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# On the cascaded Weibull fading channel model

Nikos C. Sagias[a], George S. Tombras[b,*]

[a]Institute of Informatics and Telecommunications, National Centre for Scientific Research–''Demokritos'', Agia Paraskevi, GR-15310 Athens, Greece

[b]Laboratory of Electronics, Department of Physics, University of Athens, Panepistimiopolis, Zografou, GR-15784 Athens, Greece

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**Abstract**

A new stochastic fading channel model called cascaded Weibull fading is introduced and the associated capacity is derived in closed form. This model is generated by the product of independent, but not necessarily identically distributed, Weibull random variables (RVs). By quantifying the convergence rate of the central limit theorem as pertaining to the multiplication of Weibull distributed RVs, the statistical basis of the lognormal distribution is investigated. By performing Kolmogorov–Smirnov tests, the null hypothesis for this product to be approximated by the lognormal distribution is studied. Another null hypothesis is also examined for this product to be approximated by a Weibull distribution with properly adjusted statistical parameters.


*Keywords:* Fading channels; Weibull distribution; Lognormal distribution; Cascaded fading; Shannon capacity; Kolmogorov–Smirnov test; Central limit theorem (CLT); Stochastic models; Product of variates

## 1. Introduction

Due to the existence of a great variety of fading environments, several distributions have been proposed for channel modeling of fading envelopes under short, long, as well as mixed fading conditions [1]. Recently, attention has been given to the so-called ''multiplicative'' stochastic models. Such models do not separate the fading in several parts but rather study the phenomenon as a whole. A physical interpretation for these

*Corresponding author.
*E-mail addresses:* nsagias@ieee.org (N.C. Sagias), gtombras@cc.uoa.gr (G.S. Tombras).
*URL:* http://www.iit.demokritos.gr/~nsagias.

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models has been given by considering the received signals as being generated by the product of a large number of rays reflected via $N$ statistically independent scatterers [2]. For example, for $N = 2$, the so-called *double Rayleigh* (i.e., Rayleigh$\times$Rayleigh) fading stochastic model has been found to be suitable when both transmitter and receiver are moving [3]. It is interesting to note that the *double Rayleigh* model has been recently used for keyhole channel modeling of multiple-input multiple-output (MIMO) systems [4,5]. Extending this model by characterizing the fading between each pair of the transmit and receive antennas in the presence of the keyhole as Nakagami-$m$, the *double Nakagami-m* (i.e., Nakagami-$m\times$Nakagami-$m$) fading model has been considered [6]. Also, for higher values of $N$, Coulson et al. have studied the distribution of the product of $N$ correlated Rayleigh distributed random variables (RVs) via computer simulations [7]. Interestingly enough, such products of (not necessarily correlated) RVs have been also found to be useful in other scientific fields of communications. For example, they have been used for analyzing the performance of multihop systems [8,9] as well as to derive closed-form upper bounds for the distribution of the sum of RVs [10].

As far as long term fading conditions are considered, it is known that the probability density function (PDF) of the fading envelopes can be modeled by the well-known lognormal distribution [11]. Despite its wide acceptance in wireless communication theory, this distribution is empirical and it is hard to be justified [2]. Understanding the origin of the lognormal distribution is important and it can be tackled by investigating its statistical basis as pertaining to the multiplication of RVs. By applying the central limit theorem (CLT) to the logarithm of a product of a large number of RVs, it can be easily shown that it tends toward the normal (Gaussian) distribution. Consequently, the distribution of this product tends towards the lognormal distribution [12, pp. 220–221]. In [7], by performing Kolmogorov–Smirnov (K–S) tests, the convergence of CLT toward the lognormal distribution has been investigated and it has been deduced that more than 30 RVs ($N > 30$) should be multiplied, for this product to converge. This observation has also been reported in an independent work by Andersen [2]. In another paper, Fante has investigated the validity of the CLT for the magnitude of the sum of $N$ $K$-distributed complex RVs [13] (having uniformly distributed phase). In that work, it has been demonstrated that an order of 200 RVs must be summed to obtain amplitudes that can be approximated by the Weibull distribution. The Weibull distribution has been extensively used for modeling several types of radar clutter as well as mobile fading channels. However, to the best of the authors' knowledge, the (multiplicative) cascaded Weibull fading stochastic model, generated by the product of independent but not necessarily identically distributed Weibull RVs, has never been addressed before.

