Optimal Combining for Optical Wireless Systems With Amplification: The χ^2 Noise Regime

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***Abstract*— We present novel analytical results on the optimal combiner (OC) for pre-amplified optical wireless receivers under chi-squared noise. The results show that the OC architecture that minimizes the average bit-error rate (BER) is determined by the level of the signal-spontaneous beating noise. Moreover, the OC provides increased gain to the diversity branches with the higher energies, up to the point where the beating noise becomes detrimental to the BER. The performance of the OC is simulated for practical receiver arrangements and it is shown that it performs better than the maximal-ratio and equal-gain combiners. The performance improvement amounts to an energy gain of less than 1 dB for implementations that utilize narrow optical filters and a modest number of diversity branches.**

***Index Terms*— Bit-error rate, diversity, optical wireless communications, optical amplifiers, optimal combiner.**

I. Introduction

THE ongoing quest for increasing wireless capacity has drawn considerable attention to optical wireless communication (OWC) systems, which rely on well-established optical technologies to deliver broadband links in environments that range from residences [1] and cities [2], to the deep space [3] and under the oceans [4]. Unlike their fiber-based counterparts, OWC links are impaired by random phenomena that include the pressure- and temperature-dependent refractive index of the transmission medium, and misalignments due to building sways and satellite vibrations, to name a few, and which introduce stochastic fluctuations in the received optical power [5]. As in any wireless system, these fluctuations adversely affect the operation of the OWC systems and a number of techniques have been proposed to limit their impact, including the concurrent utilization of diversity reception and optical amplification [6]–[10].

In previous works, we showed that optical amplification combined with spatial diversity is well-suited to mitigate the impact of random fluctuations that owe to small and large scale scattering, typically referred to as fading, and that an optimal combiner (OC) structure exists [10]. These works rely on the assumption that the noise at each optical receiver can be modelled as a Gaussian random variable (RV) [11], thus the overall noise in the combiner is also Gaussian. A more accurate modelling, however, treats noise as a non-central χ^2 RV [12] and the overall noise statistics at the combiner output become intractable since, to the best of our knowledge, the weighted sum of non-central χ^2 RVs does not have a unified closed-form expression for the probability density function (pdf). As a result, existing works on amplification and spatial diversity that also consider χ^2 noise statistics focus on the equal-gain combiner (EGC) [8], which is straightforward to implement and also enables the closed-form manipulation of optical noise, given that the sum of equally weighted χ^2 RVs is also a χ^2 RV. Despite its simplicity, the EGC is not the OC architecture for this specific application [13], and it is also not known to what extent the OC outperforms it.

In the current work we derive analytical results for a spatial combiner that is optimal in the sense that it minimizes the error probability at any given channel fading state. The analytical results are presented in Section II and enable the calculation of the variable gains (weights) that must be applied on each receiving branch after opto-electronic conversion. We show that the optimal strategy is to prioritize the branches with the highest energy signal up to the point where their contribution on the overall signal-spontaneous beating becomes detrimental to the error probability. As such, the OC only operates similarly to the EGC only when the branch energies are high and the optical noise is low, while it reverts to a maximal-ratio combiner (MRC) when the branch energies are low. We then assess in Section III the average bit-error rate (BER) performance of the OC in an intense fading environment. The results show that all three combiner structures under examination perform close to each other for practical receiver arrangements with a limited number of branches and relatively narrow optical and electrical filters. The observed link gain that corresponds to utilizing the OC instead of the EGC amounts to less than 1 dB for all considered setups.

II. Optimal Combiner Architecture

The setup of the OWC receiving system under study is briefly presented in Fig. 1. The system consists of L receiving branches and a fixed-gain optical amplifier is utilized at each branch to boost the signal energy prior to opto-electronic conversion on photodiodes (PDs). The resulting electrical signals X_ℓ, $\ell = 1, 2, \ldots, L$ are multiplied by variable gains w_ℓ and their linear combination

$$Z = \sum_{\ell=1}^{L} w_\ell \, X_\ell \,, \tag{1}$$

is fed to the digital decision circuit. Due to the existence of the amplifiers, the optical signals are corrupted by noise and

