gamma_{(2)} \tag{7}$$

where

$$\gamma_{(l)} = Y_{(l)}^2 \frac{E_s}{N_0} \tag{8}$$

with $l=1$ and 2. Using the binomial theorem [21, Eq. (1.111)], the nth moment of $\gamma_{\rm gsc}$, $\mu_n = E\langle\gamma_{\rm gsc}^n\rangle$, can be expressed as

$$\mu_n = E\left\langle\left(\gamma_{(1)}+\gamma_{(2)}\right)^n\right\rangle = \sum_{p=0}^{n}\binom{n}{p} E\left\langle \gamma_{(1)}^p \gamma_{(2)}^{n-p}\right\rangle. \tag{9}$$

By substituting (6) in (9), the following closed-form expression for the moments of the GSC(2,L) output SNR can be derived:

$$\begin{aligned}\mu_n = &\sum_{p=0}^{n}\binom{n}{p}\sum_{n_1=1}^{L}\sum_{\substack{n_2=1\\ n_2\neq n_1}}^{L}\frac{\Gamma(d_p+d_{n-p})}{\bar\gamma_{n_1}\bar\gamma_{n_2} d_{n-p}}\\ &\times\left[\left(\frac{\bar\gamma_{n_1}\bar\gamma_{n_2}}{\bar\gamma_{n_1}+\bar\gamma_{n_2}}\right)^{d_p+d_{n-p}} g_1\left(p, n-p, \{\bar\gamma_i\}_{i=1}^{L}\right)\right.\\ &+\sum_{k=3}^{L}(-1)^k\sum_{\lambda_3=3}^{L-k+3}\sum_{\lambda_4=\lambda_3+1}^{L-k+4}\cdots\sum_{\lambda_k=\lambda_{k-1}+1}^{L}\\ &\left.\times\frac{g_2\left(p,n-p,\{\bar\gamma_i\}_{i=1}^{L}\right)}{\left(\bar\gamma_{n_1}^{-1}+\bar\gamma_{n_2}^{-1}+\sum_{t=3}^{k}\bar\gamma_{\lambda_t}^{-1}\right)^{d_p+d_{n-p}}}\right]\end{aligned} \tag{10}$$

where the average SNR of the ℓth input branch is given by

$$\bar\gamma_\ell = \Gamma(d_2)\omega_\ell^{2/\beta}\frac{E_s}{N_0}. \tag{11}$$

III. Performance Analysis

In this section, using the previously derived closed-form expression for the moments of the GSC(2,L) output SNR, the average output SNR, A_F and several other performance quality indicators are obtained in closed form. Furthermore, by using Pade´ approximants, the ABEP and outage probability are studied.

A. Average Output SNR

The average GSC output SNR is a useful performance measure serving as an excellent indicator of the overall system's fidelity. When the receiver employs GSC, the average output SNR, $\bar\gamma_{\rm gsc}$, can be derived by setting $n=1$ in (10) as

$$\bar\gamma_{\rm gsc} = \mu_1. \tag{12}$$

Note that for i.i.d. input branches (12) reduces to a previously known expression [20, (8)].

B. Amount of Fading (A_F)

The A_F, defined as $A_F \triangleq {\rm var}(\gamma_{\rm gsc})/\bar\gamma_{\rm gsc}^2$, is a unified measure of the severity of fading [2]. Typically, this performance criterion is independent of the average fading power. Thus, A_F can be expressed in terms of first- and second-order moments of $\gamma_{\rm gsc}$ as

$$A_F = \frac{\mu_2}{\mu_1^2} - 1 \tag{13}$$

where μ_1 and μ_2 can be obtained using (10). Note again here that for i.i.d. input branches, (13) reduces to a previously known expression [20, (9)].

It is important to underline that higher order moments (i.e., μ_i with $i \geq 3$) are useful in signal processing algorithms for signal detection, classification, and estimation since they play a fundamental role for the analysis of the performance of wideband communications systems in the presence of fading [23]. In this sense, (10) can be used to study related higher order metrics, such as the kurtosis and the skewness, that characterize the distribution of $\gamma_{\rm gsc}$. Skewness, defined as $\mathcal{S} \triangleq \mu_3/\mu_2^{3/2}$, is a measure of the symmetry of a distribution. For symmetric distributions, $\mathcal{S}=0$. If $\mathcal{S}>0$, the distribution is skewed to the right. Kurtosis, defined as $\mathcal{K} \triangleq \mu_4/\mu_2^2$, is the degree of peakedness of a distribution, i.e., the PDF having a higher kurtosis has a higher peak at the center and longer tails.