In this paper, the cascaded Weibull fading channel model is introduced and its capacity is derived in closed form. Additionally, K–S tests are performed to study the convergence rate of this novel stochastic model toward the lognormal distribution. The null hypothesis for our model to be approximated by a Weibull distribution with properly adjusted statistical parameters (equating both first and second order moments) is also examined.

The remaining of the paper is organized as follows. In Section 2, the statistics of the cascaded Weibull stochastic model is presented. Section 3 provides the channel capacity of this new model, while in Section 4, the convergence rates of the cascaded Weibull fading model toward the lognormal and Weibull distributions are examined. Concluding remarks are provided in Section 5.

## 2. Statistics of the cascaded Weibull stochastic model

We consider $N \geqslant 1$ independent, not necessarily identical, Weibull RVs, $R_\ell$ ($\ell = 1, 2, \ldots, N$), each with PDF [14, Eq. (3)]

$$f_{R_\ell}(r) = \frac{\beta_\ell}{\Omega_\ell} r^{\beta_\ell - 1} \exp\left(-\frac{r^{\beta_\ell}}{\Omega_\ell}\right), \tag{1}$$

where $\Omega_\ell = \mathbb{E}\langle R_\ell^{\beta_\ell}\rangle$ is the average fading power, $\Omega_\ell^{2/\beta_\ell} = \mathbb{E}\langle R_\ell^2\rangle / \Gamma(1 + 2/\beta_\ell)$, $\Gamma(\cdot)$ is the Gamma function [15, Eq. (8.310/1)] ($\mathbb{E}\langle\cdot\rangle$ denoting expectation), and $\beta_\ell > 0$ is the fading parameter expressing the fading severity. As $\beta_\ell$ increases, the fading severity decreases, while for the special case of $\beta_\ell = 2$, Eq. (1) reduces to the well-known Rayleigh PDF [1, Eq. (2.6)]. Moreover, for the special case of $\beta_\ell = 1$, Eq. (1) reduces to the well-known exponential PDF.

**Definition 1.** The RV $Y$ is defined as the product of $N$ Weibull distributed RVs $R_\ell$, i.e.,

$$Y \triangleq \prod_{i=1}^{N} R_i \tag{2}$$

with corresponding $\beta_\ell$'s belonging to rationals.

**Theorem 1.** *The PDF of $Y$ is given by*

$$f_Y(y) = \frac{\sqrt{n}V}{y} G_{m,0}^{0,m}\left[\frac{Wn^n}{y^n} \middle| \begin{matrix} I_n(\beta_\ell; 0) \\ - \end{matrix}\right], \tag{3}$$

*where $G[\cdot]$ is the Meijer's G-function*[1] [15, Eq. (9.301)],

$$I_n(\beta_\ell; x) \triangleq \Delta\left(\frac{n}{\beta_1}; x\right), \quad \Delta\left(\frac{n}{\beta_2}; x\right), \ldots, \Delta\left(\frac{n}{\beta_N}; x\right)$$

*with $\Delta(p; x)$ defined as $\Delta(p; x) \triangleq x/p, (x+1)/p, \ldots, (x+p-1)/p$, $x$ being an arbitrary real value and $p$ a positive integer,*

$$V = \sqrt{n}(\sqrt{2}\,\pi)^{N-m} \prod_{i=1}^{N} \sqrt{\frac{n}{\beta_i}} \tag{4}$$

*and*

$$W = \frac{1}{n^n} \prod_{i=1}^{N} \left(\frac{n\,\Omega_i}{\beta_i}\right)^{n/\beta_i}. \tag{5}$$

*Moreover, $n$ and $m$ are two positive integers given by*

$$n = \prod_{i=1}^{N} k_i \tag{6}$$

[1] $G[\cdot]$ can be expressed in terms of more familiar generalized hypergeometric functions ${}_pF_q(\cdot;\cdot;\cdot)$ [15, Eq. (9.14/1)] using the transformation presented in [15, Eq. (9.303)], with $p$ and $q$ being positive integers. Note that both $G[\cdot]$ and ${}_pF_q(\cdot;\cdot;\cdot)$ are included as built-in functions in most of the popular mathematical software packages such as *Maple* or *Mathematica*.

