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Higher Order Capacity Statistics of Diversity Receivers

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Abstract A statistical analysis for the channel capacity (CC) for several diversity receivers under optimal rate adaptation with constant transmit power is provided. Independent but not necessarily identically distributed Nakagami-*m* fading channels are considered. Specifically, the moments of the CC at the output of selection combining, maximal-ratio combining, and switched and stay combining are obtained, assuming integer-order fading parameters, while for the Rayleigh model the moments of the CC at the output of equal-gain combining and generalized-selection combining are derived in closed form. Using these formulas, a new performance criterion, namely as fading figure (FF) as well as the variance, skewness, and kurtosis, are studied. Our findings show that the FF improves with an increase of the signal-to-noise ratio (SNR), the fading parameters, and/or the diversity order. Also, unlike to the variance of the error probability, the variance of the CC is a monotonic function of the average input SNR.

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1 Introduction

Diversity is an efficient means to provide a wireless link performance improvement at relatively low cost [1]. Among a wide range of diversity combining implementations, the most popular techniques are maximal-ratio combining (MRC), equal-gain combining (EGC), selection combining (SC), switched and stay combining (SSC), and a combination of MRC and SC, identified as generalized-selection combining (GSC) [2]. This latter type has been proposed to bridge the gap between the two extreme cases (MRC and SC) balancing between performance maximizing and overall complexity minimizing.

A well-known performance metric, which provides an upper-bound for maximum errorless transmission rate in a given Gaussian environment, is the Shannon channel capacity (CC) [3], which has been already studied for several channel models with independent or correlated fading statistics and/or with or without diversity [4–15]. For example, in [4,5,10], and [14,16,17], the average CC of Rayleigh, Rice, Weibull, and Nakagami-*m* fading has been derived, respectively. Several average CC expressions with diversity over independent Rayleigh fading channels have been presented in [9], while over correlated in [11,15,18]. Recently, significant interest has been indicated not only for the average value of various performance metrics, but also for theirs higher order moments. For example, some efforts have been made to obtain the variance and moments of both the signal-to-noise ratio (SNR) [19–22] and the error probability [23,24]. More importantly, for the CC, which is of primary importance for multiple-input multiple-output (MIMO) systems [25–27], the variance and the moments in a correlated Rayleigh fading environment have been recently extracted in closed form [28]. However to the best of the authors' knowledge with except for [29], it seems that relatively less attention has been paid to the moments, as well as variances, and deviations of the CC provided by multichannel receivers. Without an accompanying moments analysis together with the mean analysis, the knowledge of representative fading models is at least incomplete.

In this paper, an analysis for the statistics of the output CC of several diversity receivers is presented under optimal rate adaptation (ORA) with constant transmit power. In this context, the moments of the CC at the output of EGC and GSC receivers are derived in closed form, when they operate over independent but not necessarily identically distributed (id) Rayleigh fading channels. The same metrics are also extracted for MRC, SC, and SSC in Nakagami-*m* fading with integer-order fading parameters. Based on these formulae, a new performance criterion, namely as fading figure (FF), as well as the variance, skewness, and kurtosis, are studied.

The remainder of this paper is organized as follows. The next section presents the statistical background and the channel models. In Sects. 3 and 4, the moments of the CC for several multi-branch diversity receivers over Rayleigh and Nakagami-*m* are obtained in closed form, respectively. In Sect. 5, some numerical results are presented, while the paper concludes with a summary given in Sect. 6.

2 Statistical Background and Channel Models

In this section, we consider the adaptive transmission scheme where ORA with constant transmit power is applied. This scheme entails variable-rate transmission relative to the channel, but is rather practical since the transmit power remains constant. Next, some elementary expressions characterizing the distribution of the CC in terms of moments are presented, while the Nakagami-*m* fading channel models is introduced.

By considering a signal transmission of bandwidth B_w over the additive white Gaussian noise (AWGN) channel, the spectral efficiency (SE) (or normalized to B_w CC) is given by [4, eq. (1)]

$$S_e = \frac{C_c}{B_w} = \log_2\left(1 + \frac{E_s}{N_0}\right) \tag{1}$$

where E_s is the average transmitted symbols energy and N_0 is the AWGN single-side power spectral density.

When the same signal is transmitted over a fading plus AWGN channel the received SNR denoted with γ will vary, and hence, $S_e = \log_2(1+\gamma)$ can be considered as a random variable (RV) of which the nth order moment can be derived as

$$\mu_n = \mathbb{E}\left\langle S_e^n \right\rangle = \int_0^\infty \log_2^n(1+\gamma) f_\gamma(\gamma)\mathrm{d}\gamma \tag{2}$$

with $f_\gamma(\cdot)$ being the probability density function (PDF) of γ, n being an arbitrary positive integer, and $\mathbb{E}\langle\cdot\rangle$ denoting averaging. Note that as it is well known, μ_1 is the mean value of S_e, $\mu_1 = \overline{S}_e$, while the variance of S_e, $\mathrm{var}(\mathrm{S_e})$, can be extracted from μ_2 as $\mathrm{var}(\mathrm{S_e}) = \mu_2 - \mu_1^2$.

2.1 Fading Figure

Having the first two moments of C_c in closed form, we define the FF as the ratio of the variance to the square mean of C_c, i.e.,

$$\eta \stackrel{\Delta}{=} \frac{\mathrm{var}\left(C_c\right)}{\mathbb{E}^2\left\langle C_c\right\rangle} = \frac{\mu_2}{\mu_1^2} - 1 \tag{3}$$

which is independent of B_w. For the AWGN channel where $\mu_2 = \mu_1^2, \eta = 0$, meaning that the lower the value of η, the “better” the channel is. Note also that the above definition is similar to the amount of fading which is formulated using the first two moments of the received SNR [22].

2.2 Kurtosis and Skewness

Two important metrics, which characterize a distribution, are the kurtosis and the skewness. The kurtosis, defined as

$$\mathscr{K} \stackrel{\Delta}{=} \frac{\mathbb{E}\left\langle (S_e - \mathbb{E}\left\langle S_e\right\rangle)^4\right\rangle}{\left(\mathbb{E}\left\langle S_e^2\right\rangle - \mathbb{E}^2\left\langle S_e\right\rangle\right)^2} = \frac{\mu_4 - \mu_1^4}{\left(\mu_2 - \mu_1^2\right)^2} \tag{4}$$

is a measure of the peakedness of a distribution. For high values of the kurtosis, the PDF is called leptokurtic, while for low (near to one), platykurtic. Leptokurtic RVs have typically a spiky PDF with heavy tails, i.e., the higher kurtosis is the lower is the concentration around its mean. The skewness is defined as

$$\mathscr{S} \triangleq \frac{\mathbb{E}\left\langle (S_e - \mathbb{E}\left\langle S_e \right\rangle)^3 \right\rangle}{\left(\mathbb{E}\left\langle S_e^2 \right\rangle - \mathbb{E}^2\left\langle S_e \right\rangle\right)^{3/2}} = \frac{\mu_3 - \mu_1^3}{\left(\mu_2 - \mu_1^2\right)^{3/2}} \tag{5}$$

and is a measure of the symmetry of a distribution. For symmetric distributions, $\mathscr{S} = 0$. If $\mathscr{S} > 0$, the distribution is skewed to the right.