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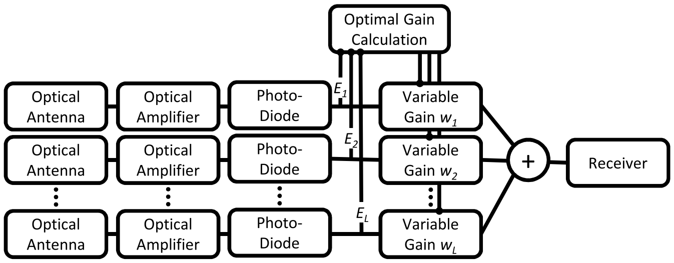

Fig. 1. Optical wireless receiver with amplification and spatial diversity.

hence the output of each PD equals to

$$X_\ell = \sum_{j=1}^{2m} \left(s_{\ell j} + n_{\ell j}\right)^2, \tag{2}$$

where $E_\ell = \sum_{j=1}^{2m} s_{\ell j}^2$ stands for the energy of the ℓth amplifier output, $n_{\ell j}$'s are filtered additive white Gaussian noise (AWGN) processes that originate from the optical amplifier noise and m is equal to the ratio of the optical B_o and electrical B_e bandwidths [14].

A. Statistics of the Output Electrical Signal

Assuming that the optical noise dominates the receiver and that the electrical noise can be ignored, X_ℓ is a non-central χ^2 RV with $2m$ degrees of freedom. The Laplace transform of the pdf of X_ℓ equals to

$$F_{X_\ell}(s, E_\ell, m) = \frac{1}{(1 + s\,N_0)^m} \exp\left(\frac{-s\,E_\ell}{1 + s\,N_0}\right), \tag{3}$$

where N_0 is the optical noise spectral density [12]. We also assume that the optical signals that arrive at each branch are uncorrelated, which holds true if the spatial separation between branches is of the order of a few cm. Then RVs X_ℓ are also uncorrelated and the Laplace transform of the pdf of the combiner output Z equals

$$F_Z\,(s, \boldsymbol{E}, \boldsymbol{m}, \boldsymbol{w}) = \prod_{\ell=1}^{L} F_{X_\ell}(w_\ell\, s, E_\ell, m_\ell). \tag{4}$$

$\boldsymbol{E}$, $\boldsymbol{m}$ and $\boldsymbol{w}$ are vector form representations of the branch energies, degrees of freedom and combiner gains. In general, both the branch energies E_ℓ and gains w_ℓ will not be equal to each other and will depend on the fading state of the OWC channel, while a common value m is expected for the degrees of freedom of all branches.

B. Optimal Threshold and Gains

For the rest of this analysis an On-Off keying (OOK) modulation format is considered. The corresponding probability of error P_e of the arrangement is given by

$$P_e = \frac{1}{2}\int_{z_{\text{th}}}^{\infty} f_Z(z, \boldsymbol{0}, \boldsymbol{m}, \boldsymbol{w})\,\mathrm{d}z + \frac{1}{2}\int_{0}^{z_{\text{th}}} f_Z(z, \boldsymbol{E}, \boldsymbol{m}, \boldsymbol{w})\,\mathrm{d}z, \tag{5}$$

since no energy is received during 0' bits. The optimal decision threshold z_{th} is calculated by minimizing P_e, which results in equating the combiner pdfs for the '0' and '1' bits

$$f_Z\,(z_{\text{th}}, \boldsymbol{0}, \boldsymbol{m}, \boldsymbol{w}) = f_Z\,(z_{\text{th}}, \boldsymbol{E}, \boldsymbol{m}, \boldsymbol{w}). \tag{6}$$

The optimal gains are obtained by differentiating (5) as

$$\int_0^{z_{\text{th}}} \frac{\partial f_Z(z, \boldsymbol{0}, \boldsymbol{m}, \boldsymbol{w})}{\partial w_\ell}\,\mathrm{d}z = \int_0^{z_{\text{th}}} \frac{\partial f_Z(z, \boldsymbol{E}, \boldsymbol{m}, \boldsymbol{w})}{\partial w_\ell}\,\mathrm{d}z. \tag{7}$$

It is, however, straightforward to verify from (3) and (4) that

$$\frac{\partial F_Z(s, \boldsymbol{E}, \boldsymbol{m}, \boldsymbol{w})}{\partial w_\ell} = -F_Z\,(s, \boldsymbol{E}, \boldsymbol{m}, \boldsymbol{w}) \times s\,\frac{E_\ell + m_\ell\,N_0\,(1 + w_\ell\,s\,N_0)}{(1 + w_\ell\,s\,N_0)^2}. \tag{8}$$