C. Average Bit-Error Probability (ABEP)

One very convenient approach to evaluate the ABEP of several signaling schemes transmitted on generalized fading channels is to use the MGF-based approach [2]. For the Weibull fading channel, if one follows the analysis presented in [5], it is very difficult to derive a closed form expression for the MGF of the GSC output SNR. This happens since for the evaluation of this MGF, some integrals with finite limits of the form $\int_0^z x^{\xi-1}\exp(-C\,x - D\,x^{\xi})dx$ appear, where z, C, D, and ξ are positive constant values. Such integrals are very difficult, if not impossible, to be solved analytically so that the MGF of $\gamma_{\rm gsc}$ could be obtained in closed form. Another straightforward method for the evaluation of the ABEP is the PDF-based approach [2]. However, this method also cannot be used, since no analytical expression is readily available for the PDF of the output SNR for GSC receivers operating in i.n.d. Weibull fading. As an alternative and efficient way to approximate the MGF, and consequently evaluate the ABEP, the Pade´ approximants method [24] has been used in the past to study the performance of EGC receivers [25]. The main advantage of this approach, which will be used in this paper, is that due to the form of the produced rational approximation, the error rates can be calculated directly using simple expressions. By definition, the MGF of $\gamma_{\rm gsc}$ is

$$\mathcal{M}_{\gamma_{\rm gsc}}(s) \triangleq E\langle \exp(s\,\gamma_{\rm gsc})\rangle \tag{14}$$

and, capitalizing on the closed-form expression for μ_n [see (10)], (14) can be represented as a formal power series (e.g., Taylor) as

$$\mathcal{M}_{\gamma_{\rm gsc}}(s) = \sum_{n=0}^{\infty} \frac{\mu_n}{n!}\, s^n. \tag{15}$$

Although μ_n can be evaluated in closed-form, the infinite series do not always converge. However, using Pade´approximants, only a finite number of terms W can be used, thus truncating the series in (15). A Pade´approximant to the MGF is a rational function of a specified order B for the denominator and A for the nominator, whose power series expansion agrees with the Wth-order $(W = A + B)$ power expansion of $\mathcal{M}_{\gamma_{\rm gsc}}(s)$, i.e.,

$$R_{[A/B]}(s) = \frac{\sum_{i=0}^{A} c_i s^i}{1+\sum_{i=1}^{B} b_i s^i} = \sum_{n=0}^{A+B} \frac{\mu_n}{n!} s^n + O\left(s^{N+1}\right) \tag{16}$$

with $O(s^{N+1})$ is the remainder after truncation, b_i and c_i are real constants [25]. Hence, the first $(A + B)$th-order moments need to be evaluated so that the approximant $R_{[A/B]}(s)$ is calculated. In our analysis, $\mathcal{M}_{\gamma_{\rm gsc}}(s)$ is approximated using subdiagonal $(R_{[A/A+1]}(s))$ Pade´approximants $(B = A + 1)$, since it is only for such order of approximants that the convergence rate and the uniqueness can be assured [24], [25]. By obtaining accurate approximation expressions for the MGF of GSC(2,L) output SNR, direct calculation of the ABEP for noncoherent binary frequencyshift keying and differential binary phaseshift keying (DBPSK) is possible. For example, the ABEP of DBPSK is given by $\bar{P}_{be}(E) = 0.5\,\mathcal{M}_{\gamma_{\rm gsc}}(-1)$ [2]. Furthermore, for other signaling formats such as BPSK, M-PSK, M-ary quadrature amplitude modulation (M-QAM), and M-DPSK, single integrals with finite limits and integrands composed of exponential and trigonometric functions have to be readily evaluated via numerical integration [2]. Note that using Pade´ with the MGF-based approach is an efficient approach due to the known expressions for the moments of the GSC(2,L) output SNR (10).