*and*

$$m = n\sum_{i=1}^{N}\frac{1}{\beta_i} \tag{7}$$

*under the constraints that* $k_\ell$ *and*

$$l_\ell = \frac{1}{\beta_\ell}\prod_{i=1}^{\ell}k_i \tag{8}$$

*are minimum positive integers.*

**Proof.** See [16].

Based on Eq. (8), the $N$ pairs $(k_\ell, l_\ell)$ can be found as follows: Depending upon the value of $\beta_1$, $k_1$ and $l_1$ are two minimum positive integers such that $l_1 = k_1/\beta_1$ holds (e.g., for $\beta_1 = 2.5$, $k_1 = 5$ and $l_1 = 2$). Next, depending upon the value of $\beta_2$, $(k_2, l_2)$ is another pair of minimum positive integers such that $l_2 = k_1 k_2/\beta_2$ holds, and finally, a similar procedure is performed till finding the $N$th pair of minimum positive integers $(k_N, l_N)$ such that Eq. (8) holds.

Some special cases of Eq. (3) are as follows: For $N = 1$, and using [15, Eq. (9.31/2)] and [17, Eq. (07.34.03.0046.01)], Eq. (3) simplifies to Eq. (1). For identical and integer order fading parameters, i.e., $\beta_\ell = \beta\ \forall\ell$, with $\beta \in \mathbb{N}$ ($\mathbb{N}$ denotes the set of natural numbers), $n = \beta$, $m = N$ ($l_1 = 1$, $k_1 = \beta$, and $k_\ell = l_\ell = 1\ \forall\ell\geqslant 2$), and hence, Eq. (3) can be expressed as

$$f_Y(y) = \frac{\beta}{y}G_{N,0}^{0,N}\left[\frac{\prod_{i=1}^{N}\Omega_i}{y^\beta}\middle|\begin{matrix}0,0,\ldots,0\\ -\end{matrix}\right]. \tag{9}$$

The above PDF for $N = 2$ reduces to

$$f_Y(y) = \frac{2\beta}{\Omega_1\Omega_2}y^{\beta-1}K_0\left(2\sqrt{\frac{y^\beta}{\Omega_1\Omega_2}}\right) \tag{10}$$

with $K_0(\cdot)$ being the zeroth order modified Bessel function of the second kind, while for $N = 3$ and 4, two useful formulae composed by familiar functions such as Bessel or hypergeometric can be obtained, using [17, Eqs. (07.34.03.0197.01) and (07.34.03.0222.01)], respectively, but they are not given here due to their not so compact form. Note also that for $\beta = 2$ and $\Omega_1 = \Omega_2 = \Omega$, Eq. (10) reduces to a known result [10, Eq. (18)], which is the PDF of the *double Rayleigh* stochastic model.

## 3. Channel capacity

In this section, first the statistics of the second power of the cascaded Weibull fading envelope is analyzed and then used to derive the Shannon capacity of our new cascaded channel model.