2.3 Channel Model

We consider L flat Nakagami-m fading channels corrupted by AWGN. The instantaneous SNR per symbol and its average value in the ℓth ($1 \leq \ell \leq L$) input branch are denoted as γ_ℓ and $\overline{\gamma}_\ell$, respectively.[1]

By assuming integer-order fading parameters ($m_\ell = 1, 2, 3, \ldots$), the PDF and the cumulative distribution function (CDF) of γ_ℓ are given by [22]

$$f_{\gamma_\ell}(\gamma) = \left(\frac{m_\ell}{\overline{\gamma}_\ell}\right)^{m_\ell} \frac{\gamma^{m_\ell - 1}}{(m_\ell - 1)!} \exp\left(-m_\ell \frac{\gamma}{\overline{\gamma}_\ell}\right) \tag{6a}$$

and

$$F_{\gamma_\ell}(\gamma) = 1 - \exp\left(-\gamma \frac{m_\ell}{\overline{\gamma}_\ell}\right) \sum_{l=0}^{m_\ell - 1} \frac{1}{l!} \left(\gamma \frac{m_\ell}{\overline{\gamma}_\ell}\right)^l \tag{6b}$$

respectively. Note that for $m_\ell = 1$, the Rayleigh fading channel model is considered.

3 GSC and EGC Receivers over Rayleigh Fading

In this section, μ_n is derived for equal-gain and generalized-selection diversity receivers operating over a Rayleigh fading environment. Also, the first two moments of the CC are especially extracted.

3.1 Equal-Gain Diversity

For a dual-branch EGC, the instantaneous output SNR is [19]

$$\gamma_{\text{egc}} = \frac{1}{2}\left(\sqrt{\gamma_1} + \sqrt{\gamma_2}\right)^2. \tag{7}$$

For Rayleigh fading, the PDF of γ_{egc} is given by[2]

$$\begin{aligned} f_{\gamma_{\text{egc}}}(\gamma) = {} & \frac{\sqrt{2\overline{\gamma}_1\overline{\gamma}_2}}{\left(\overline{\gamma}_1 + \overline{\gamma}_2\right)^{3/2}} \exp\left(-\frac{2\gamma}{\overline{\gamma}_1 + \overline{\gamma}_2}\right) \sum_{i=0}^{1}\sum_{j=0}^{1} \left(\frac{2}{\overline{\gamma}_1 + \overline{\gamma}_2}\right)^{1-(i+j)/2} \left(\frac{\overline{\gamma}_1}{\overline{\gamma}_2}\right)^{(j-i)/2} \\ & \times \gamma^{(1-i-j)/2} \left[(-1)^i \gamma\left(\frac{1+i+j}{2}, \frac{2\gamma}{\overline{\gamma}_1 + \overline{\gamma}_2}\frac{\overline{\gamma}_1}{\overline{\gamma}_2}\right)\right. \\ & \left. + (-1)^j \gamma\left(\frac{1+i+j}{2}, \frac{2\gamma}{\overline{\gamma}_1 + \overline{\gamma}_2}\frac{\overline{\gamma}_2}{\overline{\gamma}_1}\right)\right] \end{aligned} \tag{8}$$

1 When considering single-branch receivers ($L = 1$) or id channels ($\overline{\gamma}_\ell = \overline{\gamma}$ and $m_\ell = m$), we just neglect the subscript ℓ.

2 Note that $\gamma(\cdot, \cdot)$ is different that the SNR symbols, γ_ℓ.

with $\gamma(\cdot,\cdot)$ being the lower incomplete gamma function [30, eq. (8.350/1)], $\gamma(a,x) = \Gamma(a) - \Gamma(a,x)$ $(a, x \geq 0)$. Using an infinite series representation [30, eq. (8.354/1)]

$$\gamma(a,x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(a+k)k!} x^{a+k} \tag{9}$$

(8) can be expressed as

$$\begin{aligned} f_{\gamma_{\rm egc}}(\gamma) = & \frac{2\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}{(\overline{\gamma}_1+\overline{\gamma}_2)^2} \exp\left(-\frac{2\gamma}{\overline{\gamma}_1+\overline{\gamma}_2}\right) \\ & \times \sum_{i=0}^{1}\sum_{j=0}^{1}\left[\sum_{a=0}^{\infty} \frac{(-1)^{a+i}}{\left(\frac{1+i+j}{2}+a\right)a!}\left(\frac{2\gamma}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{a+1}\left(\frac{\overline{\gamma}_1}{\overline{\gamma}_2}\right)^{a+j+1/2}\right. \\ & \left. + \sum_{b=0}^{\infty} \frac{(-1)^{b+j}}{\left(\frac{1+i+j}{2}+b\right)b!}\left(\frac{2\gamma}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{b+1}\left(\frac{\overline{\gamma}_2}{\overline{\gamma}_1}\right)^{b+i+1/2}\right]. \end{aligned} \tag{10}$$

By substituting the above PDF in (2), the nth order moment of the EGC output SE over Rayleigh fading can be obtained as

$$\begin{aligned} \mu_n = & \frac{n!}{\ln^n(2)} \frac{2\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}{(\overline{\gamma}_1+\overline{\gamma}_2)^2} \exp\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right) \\ & \times \sum_{i=0}^{1}\sum_{j=0}^{1}\left[\sum_{a=0}^{\infty}\sum_{k=0}^{a+1} \frac{(-1)^{i-k+1}}{\left(\frac{1+i+j}{2}+a\right)a!}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{a-k}\left(\frac{\overline{\gamma}_1}{\overline{\gamma}_2}\right)^{a+j+1/2}\right. \\ & \times \binom{a+1}{k} G_{n+1,n+2}^{n+2,0}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\middle|\begin{matrix}1,1,\ldots,1\\0,0,\ldots,0,k+1\end{matrix}\right) \\ & + \sum_{b=0}^{\infty}\sum_{h=0}^{b+1} \frac{(-1)^{i-h+1}}{\left(\frac{1+i+j}{2}+b\right)b!}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{b-h}\left(\frac{\overline{\gamma}_2}{\overline{\gamma}_1}\right)^{b+i+1/2} \\ & \left. \times \binom{b+1}{h} G_{n+1,n+2}^{n+2,0}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\middle|\begin{matrix}1,1,\ldots,1\\0,0,\ldots,0,h+1\end{matrix}\right)\right]. \end{aligned} \tag{11}$$