For ease of notation we re-write the last equation as

$$\frac{\partial F_Z(s, \boldsymbol{E}, \boldsymbol{m}, \boldsymbol{w})}{\partial w_\ell} = -s\,E_\ell\,F_Z\left(s, \boldsymbol{E}, \boldsymbol{m}_\ell^{2}, \boldsymbol{w}\right) - s\,m_\ell\,N_0\,F_Z\left(s, \boldsymbol{E}, \boldsymbol{m}_\ell^{1}, \boldsymbol{w}\right), \tag{9}$$

where vector $\boldsymbol{m}_\ell^{k}$ is obtained from $\boldsymbol{m}$ after increasing the degrees of freedom of branch ℓ to $2\,(m + k)$. Given the derivative properties of the Laplace transform we find that

$$\frac{\partial f_Z(z, \boldsymbol{E}, \boldsymbol{m}, \boldsymbol{w})}{\partial w_\ell} = -E_\ell\,\frac{\partial f_Z(z, \boldsymbol{E}, \boldsymbol{m}_\ell^{2}, \boldsymbol{w})}{\partial z} - m_\ell\,N_0\,\frac{\partial f_Z(z, \boldsymbol{E}, \boldsymbol{m}_\ell^{1}, \boldsymbol{w})}{\partial z}, \tag{10}$$

and after combining (7) and (10), the optimal gains $\boldsymbol{w}_{\textbf{opt}}$ are calculated from

$$f_Z\left(z_{\text{th}}, \boldsymbol{0}, \boldsymbol{m}_\ell^{1}, \boldsymbol{w}_{\textbf{opt}}\right) = f_Z\left(z_{\text{th}}, \boldsymbol{E}, \boldsymbol{m}_\ell^{1}, \boldsymbol{w}_{\textbf{opt}}\right) + \frac{E_\ell}{m_\ell\,N_0}\,f_Z\left(z_{\text{th}}, \boldsymbol{E}, \boldsymbol{m}_\ell^{2}, \boldsymbol{w}_{\textbf{opt}}\right). \tag{11}$$

As a result, both the decision threshold and OC gains can be found after solving non-linear equations that involve the knowledge of the pdf of a sum of χ^2 RVs with unequal shape parameters E_ℓ and degrees of freedom $\boldsymbol{m}_\ell^{k}$.

C. Numerical Results

A closed-form solution for the pdf of the sum of non-central χ^2 RVs is not generally known, but it is possible to numerically invert the Laplace transform of (4) and solve for the corresponding threshold and gain equations with conventional root-finding algorithms [15]. The calculated OC gain ratio $w_{1,\text{opt}}/w_{2,\text{opt}}$ is presented in Fig. 2 for a two-branch combiner and a range of optical signal-to-noise ratios (OSNRs) E_ℓ/N_0. For comparison purposes, the figure also includes the gain ratio of both the EGC, which constitutes a simple combiner implementation, and the MRC, which is the OC in receivers that are dominated by Gaussian thermal noise. The EGC gain ratio equals unity, while the MRC gain ratio increases linearly with the OSNR ratio following $w_1/w_2 = OSNR_1/OSNR_2$.

The numerical results show that the combiner gains depend strongly on the OSNRs of the branches, and the OC operates

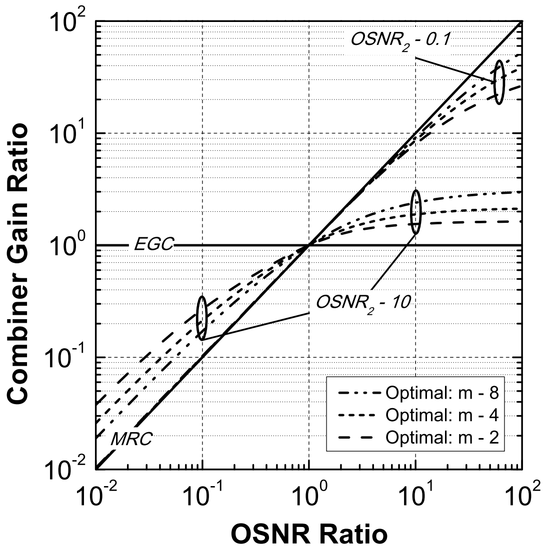

Fig. 2. Dual-branch optimal combiner gain ratio $w_{1,\mathrm{opt}}/w_{2,\mathrm{opt}}$ as a function of the OSNRs of the two branches $OSNR_1 = E_1/N_0$ and $OSNR_2 = E_2/N_0$.