### 3.1. Statistics of the signal-to-noise ratio

Let us consider a digital communication system operating over the previously described cascaded Weibull fading channel model. The instantaneous signal-to-noise ratio (SNR) at the input of the receiver is given by

$$\Lambda = \frac{E_{\rm s}}{N_0} Y^2, \tag{11}$$

where $E_{\rm s}$ is the transmitted symbol's average energy and $N_0$ is the single-sided additive white Gaussian noise (AWGN) power spectral density, while the corresponding average SNR is

$$\overline{\Lambda} = \mathbb{E}\langle Y^2 \rangle \frac{E_{\rm s}}{N_0} = \frac{E_{\rm s}}{N_0} \prod_{i=1}^{N} \Omega_i^{2/\beta_i} \Gamma\left(1 + \frac{2}{\beta_i}\right). \tag{12}$$

Based on an interesting property of the Weibull distribution, that the $n$th power of a Weibull distributed RV with parameters $(\beta_\ell, \Omega_\ell)$ is another Weibull distributed RV with parameters $(\beta_\ell/n, \Omega_\ell)$ [18, Theorem 1], it can be easily concluded that $\Lambda$ is also a Weibull RV with parameters $(\beta_\ell/2, (\Xi_\ell \overline{\Lambda}_\ell)^{\beta_\ell/2})$, where $\overline{\Lambda}_\ell = \mathbb{E}\langle R_\ell^2 \rangle E_{\rm s}/N_0$ $(\Lambda_\ell = R_\ell^2 E_{\rm s}/N_0)$ and $\Xi_\ell = 1/\Gamma(1 + 2/\beta_\ell)$. Using the above mentioned property, the PDF of $\Lambda$ can be easily derived, replacing $\beta_\ell$ with $\beta_\ell/2$ and $\Omega_\ell$ with $(\Xi_\ell \overline{\Lambda}_\ell)^{\beta_\ell/2}$ helping us to study the capacity of the cascaded Weibull fading channel. Hence from Eq. (3), the PDF of $\Lambda$ can be easily derived as

$$f_\Lambda(\lambda) = \frac{\sqrt{n}\, V}{\lambda} G_{m,0}^{0,m}\left[\frac{W n^n}{\lambda^n} \,\middle|\, \begin{matrix} I_n(\beta_\ell/2; 0) \\ - \end{matrix}\right]. \tag{13}$$

Also, hereafter in this paper, $V$ and $W$ are modified as

$$V = \sqrt{n}\,(\sqrt{2\pi})^{N-m} \prod_{i=1}^{N} \sqrt{\frac{2n}{\beta_i}} \tag{14}$$

and

$$W = \frac{1}{n^n} \prod_{i=1}^{N} (\Xi_i \overline{\Lambda}_i)^n \left(\frac{2n}{\beta_i}\right)^{2n/\beta_i} \tag{15}$$

with $m = 2n\sum_{i=1}^{N}(1/\beta_i)$ and $l_\ell = 2\,(\prod_{i=1}^{\ell} k_i)/\beta_\ell$.

### 3.2. Average channel capacity

We consider an adaptive transmission scheme where optimal rate adaptation with constant transmit power is applied. This scheme entails variable-rate transmission relative to the channel, but is rather practical since the transmit power remains constant.

For a transmitted signal of bandwidth $B_{\rm w}$ over the AWGN channel, the channel capacity is given by [19, Eq. (1), 20]

$$C = B_{\rm w} \log_2\left(1 + \frac{E_{\rm s}}{N_0}\right). \tag{16}$$

When the same signal is transmitted over the cascaded Weibull fading channel, the (normalized to $B_{\rm w}$) instantaneous channel capacity, $S_e = C/B_{\rm w}$, can be considered as a RV and it can be obtained by averaging $\log_2(1+\Lambda)$ over the PDF of $\Lambda$, i.e.,

$$\overline{S}_e = \int_0^\infty \log_2(1+\lambda) f_\Lambda(\lambda)\,{\rm d}\lambda. \tag{17}$$

By substituting Eq. (13) in the above equation, and after some mathematical transformations, an integral of the form

$$\int_0^\infty \lambda^{-1}\,\ln(1+\lambda)\, G_{m,0}^{0,m}\left[\frac{W\,n^n}{\lambda^n}\middle|\begin{matrix} I_n(\beta_\ell/2;0) \\ - \end{matrix}\right]{\rm d}\lambda$$

appears. Similarly to [21, Appendix] and using [22, Eqs. (11) and (21)], $\overline{S}_e$ can be expressed in closed form as

$$\overline{S}_e = \frac{V}{\ln(2)\sqrt{n}}\, G_{2n,\,m+2n}^{m+2n,\,n}\left[\frac{1}{W\,n^n}\middle|\begin{matrix} \Delta(n;0), \Delta(n;1) \\ I_n(\beta_\ell/2;1), \Delta(n;0), \Delta(n;0) \end{matrix}\right]. \tag{18}$$

Note that for $N=1$, the above equation reduces to an already known expression [20, Eq. (17)].