Average SE: From (11), for dual-branch EGC receivers, the average value of SE is

$$\begin{aligned} \mathbb{E}\langle S_e\rangle = & \frac{1}{\ln(2)} \frac{2\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}{(\overline{\gamma}_1+\overline{\gamma}_2)^2} \exp\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right) \\ & \times \sum_{i=0}^{1}\sum_{j=0}^{1}\left[\sum_{a=0}^{\infty}\sum_{k=0}^{a+1} \frac{(-1)^{i-k+1}}{\left(\frac{1+i+j}{2}+a\right)a!}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{a-k}\left(\frac{\overline{\gamma}_1}{\overline{\gamma}_2}\right)^{a+j+1/2}\right. \\ & \times \binom{a+1}{k} G_{2,3}^{3,0}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\middle|\begin{matrix}1,1\\0,0,k+1\end{matrix}\right) \\ & + \sum_{b=0}^{\infty}\sum_{h=0}^{b+1} \frac{(-1)^{i-h+1}}{\left(\frac{1+i+j}{2}+b\right)b!}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{b-h}\left(\frac{\overline{\gamma}_2}{\overline{\gamma}_1}\right)^{b+i+1/2} \end{aligned}$$

$$\times \binom{b+1}{h} G_{2,3}^{3,0}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\middle|\begin{matrix}1,1\\0,0,h+1\end{matrix}\right)\Bigg]. \tag{12}$$

Second moment of SE: By setting $n = 2$ in (11), the average square of SE of dual-branch EGC receivers can be obtained as

$$\begin{aligned}\mathbb{E}\left\langle S_e^2\right\rangle =& \frac{1}{\ln^2(2)}\frac{4\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}{\left(\overline{\gamma}_1+\overline{\gamma}_2\right)^2}\exp\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)\\ &\times\sum_{i=0}^{1}\sum_{j=0}^{1}\left[\sum_{a=0}^{\infty}\sum_{k=0}^{a+1}\frac{(-1)^{i-k+1}}{\left(\frac{1+i+j}{2}+a\right)a!}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{a-k}\left(\frac{\overline{\gamma}_1}{\overline{\gamma}_2}\right)^{a+j+1/2}\right.\\ &\times\binom{a+1}{k}G_{3,4}^{4,0}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\middle|\begin{matrix}1,1,1\\0,0,0,k+1\end{matrix}\right)\\ &+\sum_{b=0}^{\infty}\sum_{h=0}^{b+1}\frac{(-1)^{i-h+1}}{\left(\frac{1+i+j}{2}+b\right)b!}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{b-h}\left(\frac{\overline{\gamma}_2}{\overline{\gamma}_1}\right)^{b+i+1/2}\\ &\left.\times\binom{b+1}{h}G_{3,4}^{4,0}\left(\frac{2}{\overline{\gamma}_1+\overline{\gamma}_2}\middle|\begin{matrix}1,1,1\\0,0,0,h+1\end{matrix}\right)\right].\end{aligned} \tag{13}$$

3.2 Generalized-Selection Diversity

In GSC(N, L), the N strongest branches having the highest instantaneous SNR are selected among the L available and adaptively combined [22]. The GSC reception is equivalent to MRC reception if all L branches are combined (i.e., $N = L$), while it is equivalent to SC reception, if one of the L branches is utilized, (i.e., $N = 1$).

Without loss of generality, let $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_N$. The joint PDF of this ordered set is given by [31], [22, eq. (9.470)]

$$\begin{aligned}f_{\gamma_1\gamma_2,\ldots,\gamma_N}(\gamma_1,\gamma_2,\ldots,\gamma_N) =& \sum_{n_1=1}^{L}\sum_{\substack{n_2=1\\n_2\neq n_1}}^{L}\cdots\sum_{\substack{n_N=1\\n_N\neq n_{N-1}}}^{L}\left[\prod_{i=1}^{N}f_{\gamma_{n_i}}(\gamma_i)\right]\\ &\times\prod_{\substack{l'=N+1\\n_{l'}\neq n_1,n_2,\ldots,n_N}}^{L}F_{\gamma_{n_{l'}}}(\gamma_N)\end{aligned} \tag{14}$$

where the index with the prime refers to the $L - N$ unselected channel outputs, thus excluding all unprimed indexes occurring in any outer summations. The instantaneous GSC output SNR is

$$\gamma_{\text{gsc}} = \sum_{i=1}^{N}\gamma_i. \tag{15}$$

For Rayleigh fading, the PDF of γ_{gsc} is given by

$$f_{\gamma_{\text{gsc}}}(\gamma) = \sum_{\substack{\lambda_1,\lambda_2,\ldots,\lambda_L=1 \\ \lambda_1 \neq \lambda_2 \neq \cdots \neq \lambda_L}}^{L} \left(\prod_{l=1}^{L} \frac{\overline{\gamma}_{\lambda_l}^{-1}}{\min\{l, N\}} \right) \sum_{j=1}^{L} C_j \exp\left(-a_j \gamma\right) \tag{16}$$

with $a_j = \sum_{k=1}^{j} \overline{\gamma}_{\lambda_k}^{-1} / \min\{j, N\}$ and $C_j = \prod_{\substack{n=1 \\ n \neq j}}^{L} \left(a_n - a_j\right)^{-1}$. By substituting the above PDF in (2) and, the nth order moment of the GSC(N, L) output can be obtained as

$$\mu_n = \frac{n!}{\ln^n(2)} \sum_{\substack{\lambda_1,\lambda_2,\ldots,\lambda_L=1 \\ \lambda_1 \neq \lambda_2 \neq \cdots \neq \lambda_L}}^{L} \left(\prod_{l=1}^{L} \frac{\overline{\gamma}_{\lambda_l}^{-1}}{\min\{l, N\}} \right) \sum_{j=1}^{L} \frac{C_j}{a_j} \exp\left(a_j\right) G_{n+1,n+2}^{n+2,0}\left(a_j \,\middle|\, \begin{matrix} 1,1,\ldots,1 \\ 0,0,\ldots,0,1 \end{matrix}\right). \tag{17}$$

For the specific case of SC receiver, i.e., $N = 1$, the above expression reduces to

$$\begin{aligned} \mu_n = {} & \frac{n!}{\ln^n(2)} \sum_{k=1}^{L} (-1)^{k+1} \sum_{\lambda_1=1}^{L-k+1} \sum_{\lambda_2=\lambda_1+1}^{L-k+2} \cdots \sum_{\lambda_k=\lambda_{k-1}+1}^{L} \exp\left(\sum_{i=1}^{k} \frac{1}{\overline{\gamma}_{\lambda_i}} \right) \\ & \times G_{n+1,n+2}^{n+2,0}\left(\sum_{i=1}^{k} \frac{1}{\overline{\gamma}_{\lambda_i}} \,\middle|\, \begin{matrix} 1,1,\ldots,1 \\ 0,0,\ldots,0,1 \end{matrix} \right) \end{aligned} \tag{18}$$

while for $N = L$, i.e., MRC receivers, to

$$\mu_n = \frac{n!}{\ln^n(2)} \sum_{k=1}^{L} \exp\left(\frac{1}{\overline{\gamma}_k}\right) \left(\prod_{\substack{i=1 \\ i \neq k}}^{L} \frac{\overline{\gamma}_k}{\overline{\gamma}_k - \overline{\gamma}_i} \right) G_{n+1,n+2}^{n+2,0}\left(\frac{1}{\overline{\gamma}_k} \,\middle|\, \begin{matrix} 1,1,\ldots,1 \\ 0,0,\ldots,0,1 \end{matrix} \right). \tag{19}$$