D. Outage Probability

The outage probability $P_{\rm out}$ is defined as the probability that $\gamma_{\rm gsc}$ falls below the outage threshold $\gamma_{\rm th}$ and can be expressed as

$$P_{\rm out}(\gamma_{\rm th}) \triangleq F_{\gamma_{\rm gsc}}(\gamma_{\rm th}) = \mathcal{L}^{-1}\left[\frac{\mathcal{M}_{\gamma_{\rm gsc}}(s)}{s}\right]\Bigg|_{\gamma_{\rm gsc}=\gamma_{\rm th}} \tag{17}$$

where $F_{\gamma_{\rm gsc}}(\cdot)$ is the CDF of the combiner's output SNR, and $\mathcal{L}^{-1}(\cdot)$ denotes inverse Laplace transformation. Using (15) and (16), $P_{\rm out}$ can be obtained as

$$P_{\rm out}(\gamma_{\rm th}) = \sum_{i=1}^{B} \frac{\lambda_i}{p_i} \exp(p_i \gamma_{\rm th}) \tag{18}$$

where $\{p_i\}$ are the poles of the Pade´ approximants to the MGF, which must have negative real part, and $\{\lambda_i\}$ are the residues [25].

TABLE I
ORDER OF MOMENTS $[A/A+1]$ THAT PADE´APPROXIMANTS GUARANTEEING FOR A FIVE SIGNIFICANT DIGIT ACCURACY

$\bar{\gamma}_1$ (dB)	DBPSK	BPSK	16-QAM
-5	[6/7]	[2/3]	[2/3]
0	[7/8]	[4/5]	[4/5]
5	[9/10]	[6/7]	[5/6]
10	[11/12]	[9/10]	[7/8]
15	[14/15]	[12/13]	[12/13]

IV. PERFORMANCE EVALUATION RESULTS AND DISCUSSION

In this section, numerical performance evaluation results complemented by equivalent computer simulated results are presented. These results include performance comparisons of several GSC$(2, L)$ receiver structures, employing various modulation formats and different Weibull channel conditions. Per our previous performance analysis, the following performance criteria will be used: normalized average GSC output SNR $\bar{\gamma}_{\rm gsc}/\bar{\gamma}_1$ [see (12)], A_F [see (13)], ABEP [see (16)], and $P_{\rm out}$ [see (18)]. For the multipath channel, the well-accepted exponentially decaying PDP has been considered [5], i.e., $\overline{\gamma}_\ell = \overline{\gamma}_1 \exp[-\delta(\ell-1)]$, where δ is the power decaying factor.

In order to check the convergence rate of the Pade´ approximants as given by (16), Table I provides the number of moments $W = 2A + 1$ which are needed to guarantee a five significant digit accuracy. Starting with a GSC(2,3) receiver and a Weibull fading environment with $\beta = 2.5$ and $\delta = 0.5$, Table I presents the results for three representative signals. It can be easily observed that as $\bar{\gamma}_1$ increases, W also increases. Moreover, it should pointed out that our research has shown that for other GSC(2,L), e.g., $L = 4, 5$ receivers, very similar W values, such as those listed, in Table I, have been obtained.

In order to investigate the range of values where the Weibull fading parameter β varies, we compare the more general Weibull fading model with the typical Rice mobile satellite channel model by identifying the equivalency between β and the K-factor of the Rice fading [2]. Based upon this equivalence, performance evaluation results presented for the Weibull channel can be also used for analyzing the performance of mobile satellite systems. Such comparison can be made by equating the first two moments of the input SNR for these two fading distributions. Considering that for state-of-the-art satellite systems large and small values of K are possible [19], β is plotted in