### 3.3. Numerically evaluated results

Without loss of generality, in the results that follow, $\overline{\Lambda}_0 = \overline{\Lambda}_\ell$ and $\beta = \beta_\ell\ \forall \ell$. Having numerically evaluated Eq. (18), in Fig. 1, $\overline{S}_e$ is plotted as a function of $\overline{\Lambda}_0$, for the cascaded Weibull fading channel with $N = 1, 2, 3, 4$, and $\beta = 2.5$. These results clearly show that for

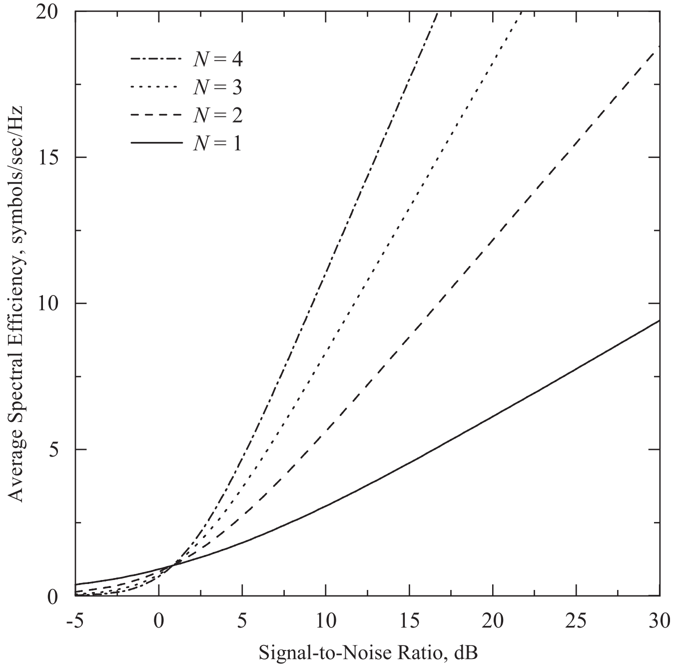


Fig. 1. Average spectral efficiency as a function of the average SNR for $\beta = 2.5$ and several values of $N$.

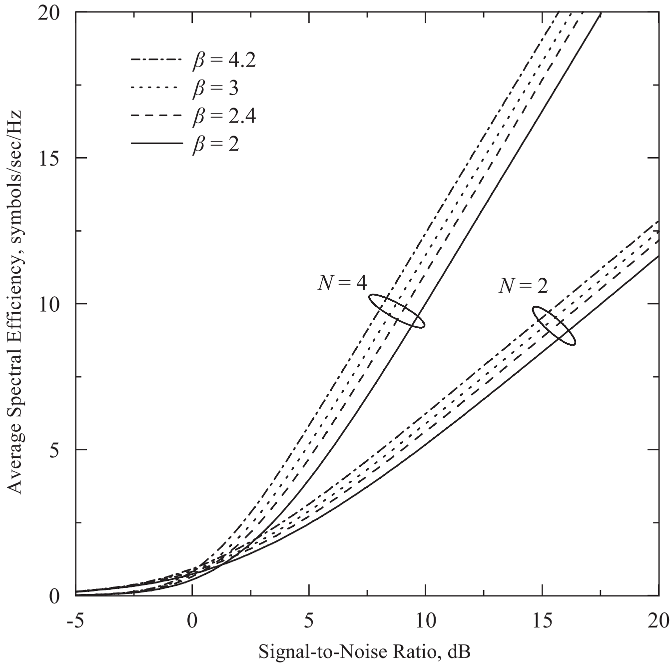


Fig. 2. Average spectral efficiency as a function of the average SNR for several values of $\beta$.