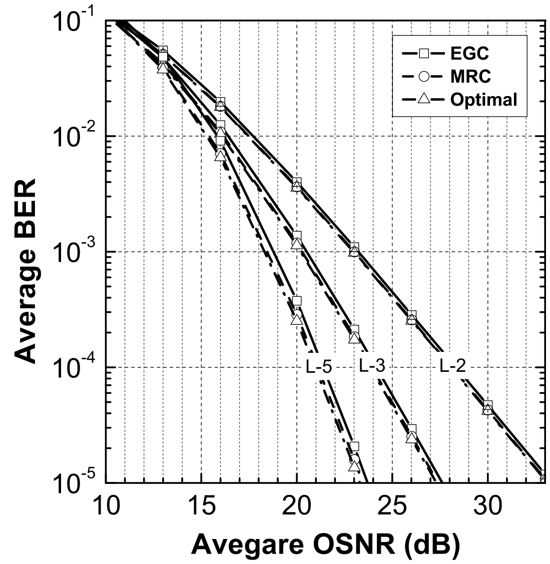

Fig. 3. Average BER performance of the OC, EGC and MRC for $m = 2$.

in close agreement with the MRC, i.e. $w_{\ell,\mathrm{opt}} = E_\ell/N_0$, when both branches receive a weak signal. Considering the plots for $OSNR_2 = 0.1$, it can be verified that the OC gains are practically identical to their MRC counterparts as long as $OSNR_1$ remains equally low. For an increasing $OSNR_1$, the gain ratio increases non-linearly with $w_{1,\mathrm{opt}}/w_{2,\mathrm{opt}} < OSNR_1/OSNR_2$, which means that the gain in the higher energy branch w_1 grows at a decreased rate with respect to the gain in the lower energy branch w_2. A symmetric behavior is observed for $OSNR_2 = 10$ and a low $OSNR_1$, since the first branch is now the weaker signal one. Finally, when both branches experience high OSNRs the OC reduces both gains and the gain ratio almost stabilizes, rendering the OC operation similar to that of the EGC.

Following the above, the OC provides a smaller relative gain to the branches with the strongest signal. This observation can be explained from the fact that a significant component of the system noise is signal dependent due to the signal-spontaneous beating on the PDs. The MRC is not optimal in the noise-dependent noise regime, since it will prioritize the branch with the strongest signal without taking into account that it also suffers from the highest noise level. In contrast, the OC limits the corresponding branch gain so as not to aggravate its noise contribution.

As far as the degrees of freedom m are concerned, it can be verified that increasing values shift the gain ratio of the OC from the EGC towards the MRC, especially in high OSNRs. A possible explanation is that the signal-independent (spontaneous-spontaneous beating) noise component increases with m, while the signal-dependent component remains unaffected [12]. Thus, branches with a low input energy become more detrimental to the OC operation, which then compensates by further increasing the gain at higher energy branches. In addition, the χ^2 noise statistics become more "Gaussian-like" with increasing degrees of freedom, and the OC is expected to perform closer to the MRC.

III. Average BER Performance and Discussion

The average BER performance of the OC is evaluated under a strong turbulence, negative-exponential (NE) fading environment, where the instantaneous branch energy E_ℓ is distributed following

$$f_{E_\ell}(E) = \frac{L}{\overline{E}_{\mathrm{in}}} \exp\left(-\frac{L\,E}{\overline{E}_{\mathrm{in}}}\right). \tag{12}$$

$\overline{E}_{\mathrm{in}}$ corresponds to the average optical energy at the combiner input. Extensive Monte-Carlo simulations were performed, since the OC gains can not be calculated in a simple analytical fashion, which also makes the calculation of the pdf of the combiner signal z intractable. The simulation results are presented in Figs. 3 and 4, where the average BER is plotted against the average OSNR $\overline{E}_{\mathrm{in}}/N_0$ for $m = 2$, 8, respectively, and $L = 2$, 3, 5.