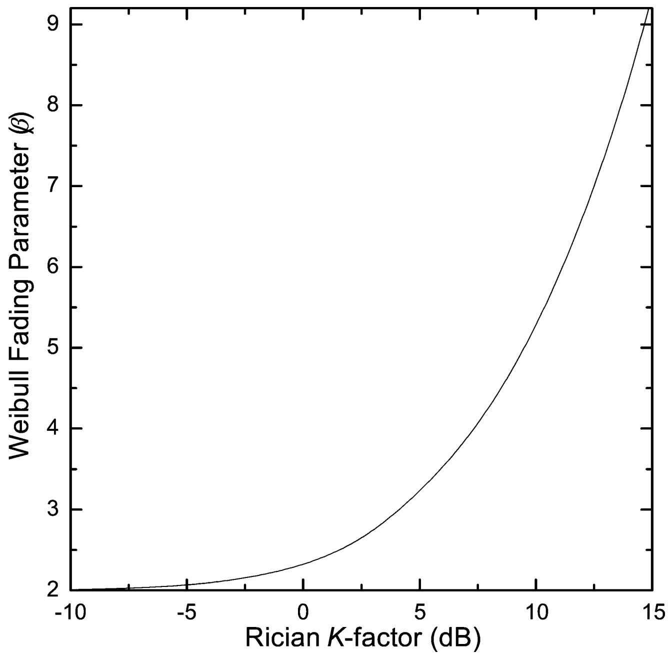

Fig. 1. Equivalency between the Weibull fading parameter β and Rician K-factor (in decibels).

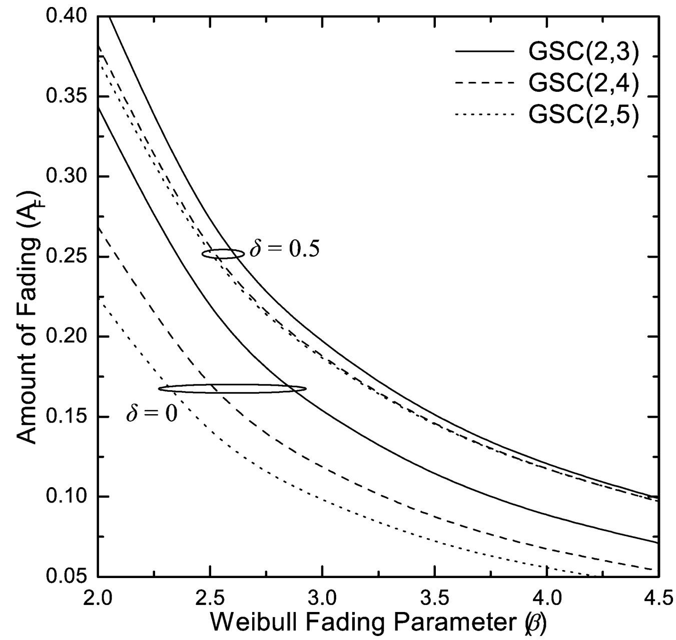

Fig. 3. A_F versus β for three GSC(2,L) receivers with $\delta = 0$ and 0.5.

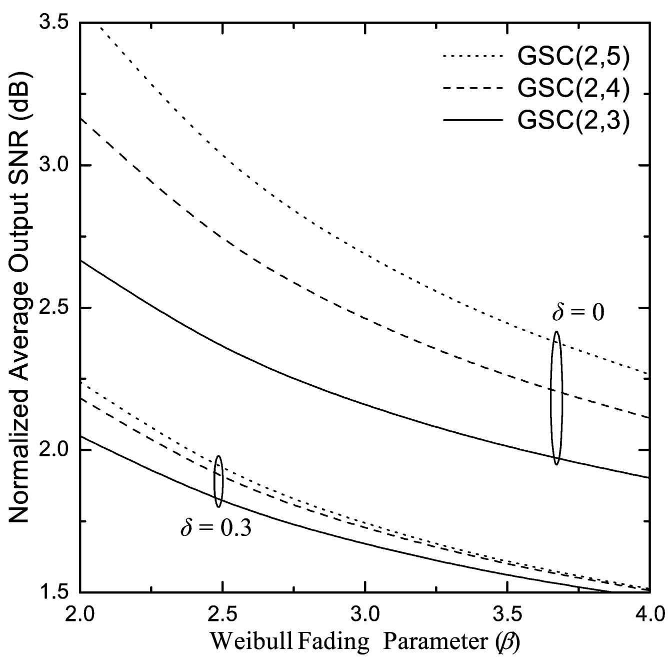

Fig. 2. $\bar{\gamma}_{\rm gsc}/\bar{\gamma}_1$ versus β for three GSC(2,L) receivers with $\delta = 0$ and 0.3.