Average SE: For GSC(N, L) receivers, the average output SE is

$$\mathbb{E}\left\langle S_e \right\rangle = \frac{1}{\ln(2)} \sum_{\substack{\lambda_1,\lambda_2,\ldots,\lambda_L=1 \\ \lambda_1 \neq \lambda_2 \neq \cdots \neq \lambda_L}}^{L} \left(\prod_{l=1}^{L} \frac{\overline{\gamma}_{\lambda_l}^{-1}}{\min\{l, N\}} \right) \sum_{j=1}^{L} \frac{C_j}{a_j} \exp\left(a_j\right) \Gamma\left(0, a_j\right). \tag{20}$$

Second moment of SE: Using (17), the average square of SE is

$$\mathbb{E}\left\langle S_e^2 \right\rangle = \frac{2}{\ln^2(2)} \sum_{\substack{\lambda_1,\lambda_2,\ldots,\lambda_L=1 \\ \lambda_1 \neq \lambda_2 \neq \cdots \neq \lambda_L}}^{L} \left(\prod_{l=1}^{L} \frac{\overline{\gamma}_{\lambda_l}^{-1}}{\min\{l, N\}} \right) \sum_{j=1}^{L} \frac{C_j}{a_j} \exp\left(a_j\right) G_{3,4}^{4,0}\left(a_j \,\middle|\, \begin{matrix} 1,1,1 \\ 0,0,0,1 \end{matrix}\right). \tag{21}$$

4 Multi-Branch Receivers over Nakagami-*m* Fading

Next the CC analysis is generalized for multi-branch receivers operating over Nakagami-*m* fading channels, with integer-order fading parameters. The receivers under consideration are MRC, SC, and SSC.

4.1 Maximal-Ratio Diversity

The instantaneous output SNR of an L-branch MRC receiver is given by

$$\gamma_{\mathrm{mrc}} = \sum_{i=1}^{L} \gamma_i. \tag{22}$$

For Nakagami-m fading with id channels ($\overline{\gamma}_\ell = \overline{\gamma}$ and $m_\ell = m$), the PDF of γ_{mrc} is given by the Erlang distribution

$$f_{\gamma_{\mathrm{mrc}}}(\gamma) = \left(\frac{m}{\overline{\gamma}}\right)^{mL} \frac{\gamma^{mL-1}}{(mL-1)!} \exp\left(-m\frac{\gamma}{\overline{\gamma}}\right) \tag{23a}$$

while for distinct channels ($\overline{\gamma}_j/m_j \neq \overline{\gamma}_i/m_i, \forall i \neq j$) by [32, eq. (6)]

$$f_{\gamma_{\mathrm{mrc}}}(\gamma) = \sum_{i=1}^{L} \sum_{k=1}^{m_i} \Xi_{i,k} f_{\gamma_i}(\gamma) \tag{23b}$$

where $f_{\gamma_\ell}(\cdot)$ is given by (6a) and $\Xi_{i,k}$ by (24)

$$\begin{aligned}
\Xi_{i,k} = \sum_{l_1=k}^{m_i} \sum_{l_2=k}^{l_1} \cdots \sum_{l_{L-2}=k}^{l_{L-3}} \Bigg[& \frac{(-1)^{R_L - m_i} \eta_i^k}{\prod_{h=1}^{L} \eta_h^{m_h}} \frac{\left(m_i + m_{1+\mathrm{U}(1-i)} - l_1 - 1\right)!}{\left(m_{1+\mathrm{U}(1-i)} - 1\right)! \, (m_i - l_1)!} \\
& \times \left(\frac{1}{\eta_i} - \frac{1}{\eta_{1+\mathrm{U}(1-i)}}\right)^{l_1 - m_i - m_{1+\mathrm{U}(1-i)}} \frac{\left(l_{L-2} + m_{L-1+\mathrm{U}(L-1-i)} - k - 1\right)!}{\left(m_{L-1+\mathrm{U}(L-1-i)} - 1\right)! \, (l_{L-2} - k)!} \\
& \times \prod_{s=1}^{L-3} \frac{\left(l_s + m_{s+1+\mathrm{U}(s+1-i)} - l_{s+1} - 1\right)!}{\left(m_{s+1+\mathrm{U}(s+1-i)} - 1\right)! \, (l_s - l_{s+1})!} \\
& \times \left(\frac{1}{\eta_i} - \frac{1}{\eta_{s+1+\mathrm{U}(s+1-i)}}\right)^{l_{s+1} - l_s - m_{s+1+\mathrm{U}(s+1-i)}} \Bigg]
\end{aligned} \tag{24}$$

with $\eta_\ell = \overline{\gamma}_\ell / m_\ell$ and $U(\cdot)$ being the well-known unit step function, where $U(x \geq 0) = 1$, else $U(x < 0) = 0$.

By substituting (23) in (2), interchanging the order of summation and integration, using [26, eq. (37)], and after some straightforward mathematical manipulations, the nth order moment of SE of MRC receiver operating over Nakagami-m fading and id channels can be obtained in closed form as

$$\begin{aligned}
\mu_n = & \frac{n!}{\ln^n(2)} \frac{\exp(m/\overline{\gamma})}{(mL-1)!} \sum_{k=0}^{mL-1} (-1)^{mL-k-1} \binom{mL-1}{k} \left(\frac{\overline{\gamma}}{m}\right)^{k+1-mL} \\
& \times G_{n+1,n+2}^{\,n+2,0}\left(\frac{m}{\overline{\gamma}} \,\middle|\, \begin{matrix} 1,1,\ldots,1 \\ 0,0,\ldots,0,k+1 \end{matrix}\right)
\end{aligned} \tag{25a}$$

while for distinct channels as

$$\begin{aligned}
\mu_n = & \frac{n!}{\ln^n(2)} \sum_{i=1}^{L} \sum_{q=1}^{m_i} \Xi_{i,q} \frac{\exp\left(m_i/\overline{\gamma}_i\right)}{(m_i-1)!} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i-1}{k} \\
& \times \left(\frac{\overline{\gamma}_i}{m_i}\right)^{k+1-m_i} G_{n+1,n+2}^{\,n+2,0}\left(\frac{m_i}{\overline{\gamma}_i} \,\middle|\, \begin{matrix} 1,1,\ldots,1 \\ 0,0,\ldots,0,k+1 \end{matrix}\right).
\end{aligned} \tag{25b}$$

For $L = 1$ and $m_\ell = 1 \forall \ell$, it can be verified that (25b) simplifies to

$$\mu_n = \frac{n!}{\ln^n(2)} \frac{\exp(m/\overline{\gamma})}{(m-1)!} \sum_{k=0}^{m-1} (-1)^{m-k-1} \binom{m-1}{k} \left(\frac{\overline{\gamma}}{m}\right)^{k+1-m} \times G_{n+1,n+2}^{n+2,0}\left(\frac{m}{\overline{\gamma}}\middle|\begin{matrix}1,1,\ldots,1\\0,0,\ldots,0,k+1\end{matrix}\right) \tag{26}$$

and (19), respectively.