a fixed value of $N$, $\overline{S}_e$ improves as $\overline{\Lambda}_0$ increases. Moreover, the higher the value of $N$, the higher $\overline{S}_e$ is obtained. The results further indicate that for a given value of $\overline{\Lambda}_0 \simeq 1.5\,\text{dB}$, a threshold for $\overline{S}_e$ exists above (below) which $\overline{S}_e$ improves (degrades) with increasing $N$. Although not shown in Fig. 1, similar thresholds have been found for other values of $\beta$. Similar findings can be also extracted from Fig. 2 where $\overline{S}_e$ is plotted as a function of $\overline{\Lambda}_0$, for the cascaded Weibull fading channel with $N = 2$, 4, and $\beta = 2$, 2.4, 3, 4.2. As $\beta$ increases, higher $\overline{S}_e$ is obtained. This occurs because by increasing $\beta$, the fading severity of the cascaded channels decreases, and hence, deep fades generated by the product of Weibull fading envelopes occur less frequently.

## 4. Approximations for the cascaded Weibull fading model

Next, we investigate the necessary conditions for the distribution of the cascaded Weibull fading model to become equivalent (denoted next with the symbol $\equiv$) to the lognormal or the Weibull distributions.

### 4.1. Lognormal approximation

Let $\mu_{Y_\mathscr{P}}$ and $\sigma^2_{Y_\mathscr{P}}$ be the mean and the variance, respectively, of a lognormal RV $X$, with cumulative distribution function (CDF) given by

$$F_X(x) = 1 - \frac{1}{2}\,\text{erfc}\left[\frac{\ln(x) - \mu_{Y_\mathscr{P}}}{\sqrt{2}\,\sigma_{Y_\mathscr{P}}}\right], \tag{19}$$

where erfc(·) is the complementary error function [15, Eq. (8.250/4)]. After taking the natural logarithm in both parts of Eq. (2), $Y_{\mathscr{P}} = \ln(Y)$ can be written as

$$Y_{\mathscr{P}} = \sum_{i=1}^{N} \ln(R_i). \tag{20}$$

For a large value of $N$ and by applying the CLT, the second part of the above equation tends to the normal (Gaussian) distribution, and consequently, $Y$ tends to be lognormal [12, pp. 220–221]. A necessary condition in order to $Y \equiv X$ is that both $Y$ and $X$ should have the same mean

$$\mu_{Y_{\mathscr{P}}} = \mathbb{E}\langle \ln(Y) \rangle \tag{21}$$

and variance

$$\sigma^2_{Y_{\mathscr{P}}} = \mathbb{E}\langle \ln^2(Y) \rangle - \mathbb{E}^2\langle \ln(Y) \rangle. \tag{22}$$

The $p$th order moment of $\ln(R_\ell)$ is

$$\mathbb{E}\langle \ln^p(R_\ell) \rangle = \int_0^\infty \ln^p(r) f_{R_\ell}(r)\, \mathrm{d}r. \tag{23}$$

By substituting Eq. (1) in the above equation, applying the transformation $z = r^{\beta_\ell}$, using [15, Eq. (4.358/1)], and after some straightforward mathematical manipulations, $\mathbb{E}\langle \ln^p(R_\ell) \rangle$ can be obtained as

$$\mathbb{E}\langle \ln^p(R_\ell) \rangle = \frac{p}{\beta_\ell \Omega_\ell} \frac{\partial^p}{\partial s^p} \left[ \Omega_\ell^s \Gamma(s) \right] \Big|_{s=1}. \tag{24}$$

Hence, with the aid of Eq. (2) and setting $p = 1$ and 2 in Eq. (24), the mean and variance of $Y_{\mathscr{P}}$ can be determined as

$$\mu_{Y_{\mathscr{P}}} = \sum_{i=1}^{N} \frac{1}{\beta_i} [\ln(\Omega_i) - \mathscr{C}] \tag{25}$$

and

$$\sigma^2_{Y_{\mathscr{P}}} = \frac{\pi^2}{6} \sum_{i=1}^{N} \frac{1}{\beta_i^2}, \tag{26}$$

respectively, where $\mathscr{C}$ is Euler's constant [15, Section 9.73].