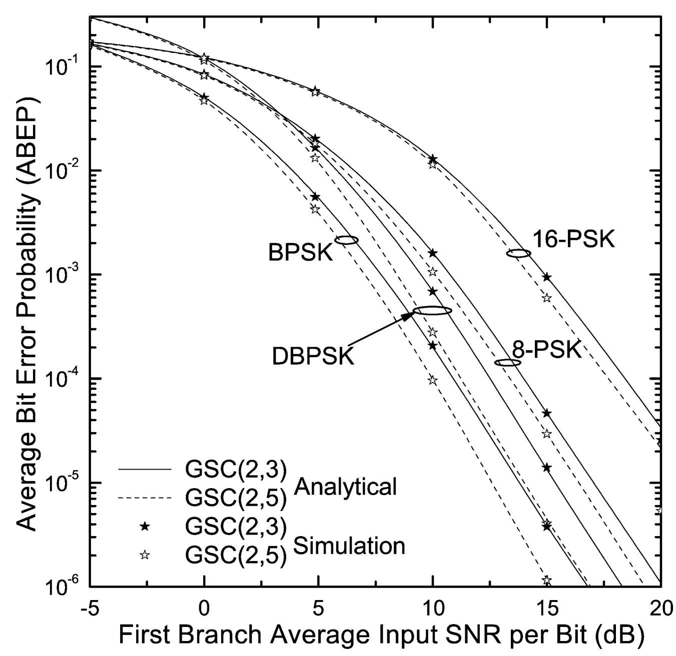

Fig. 4. ABEP of DBPSK and Gray encoded M-PSK signaling formats versus $\bar{\gamma}_{\rm gsc}/\bar{\gamma}_1$ for GSC(2,3) and GSC(2,5) receivers in Weibull fading environment with $\beta = 2.5$ and $\delta = 0.5$.

Fig. 1 as a function of K for -10 dB $< K < 15$ dB. It should be noted that the lowest value for the Weibull fading parameter in the figure ($\beta = 2$) represents a Rayleigh fading channel (i.e., $K \to -\infty$) and that the Rice distribution is not able to model worse fading environments than Rayleigh (e.g., for the Weibull channel when $0 < \beta < 2$).

Figs. 2 and 3 present $\bar{\gamma}_{\rm gsc}/\bar{\gamma}_1$ and A_F, respectively, as functions of the Weibull fading parameter β and for various values of δ. As expected, when β and/or δ increase, $\bar{\gamma}_{\rm gsc}/\bar{\gamma}_1$ also increases. It is interesting to note that $\bar{\gamma}_{\rm gsc}/\bar{\gamma}_1$ degrades more rapidly as δ increases. Furthermore, from Fig. 3, it is observed that with increasing β, A_F decreases, while when δ increases, A_F increases, and the gap among the curves for GSC(2,3), GSC(2,4), and GSC(2,5) is reduced.

In Fig. 4, the ABEP of GSC(2,3) and GSC(2,5) is compared for DBPSK and for Gray-encoded M-PSK signaling constellations, assuming $\beta = 2.5$ and $\delta = 0.5$. As expected, the ABEP improves with an increase in the diversity order, and as M

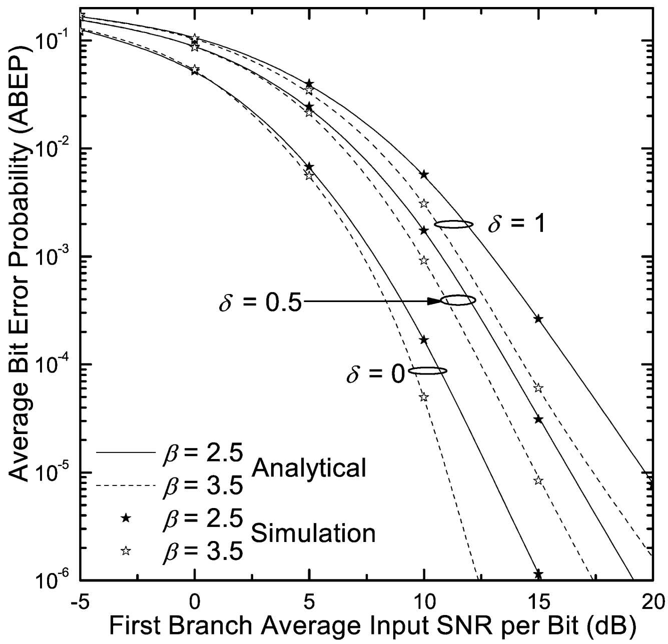

Fig. 5. ABEP of Gray encoded squared 16-QAM signaling format versus $\bar{\gamma}_{\rm gsc}/\bar{\gamma}_1$ for a GSC(2,4) receiver in a Weibull fading environment with various values of β and δ.