Average SE: For id channels, the average SE yields

$$\mathbb{E}\langle S_e\rangle = \frac{1}{\ln(2)} \frac{\exp(m/\overline{\gamma})}{(mL-1)!} \sum_{k=0}^{mL-1} (-1)^{mL-k-1} \binom{mL-1}{k} \left(\frac{\overline{\gamma}}{m}\right)^{k+1-mL} \times G_{2,3}^{3,0}\left(\frac{m}{\overline{\gamma}}\middle|\begin{matrix}1,1\\0,0,k+1\end{matrix}\right) \tag{27a}$$

while for distinct channels

$$\mathbb{E}\langle S_e\rangle = \frac{1}{\ln(2)} \sum_{i=1}^{L}\sum_{q=1}^{m_i} \Xi_{i,q} \frac{\exp\left(m_i/\overline{\gamma}_i\right)}{(m_i-1)!} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i-1}{k} \times \left(\frac{\overline{\gamma}_i}{m_i}\right)^{k+1-m_i} G_{2,3}^{3,0}\left(\frac{m_i}{\overline{\gamma}_i}\middle|\begin{matrix}1,1\\0,0,k+1\end{matrix}\right). \tag{27b}$$

Second moment of SE: For id channels, the average square of SE can be obtained as

$$\mathbb{E}\left\langle S_e^2\right\rangle = \frac{2}{\ln^2(2)} \frac{\exp(m/\overline{\gamma})}{(mL-1)!} \sum_{k=0}^{mL-1} (-1)^{mL-k-1} \binom{mL-1}{k} \left(\frac{\overline{\gamma}}{m}\right)^{k+1-mL} \times G_{3,4}^{4,0}\left(\frac{m}{\overline{\gamma}}\middle|\begin{matrix}1,1,1\\0,0,0,k+1\end{matrix}\right) \tag{28a}$$

while for distinct channels as

$$\mathbb{E}\left\langle S_e^2\right\rangle = \frac{2}{\ln^2(2)} \sum_{i=1}^{L}\sum_{q=1}^{m_i} \Xi_{i,q} \frac{\exp\left(m_i/\overline{\gamma}_i\right)}{(m_i-1)!} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i-1}{k} \times \left(\frac{\overline{\gamma}_i}{m_i}\right)^{k+1-m_i} G_{3,4}^{4,0}\left(\frac{m_i}{\overline{\gamma}_i}\middle|\begin{matrix}1,1,1\\0,0,0,k+1\end{matrix}\right). \tag{28b}$$

4.2 Selection Diversity

By considering an L-branch SC receiver, the instantaneous SNR per symbol γ_ℓ at its output will be the one with the highest instantaneous value among the L branches [33], i.e.,

$$\gamma_{\rm sc} = \max\{\gamma_\ell\}. \tag{29}$$

The CDF of γ_{sc} equals to the probability $\Pr(\cdot)$ that the SNR signal levels of all branches fall below a certain level, which using (6b) can be expressed as

$$F_{\gamma_{\text{sc}}}(\gamma) = \prod_{k=1}^{L} \Pr(\gamma_{\text{sc}} \leq \gamma) = \prod_{k=1}^{L} F_{\gamma_k}(\gamma). \tag{30}$$

Using the binomial identity [30, eq. (1.111)], the CDF of γ_{sc} with id channels can be written as

$$F_{\gamma_{\text{sc}}}(\gamma) = 1 + \sum_{d=0}^{L-1} \binom{L}{d} (-1)^{L-d} \exp\left[-m(L-d)\frac{\gamma}{\overline{\gamma}}\right] \left[\sum_{l=0}^{m-1} \frac{1}{l!}\left(m\frac{\gamma}{\overline{\gamma}}\right)^{l}\right]^{L-d} \tag{31}$$

in which by applying the multinomial identity [34, eq. (24.1.2)] in the last term of the above product, yields

$$F_{\gamma_{\text{sc}}}(\gamma) = 1 + L! \sum_{d=0}^{L-1} \frac{(-1)^{L-d}}{d!} \exp\left[-m(L-d)\frac{\gamma}{\overline{\gamma}}\right] \times \sum_{\substack{k_0,k_1,\ldots,k_{m-1}=0\\ k_0+k_1+\cdots+k_{m-1}=L-d}}^{L-d} \left[\prod_{l=0}^{m-1} \frac{(m/\overline{\gamma})^{lk_l}}{k_l!(l!)^{k_l}}\right] \gamma^{\xi} \tag{32}$$

where $\xi = \sum_{l=1}^{m-1} lk_l$. By taking the first derivative of the above equation with respect to γ, the PDF of γ_{sc} can be expressed as

$$f_{\gamma_{\text{sc}}}(\gamma) = L! \sum_{d=0}^{L-1} \frac{(-1)^{L-d-1}}{d!} \exp\left[-m(L-d)\frac{\gamma}{\overline{\gamma}}\right] \times \sum_{\substack{k_0,k_1,\ldots,k_{m-1}=0\\ k_0+k_1+\cdots+k_{m-1}=L-d}}^{L-d} \left[\prod_{l=0}^{m-1} \frac{(m/\overline{\gamma})^{lk_l}}{k_l!(l!)^{k_l}}\right] \left(\frac{L-d}{\overline{\gamma}/m}\gamma^{\xi} - \xi\gamma^{\xi-1}\right). \tag{33}$$

By substituting (33) in (2), interchanging the order of summations and integration, and after some mathematical manipulations, the nth order moment of the SC output SE over id Nakagami-m fading can be obtained in closed form as

$$\mu_n = \frac{n!L!}{\ln^n(2)} \sum_{d=0}^{L-1} \frac{(-1)^{L-d-1}}{d!} \exp\left(\frac{L-d}{\overline{\gamma}/m}\right) \sum_{\substack{k_0,k_1,\ldots,k_{m-1}=0\\ k_0+k_1+\cdots+k_{m-1}=L-d}}^{L-d} \left[\prod_{l=0}^{m-1} \frac{(m/\overline{\gamma})^{lk_l}}{k_l!(l!)^{k_l}}\right] \times \left[\sum_{k=0}^{\xi}(-1)^{\xi-k}\binom{\xi}{k}\left(\frac{\overline{\gamma}/m}{L-d}\right)^{k} G_{n+1,n+2}^{n+2,0}\left(\frac{m/\overline{\gamma}}{L-d}\,\middle|\,\begin{matrix}1,1,\ldots,1\\0,0,\ldots,0,k+1\end{matrix}\right) + \xi\sum_{k=0}^{\xi-1}(-1)^{\xi-k}\binom{\xi-1}{k}\left(\frac{\overline{\gamma}/m}{L-d}\right)^{k+1} G_{n+1,n+2}^{n+2,0}\left(\frac{m/\overline{\gamma}}{L-d}\,\middle|\,\begin{matrix}1,1,\ldots,1\\0,0,\ldots,0,k+1\end{matrix}\right)\right]. \tag{34}$$