**Definition 2.** We define $\mathrm{H}_{11}$ as the null hypothesis that generated samples of $Y$ belong to the CDF of the lognormal distribution.

### *4.2. Weibull approximation*

Let $Z$ be a Weibull distributed RV with fading and shaping parameters $b$ and $\Psi$, respectively. Like as in the previous subsection, a necessary condition in order $Y \equiv Z$ is that both should have the same mean and variance. By equating the first two moments of $Y$ and $Z$, i.e., $\mathbb{E}\langle Y^p \rangle = \mathbb{E}\langle Z^p \rangle$, with $p = 1$ and 2, the non-linear set of equations

$$\prod_{i=1}^{N} \Omega_i^{1/\beta_i} \Gamma\left(1 + \frac{1}{\beta_i}\right) = \Psi^{1/b} \Gamma\left(1 + \frac{1}{b}\right), \tag{27}$$

$$\prod_{i=1}^{N} \Omega_i^{2/\beta_i} \Gamma\left(1 + \frac{2}{\beta_i}\right) = \Psi^{2/b} \Gamma\left(1 + \frac{2}{b}\right) \tag{28}$$

should be numerically solved.

**Definition 3.** We define $H_{12}$ as the null hypothesis that generated samples of $Y$ belong to the CDF of the Weibull distribution.

### 4.3. K–S goodness-of-fit test

In order to measure the overall difference between two CDFs, a number of statistics, such as the absolute value of the area between them or their integrated mean square difference, can be applied. The K–S statistic $\mathscr{T}$ is a particularly simple measure which is defined as the maximum value of the absolute difference between the two CDFs of $X$ (or $Z$) and $Y$. Thus, for comparing one data set with CDF $F_Y$ to the known CDF $F_X$ (or $F_Z$), the K–S statistic is

$$\mathscr{T} = \max |F_X(x) - F_Y(x)|. \tag{29}$$

To test the null hypotheses $H_{11}$ (or $H_{12}$) that observed data of $Y$ belong to analytical CDF $F_X$ (or $F_Z$), respectively, the K–S goodness-of-fit test compares $\mathscr{T}$ to a critical level $\mathscr{T}_{\max}$, as

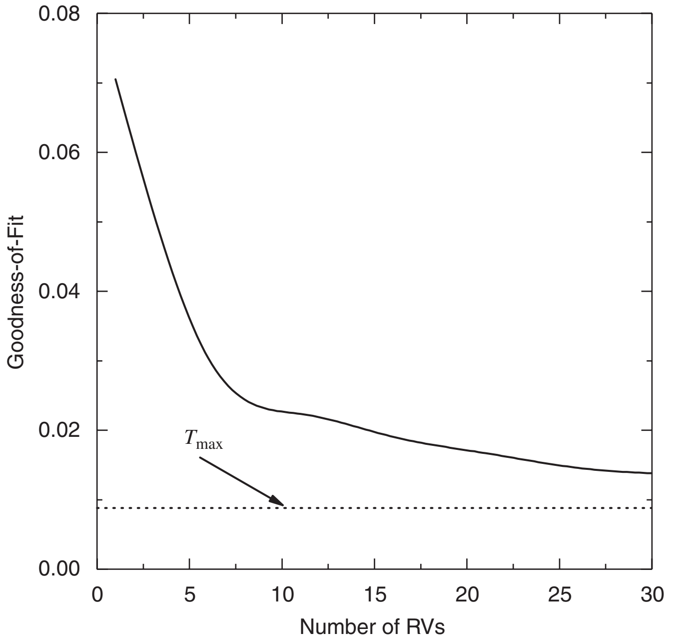


Fig. 3. Hypothesis testing distribution of the product of Weibull distributed RVs. Comparison of K–S tests (for $H_{11}$: $Y$ be lognormal) referred to 5% significance level.