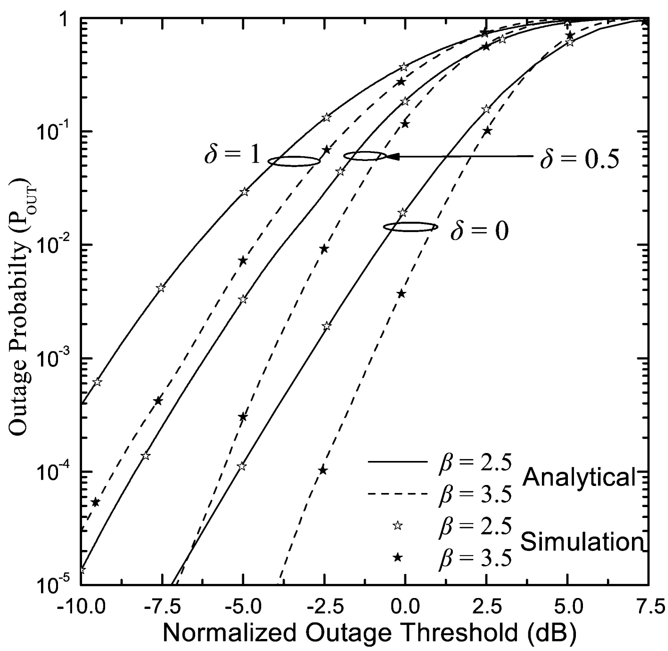

Fig. 6. $P_{\rm out}$ versus $\bar{\gamma}_1$ for a GSC(2,4) receiver in Weibull fading environment with $\beta = 2.5$ and 3.5.

increases, the ABEP performance is degrading. Similar behavior is also observed in Fig. 5, for the ABEP of 16-QAM signaling with Gray-encoding, of a GSC(2,4) receiver, which is also plotted as a function of $\bar{\gamma}_1$ and for several values of β and δ. The ABEP improves with an increase of β, while as δ decreases, it also decreases. Finally, Fig. 6 shows $P_{\rm out}$ versus $\bar{\gamma}_1$ for several values of β and δ. It can easily be observed that $P_{\rm out}$ increases with an increase of δ, while as β increases, $P_{\rm out}$ decreases, approaching the performance of a nonfading channel. For comparison purposes, computer simulation performance results are also included in Figs. 4–6, verifying in all cases the validity of the proposed theoretical approach.

V. Conclusion

In this paper, the performance of a class of GSC receivers over nonidentical Weibull fading diversity channels has been studied by means of analysis and computer simulation. Using a novel statistical theorem for the product moments of ordered statistical RVs, analytical expressions for the moments of the output SNR were derived, and important performance metrics, such as average output SNR and A_F, have been obtained in closed form. Based on the MGF approach, the outage and the error performance have been studied, using the Pade´ approximants theory. Various numerical performance evaluation results complemented by equivalent computer simulations have been presented for several propagation environments.

Appendix

Proof of Theorem I

From (5), the higher order joint moments can be obtained as

$$
\begin{aligned}
&E\left\langle Y_{(1)}^m Y_{(2)}^n \right\rangle \\
&= \int_0^\infty \int_0^{y_1} y_1^m\, y_2^n\, f_{Y_{(1)}Y_{(2)}}(y_1, y_2)\, dy_2\, dy_1 \\
&= \sum_{n_1=1}^{L} \sum_{\substack{n_2=1\\ n_2\neq n_1}}^{L} \int_0^\infty \int_0^{y_1} y_1^m y_2^n f_{Y_{n_1}}(y_1) f_{Y_{n_2}}(y_2) \\
&\quad \times \prod_{n'=3}^{L} F_{Y_{n'}}(y_2) dy_2 dy_1 \qquad \text{(A1)}
\end{aligned}
$$