Average SE: The average SE is the first moment of S_e and can be simplified as

$$\mathbb{E}\left\langle S_e\right\rangle = \frac{L!}{\ln(2)}\sum_{d=0}^{L-1}\frac{(-1)^{L-d-1}}{d!}\exp\left(\frac{L-d}{\overline{\gamma}/m}\right)\sum_{\substack{k_0,k_1,\ldots,k_{m-1}=0\\ k_0+k_1+\cdots+k_{m-1}=L-d}}^{L-d}\left[\prod_{l=0}^{m-1}\frac{(m/\overline{\gamma})^{lk_l}}{k_l!(l!)^{k_l}}\right]$$
$$\times\left[\sum_{k=0}^{\xi}(-1)^{\xi-k}\binom{\xi}{k}\left(\frac{\overline{\gamma}/m}{L-d}\right)^{k}G_{2,3}^{3,0}\left(\frac{m/\overline{\gamma}}{L-d}\middle|\begin{matrix}1,1\\0,0,k+1\end{matrix}\right)\right.$$
$$\left.+\xi\sum_{k=0}^{\xi-1}(-1)^{\xi-k}\binom{\xi-1}{k}\left(\frac{\overline{\gamma}/m}{L-d}\right)^{k+1}G_{2,3}^{3,0}\left(\frac{m/\overline{\gamma}}{L-d}\middle|\begin{matrix}1,1\\0,0,k+1\end{matrix}\right)\right]. \quad (35)$$

Second moment of SE: Using (34), the second moment of S_e is

$$\mathbb{E}\left\langle S_e^2\right\rangle = \frac{2L!}{\ln^2(2)}\sum_{d=0}^{L-1}\frac{(-1)^{L-d-1}}{d!}\exp\left(\frac{L-d}{\overline{\gamma}/m}\right)\sum_{\substack{k_0,k_1,\ldots,k_{m-1}=0\\ k_0+k_1+\cdots+k_{m-1}=L-d}}^{L-d}\left[\prod_{l=0}^{m-1}\frac{(m/\overline{\gamma})^{lk_l}}{k_l!(l!)^{k_l}}\right]$$
$$\times\left[\sum_{k=0}^{\xi}(-1)^{\xi-k}\binom{\xi}{k}\left(\frac{\overline{\gamma}/m}{L-d}\right)^{k}G_{4,4}^{4,0}\left(\frac{m/\overline{\gamma}}{L-d}\middle|\begin{matrix}1,1,1\\0,0,0,k+1\end{matrix}\right)\right.$$
$$\left.+\xi\sum_{k=0}^{\xi-1}(-1)^{\xi-k}\binom{\xi-1}{k}\left(\frac{\overline{\gamma}/m}{L-d}\right)^{k+1}G_{3,4}^{4,0}\left(\frac{m/\overline{\gamma}}{L-d}\middle|\begin{matrix}1,1,1\\0,0,0,k+1\end{matrix}\right)\right]. \quad (36)$$

4.3 Switched Diversity

We consider a dual-branch SSC receiver operating over a flat fading environment. By denoting with γ_τ as the common switching threshold at both diversity branches, the PDF of $\gamma_{\rm ssc}$ is [35, eq. (4)]

$$f_{\gamma_{\rm ssc}}(\gamma) = \begin{cases}\frac{P_1\,P_2}{P_1+P_2}\,\sum_{i=1}^{2} f_{\gamma_\ell}(\gamma)\,, \gamma\le\gamma_\tau\\ \frac{P_1\,P_2}{P_1+P_2}\,\sum_{i=1}^{2} f_{\gamma_\ell}(\gamma)\left(1+\frac{1}{P_i}\right), \gamma>\gamma_\tau\end{cases} \quad (37)$$

where $P_\ell = F_{\gamma_\ell}(\gamma_\tau)$. For id channels, the common optimum switching threshold, γ_τ^*, for maximum average SE is [36]

$$\gamma_\tau^* = 2^{\mathbb{E}\langle S_e\rangle} - 1 \quad (38)$$

while for non id channels, γ_τ^* can be derived using any numerical method available in most of the well-known mathematical software packages, such as Mathematica and Maple.

By substituting (37) in (2) and taking into consideration (6a) and (6b), the nth order moment of the SSC output SE can be obtained as

$$\mu_n = \frac{1}{\ln^n(2)} \frac{P_1\,P_2}{P_1+P_2} \sum_{i=1}^{2} \Bigg[n! \left(1+\frac{1}{P_i}\right) \frac{\exp\left(m_i/\overline{\gamma}_i\right)}{(m_i-1)!} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i-1}{k} \\ \times \left(\frac{\overline{\gamma}_i}{m_i}\right)^{k+1-m_i} G_{n+1,n+2}^{\,n+2,0}\left(\frac{m_i}{\overline{\gamma}_i}\,\middle|\, {}^{1,1,\ldots,1}_{0,0,\ldots,0,k+1}\right) \\ - \left(\frac{m_i}{\overline{\gamma}_i}\right)^{m_i} \frac{1/P_i}{(m_i-1)!} \int_0^{\gamma_\tau} \ln^n(1+x)\, x^{m_i-1} \exp\left(-x\,\frac{m_i}{\overline{\gamma}_i}\right) \mathrm{d}x \Bigg]. \tag{39}$$

The above equation includes an integral with finite limits, which can be easily evaluated via numerical integration.

Average SE: Using (39), the average output SE of SSC can be obtained in closed form as

$$\mathbb{E}\langle S_e\rangle = \frac{1}{\ln(2)} \frac{P_1\,P_2}{P_1+P_2} \sum_{i=1}^{2} \Bigg[\left(1+\frac{1}{P_i}\right) \frac{\exp\left(m_i/\overline{\gamma}_i\right)}{(m_i-1)!} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \binom{m_i-1}{k} \\ \times \left(\frac{\overline{\gamma}_i}{m_i}\right)^{k+1-m_i} G_{2,3}^{3,0}\left(\frac{m_i}{\overline{\gamma}_i}\,\middle|\, {}^{1,1}_{0,0,k+1}\right) \\ - \left(\frac{m_i}{\overline{\gamma}_i}\right)^{m_i} \frac{1/P_i}{(m_i-1)!} \int_0^{\gamma_\tau} \ln(1+x)\, x^{m_i-1} \exp\left(-x\,\frac{m_i}{\overline{\gamma}_i}\right) \mathrm{d}x \Bigg]. \tag{40}$$