The figures show that the OC always achieves a better BER than the MRC. The energy gain that is observed increases with the number of branches, but the BER discrepancy between the two combiners becomes less important with increasing m. This is consistent with the analysis of the previous section, since increasing m results in noise statistics that resemble a Gaussian distribution and the MRC is the (almost) OC. The maximum expected benefit from utilizing the OC instead of the MRC is observed for $m = 2$, where an energy gain of less than 0.5 dB is predicted for $L = 5$ branches.

A second interesting result relates to the comparison between the OC and the EGC. As it becomes evident from the figures, the discrepancy between the EGC and the OC is less than 1 dB, while the minimum difference is observed for $m = 2$. Therefore, if narrow optical filters are utilized to reject the amplifier noise the benefit is two-fold: (*a*) the system average BER is improved, as expected, and (*b*) the combiner operation is simplified and the EGC is adequate to provide almost optimal operation. This last result has also been observed in an OWC system operating under Gaussian

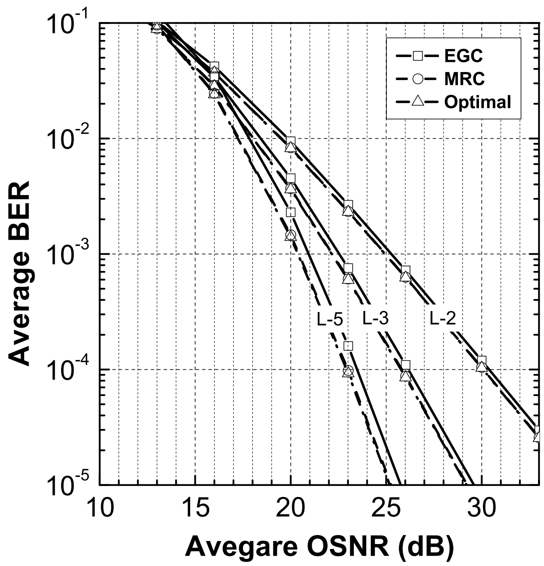

Fig. 4. Average BER performance of the OC, EGC and MRC for $m = 8$.

noise [10], while similar results are expected for less intense fading conditions where the OC is expected to impart an even smaller performance improvement.

In view of the simulation results and the performance similarities that the OC exhibits with the EGC and the MRC, it is of interest to discuss the complexity it adds. As it becomes evident from Section II, the OC requires (a) information on the channel state of all L receiving branches, which can be obtained by using channel estimation techniques [16], and (b) the solution of $L + 1$ equations for the optimal threshold and branch gains, which can be performed in software or hardware. The MRC is somewhat simpler, as the branch gains are calculated without solving the corresponding equations, but it also requires channel estimation on all L branches and a calculation of the optimal threshold by inverting (4). Thus the MRC reduces the problem size, but does not actually simplify the problem itself. Finally, the EGC is simpler since the noise statistics of the signal sum follows a non-central χ^2 distribution with $L m$ degrees of freedom and an equivalent signal energy of $E = \sum_{\ell=1}^{L} E_\ell$. Therefore, the EGC does not completely avoid channel estimation, as it requires knowledge of the sum energy, but the optimal threshold can be found by equating two χ^2 pdfs, which leads to both a size reduction and a simplification in equation solving. The OC implementation seems to be more appealing than the EGC whenever large optical bandwidths are required, for example so as to mitigate impairments that have been ignored in the presented analysis, such as non-ideal filter shapes or finite extinction ratios [17]. If the optical bandwidth can be kept to a minimum, the EGC is a well suited alternative, especially if the OWC system is further assisted against fading, for example by the use of efficient coding techniques that are expected to further bridge the gap between the two combiners [18].

IV. Conclusion

We have presented analytical relations for calculating the OC gains in OWC systems with spatial diversity and amplification. The OC operates similarly to the MRC when the OSNRs of the diversity branches are low, but becomes more EGC-like with increasing OSNRs so as to reduce the deleterious impact of signal-noise beating. The optical noise levels also affect the combiner operation, and its response becomes almost identical to the MRC one for increased noise powers, where the χ^2 distribution can be sufficiently approximated by a Gaussian. The average BER performance of OC has been assessed with Monte-Carlo simulations. The results show that all three combiner structures perform very close to each other, and that the EGC is almost optimal in low noise OWC systems with a limited number of diversity branches.

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