where the index n' refers to the $L-2$ unselected channel outputs. Since

$$
\begin{aligned}
&\prod_{k=a}^{L}(1-t_k) = 1 \\
&\quad + \sum_{k=a}^{L}(-1)^{k-a+1} \\
&\quad \times \sum_{\lambda_a=a}^{L-k+a} \sum_{\lambda_{a+1}=\lambda_a+1}^{L-k+a+1} \cdots \sum_{\lambda_k=\lambda_{k-1}+1}^{L} \prod_{n=a}^{k} t_{\lambda_n} \qquad \text{(A2)}
\end{aligned}
$$

with $t_k = \exp(-y_2^\beta/\omega_{n_k})$ and $a = 3$, using (2), (3), and(A2), (A1) can be expressed as in

$$
\begin{aligned}
&E\left\langle Y_{(1)}^m Y_{(2)}^n \right\rangle \\
&= \sum_{n_1=1}^{L} \sum_{\substack{n_2=1\\ n_2\neq n_1}}^{L} \frac{\beta^2}{\omega_{n_1}\omega_{n_2}} \left[\int_0^\infty y_1^{m+\beta-1}\right.
\end{aligned}
$$

$$\times \exp\left(-\frac{y_1^\beta}{\omega_{n_1}}\right)\int_0^{y_1} y_2^{n+\beta-1}\exp\left(-\frac{y_2^\beta}{\omega_{n_2}}\right)dy_2\,dy_1$$
$$+\sum_{k=3}^{L}(-1)^{k-2}\sum_{\lambda_3=3}^{L-k+3}\sum_{\lambda_4=\lambda_3+1}^{L-k+4}\cdots$$
$$\times\sum_{\lambda_k=\lambda_{k-1}+1}^{L}\int_0^\infty y_1^{m+\beta-1}$$
$$\times \exp\left(-\frac{y_1^\beta}{\omega_{n_1}}\right)\int_0^{y_1} y_2^{n+\beta-1}\exp\left(-\frac{y_2^\beta}{\omega_{n_2}}\right)$$
$$\left.\times\,\exp\left(-\sum_{t=3}^{k}\frac{y_2^\beta}{\omega_{\lambda_t}}\right)dy_2\,dy_1\right]. \quad \text{(A3)}$$

The double integrals in (A3) are of the form

$$\Upsilon=\int_0^\infty y_1^{m+\beta-1}\exp\left(-\frac{1}{\omega_{n_1}}\,y_1^\beta\right)\times\int_0^{y_1} y_2^{n+\beta-1}\exp\left(-\xi\,y_2^\beta\right)dy_2\,dy_1. \quad \text{(A4)}$$

Using the definition of the incomplete lower Gamma function $\gamma(\cdot,\cdot)$ [21 (3.381/1)] and after applying the transformation $z_i = y_i^\beta (i = 1$ and $2)$, (A4) simplifies to

$$\Upsilon=\frac{\xi^{-d_n}}{\beta}\int_0^\infty z_1^{d_m-1}\times\exp\left(-\frac{1}{\omega_{n_1}}\,z_1\right)\gamma(d_n,\xi\,z_1)dz_1 \quad \text{(A5)}$$

with $d_x \triangleq 1 + x/\beta$, where $x \in \Re$. Furthermore, using [21 (6.455/2)], Υ can be obtained in closed form as

$$\Upsilon=\frac{1}{\beta^2}\frac{\Gamma(d_m+d_n)}{d_n\,(1/\omega_{n_1}+\xi)^{d_m+d_n}}\times{}_2F_1\left(1,d_m+d_n;d_n+1;\frac{\xi}{1/\omega_{n_1}+\xi}\right). \quad \text{(A6)}$$

With the aid of the above equation, it is not difficult to recognize that (A3) becomes (6).