Second moment of SE: The average output square SE of SSC can be formulated as

$$\mathbb{E}\left\langle S_e^2\right\rangle = \frac{1}{\ln^2(2)} \frac{P_1\,P_2}{P_1+P_2} \sum_{i=1}^{2} \Bigg[2 \left(1+\frac{1}{P_i}\right) \frac{\exp\left(m_i/\overline{\gamma}_i\right)}{(m_i-1)!} \sum_{k=0}^{m_i-1} (-1)^{m_i-k-1} \\ \times \binom{m_i-1}{k} \left(\frac{\overline{\gamma}_i}{m_i}\right)^{k+1-m_i} G_{3,4}^{4,0}\left(\frac{m_i}{\overline{\gamma}_i}\,\middle|\, {}^{1,1,1}_{0,0,0,k+1}\right) \\ - \left(\frac{m_i}{\overline{\gamma}_i}\right)^{m_i} \frac{1/P_i}{(m_i-1)!} \int_0^{\gamma_\tau} \ln^2(1+x)\, x^{m_i-1} \exp\left(-x\,\frac{m_i}{\overline{\gamma}_i}\right) \mathrm{d}x \Bigg]. \tag{41}$$

5 Numerical Results

As typical examples, we numerically evaluate some of the above mentioned expressions for the FF, variance of SE, kurtosis, and skewness. In Figs. 1, 2, 3, 4, 5 and 6, Rayleigh channels are considered, while in Figs. 7 and 8, Nakagami-m. By using (18), in Fig. 1, η is plotted as a function of $\overline{\gamma}$ for id input SNRs and several values of L. It can be easily recognized that η reduces as $\overline{\gamma}$ increases, showing improved performance. Furthermore, η improves as L increases.

By considering an exponentially decaying power delay profile

$$\overline{\gamma}_\ell = \overline{\gamma}_1 \exp[-(\ell-1)\delta] \tag{42}$$

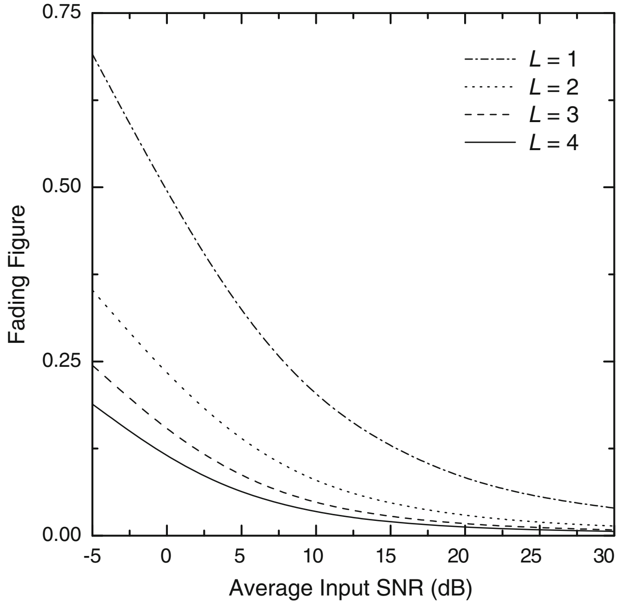


Fig. 1 Fading figure of an SC over Rayleigh fading as a function of the average input SNR

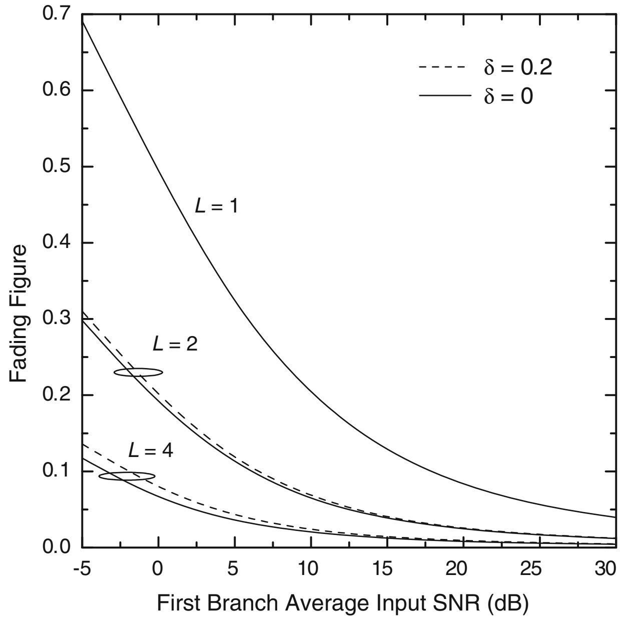


Fig. 2 Fading figure of multi-branch MRC receivers over Rayleigh fading, as a function of the first branch average input SNR

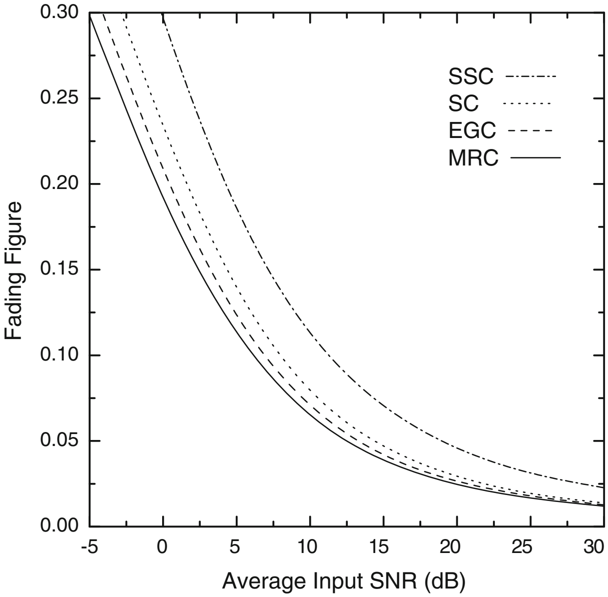


Fig. 3 Fading figure of dual-branch MRC, EGC, SC, and SSC receivers over Rayleigh fading, for id channels, as a function of the average input SNR

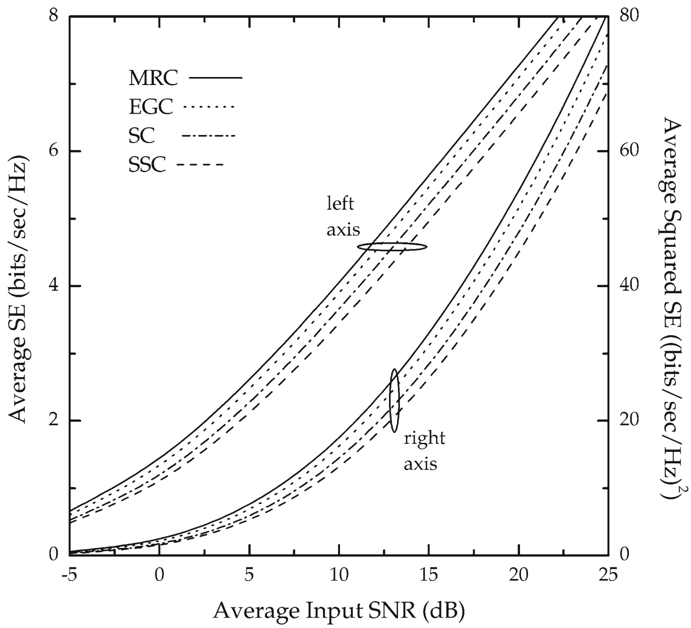


Fig. 4 Average SE and square SE of dual-branch MRC, EGC, SC, and SSC receivers over Rayleigh fading, for id channels, as a function of the average input SNR