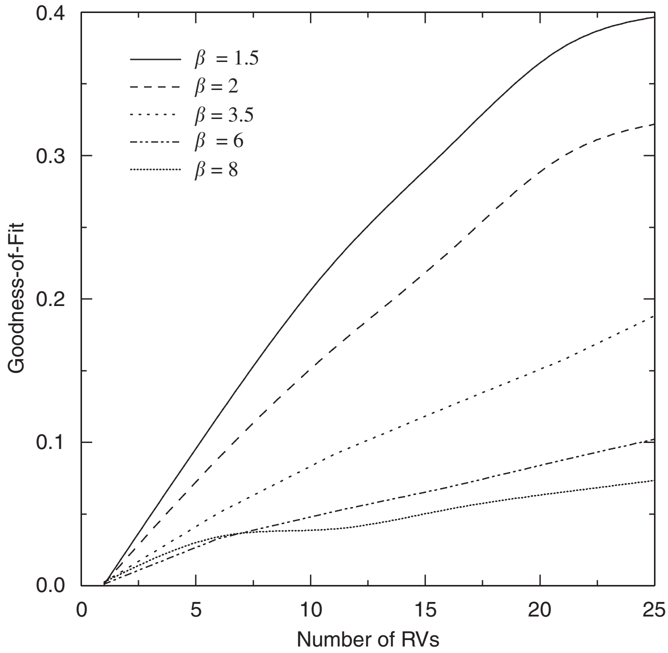


Fig. 4. Hypothesis testing distribution of the product of Weibull distributed RVs. Comparison of K–S tests (for $H_{12}$: $Y$ be Weibull) referred to 5% significance level.

a function of $N$ and a significance level $\alpha$. Any hypothesis for which $\mathscr{T} > \mathscr{T}_{max}$ is rejected with significance $1-\alpha$, while any hypothesis for which $\mathscr{T} < \mathscr{T}_{max}$ is accepted with the same level of significance.

### *4.4. Simulation results*

We compare values of $\mathscr{T}$, as calculated using Eq. (29), for a sample vector length 10 000 and significance level $\alpha = 5\%$. Note that similarly to [7], our results have been obtained by averaging the results of 30 simulation runs, each for 10 000 samples. Without loss of generality, RVs $R_\ell$ are independent and identically distributed Weibull RVs ($\Omega_\ell = \Omega$ and $\beta_\ell = \beta\ \forall \ell$). This choice is arbitrary, since there is not any empirical justification for a probably more accurate assumption [7]. Fig. 3 tests hypothesis $H_{11}$. Our simulations show that, independently of the value of $\beta$, when $1 < N \leqslant 30$, hypothesis $H_{11}$ is rejected with 95% significance ($\mathscr{T} > 0.09$) [7, Table I], although the distribution of $Y$ is clearly seen to converge toward the lognormal distribution with increasing $N > 30$. This observation agrees with [2,7]. Fig. 4 tests hypothesis $H_{12}$ for several values of $\beta$, where values of $\mathscr{T}$ are plotted for the data tested against the number of RVs $N$. For $N > 1$, $H_{12}$ is rejected with 95% significance, despite that the higher the value of $\beta$, the more it resembles the two distributions.

## 5. Conclusions

A cascaded Weibull fading stochastic model was introduced and its channel capacity was derived in closed form. Additionally, the empirical justification for the lognormal PDF in multipath fading channels was examined by quantifying the rates of convergence of the CLT for the multiplication of Weibull distributed RVs. It was shown that more than 30 multipliers are required for the product of Weibull RVs to be accurately approximated as being lognormally distributed. The hypothesis for this product to be Weibull distributed was also rejected for all $N > 1$. Experimental channel measurements would be interesting to be conducted, which may verify the suitability of the proposed cascaded stochastic model to be used in realistic wireless fading channels.

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