REFERENCES

[1] T. S. Rappaport, *Wireless Communications*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 2001.
[2] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*, 1st ed. New York: Wiley, 2001.
[3] M.-S. Alouini and M. K. Simon, "An MGF-based performance analysis of generalized selection combining over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 401–415, Mar. 2000.
[4] Y. Ma and C. Chai, "Unified error probability analysis for generalized selection combining in Nakagami fading channels," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 11, pp. 2198–2210, Nov. 2000.
[5] Y. Ma and S. Pasupathy, "Efficient performance evaluation for generalized selection combining on generalized fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 29–34, Jan. 2004.
[6] Y. Ma, Z. Wang, and S. Pasupathy, "Asymptotic gains of generalized selection combining," in *Proc. IEEE Vehicular Technology Conf.*, Orlando, FL, Oct. 2003, pp. 3149–3153.
[7] M.-S. Alouini and M. K. Simon, "Performance of coherent receivers with hybrid SC/MRC over Nakagami-m fading channels," *IEEE Trans. Veh. Technol*, vol. 48, no. 4, pp. 1155–1164, Jul. 1999.
[8] J. Cheng, C. Tellambura, and N. C. Beaulieu, "Performance analysis of digital modulations on Weibull fading channels," in *Proc. IEEE Vehicular Technology Conf.*, vol. 1, Orlando, FL, Oct. 2003, pp. 236–240.
[9] N. C. Sagias, G. K. Karagiannidis, D. A. Zogas, P. T. Mathiopoulos, and G. S. Tombras, "Performance analysis of dual selection diversity in correlated Weibull fading channels," *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1063–1067, Jul. 2004.
[10] J. Cheng, C. Tellambura, and N. C. Beaulieu, "Performance of digital linear modulations on Weibull slow-fading channels," *IEEE Trans. Commun.*, vol. 52, no. 8, pp. 1265–1268, Aug. 2004.
[11] H. Hashemi, "The indoor radio propagation channel," *Proc. IEEE*, vol. 81, no. 7, pp. 943–968, Jul. 1993.
[12] F. Babich and G. Lombardi, "Statistical analysis and characterization of the indoor propagation channel," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 455–464, Mar. 2000.
[13] N. S. Adawi *et al.*, "Coverage prediction for mobile radio systems operating in the 800/900 MHz frequency range," *IEEE Trans. Veh. Technol*, vol. 37, no. 1, pp. 3–72, Feb. 1988.
[14] N. H. Shepherd, "Radio wave loss deviation and shadow loss at 900 mhz," *IEEE Trans. Veh. Technol*, vol. VT-26, pp. 309–313, Jun. 1977.
[15] G. Tzeremes and C. G. Christodoulou, "Use of Weibull distribution for describing outdoor multipath fading," in *Proc. Antennas and Propagation Society Int. Sym.*, vol. 1, San Antonio, TX, Jun. 2002, pp. 232–235.
[16] P. A. Bello, "A generic channel simulator for wireless channels," in *Proc. IEEE Military Communications Conf.*, vol. 3, Monterey, CA, Nov. 1997, pp. 1575–1579.
[17] W. P. Qing and J. E. Allnutt, "12-GHz fade durations and intervals in the tropics," *IEEE Trans. Antennas Propagat.*, vol. 52, no. 3, pp. 693–701, Mar. 2004.
[18] E. C. de Miranda, M. S. Pontes, L. A. R. da Silva Mello, and M. P. Chaves, "Dynamic statistics of attenuation for 12 GHz satellite beacon link in Brazil," in *Proc. Int. Conf. Antennas and Propagation*, Edinburgh, U.K., Apr. 1997, pp. 251–254.
[19] G. E. Corazza, A. V. Coralli, R. Pedone, and M. Neri, in *Signal Processing for Mobile Communications Handbook*. Boca Raton, FL: CRC, 2004.
[20] M.-S. Alouini and M. K. Simon, "Performance of generalized selection combining over Weibull fading channels," in *Proc. IEEE Vehicular Technology Conf.*, Rhodes, Greece, May 2001, pp. 1735–1739.
[21] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 6th ed. New York: Academic, 2000.
[22] N. Kong and L. B. Milstein, "Average SNR of generalized diversity selection combining scheme," *IEEE Commun. Lett.*, vol. 3, no. 5, pp. 57–59, Mar. 1999.
[23] M. Z. Win, R. K. Mallik, and G. Chrisikos, "Higher order statistics of antenna subset diversity," *IEEE Trans. Wireless Commun.*, vol. 2, no. 5, pp. 871–875, Sep. 2003.
[24] G. A. Baker and P. Graves-Morris, *Padé Approximants*. Cambridge Univ., U.K.: Cambridge Press, 1996.
[25] G. K. Karagiannidis, "Moments-based approach to the performance analysis of equal gain diversity in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 52, no. 5, pp. 685–690, May 2004.

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