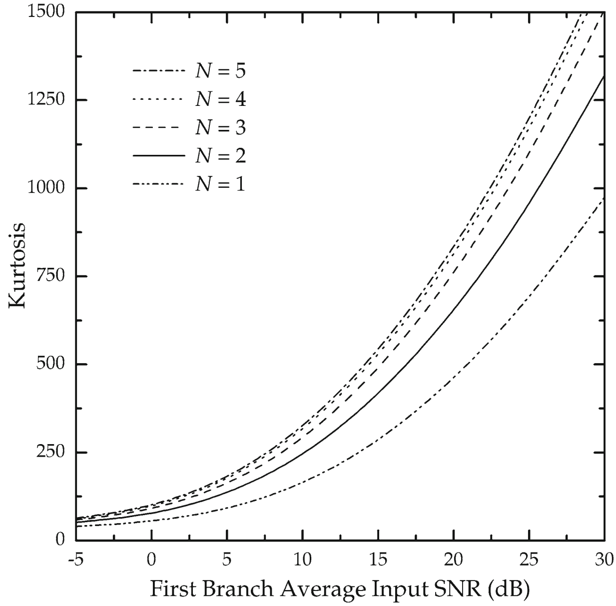


Fig. 5 Kurtosis of the SE at the output of a GSC(N, 5) for non-identical channels with $\delta = 0.2$ and Rayleigh fading, as a function of the first branch average input SNR

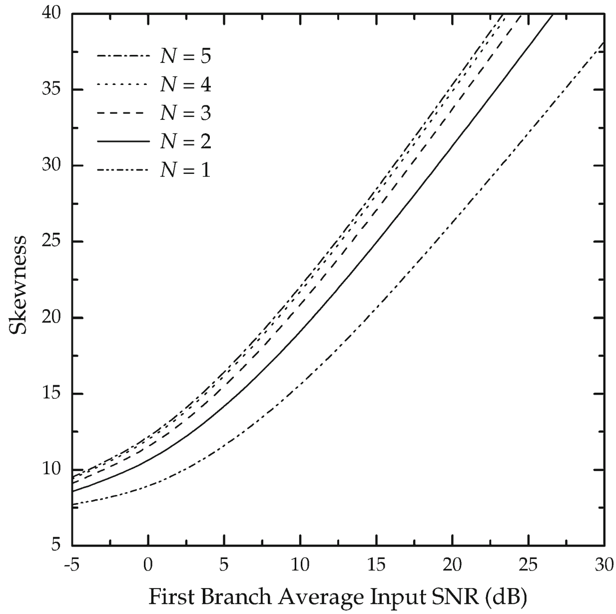
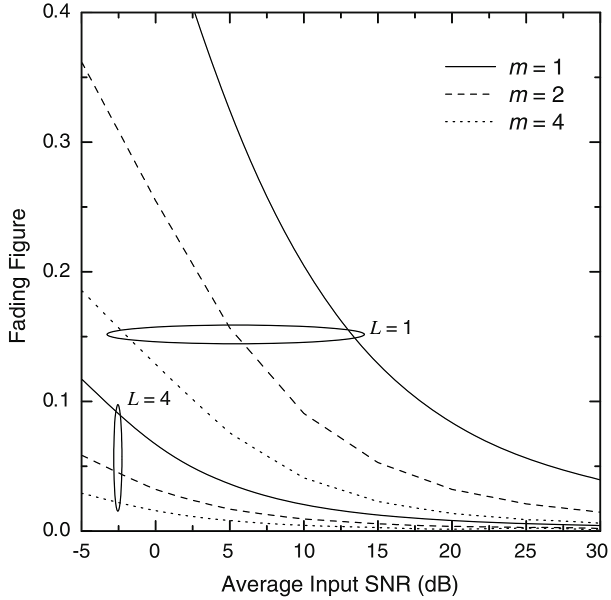


Fig. 6 Skewness of the SE at the output of a GSC(N, 5) for non-identical channels with $\delta = 0.2$ and Rayleigh fading, as a function of the first branch average input SNR

Fig. 7 Fading figure of quadruple-branch MRC in Nakagami-*m* fading, as a function of the average input SNR



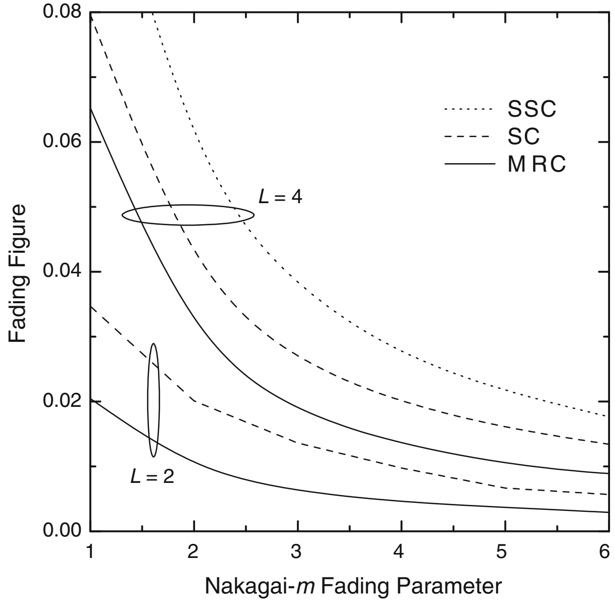


Fig. 8 Fading figure of dual- and quadruple-branch receivers in Nakagami-*m* fading, as a function of the fading parameter for $\overline{\gamma} = 10\,\text{dB}$

with δ being the power decaying factor, similar results for η at the MRC output are presented in Fig. 2, where both id and non id ($\delta = 0.2$) channels are considered. From this figure is shown that as expected, η increases as δ increases.

A comparison among dual-branch MRC, EGC, SC, and SSC is performed in Figs. 3 and 4, where the MRC provides the best performance. Specifically, Fig. 4 clearly shows that as

$\mathbb{E}\langle S_e \rangle$ increases, $\mathbb{E}\langle S_e^2 \rangle$ also increases reducing the value of η. It is interesting to be mentioned that unlike to the variance of the error probability [23], the variance of the SE monotonically increases as the average input SNR increases.

In Figs. 5 and 6, $\mathscr{K}$ and $\mathscr{S}$ are plotted, respectively, as a function of $\overline{\gamma}_1$ for a GSC(N, 5) receiver, non id channels ($\delta = 0.2$), and several values of N. From both figures is evident that the kurtosis and the skewness increase as $\overline{\gamma}_1$ and/or N increase, showing that the PDF of the SE becomes more spiky with heavy tails and asymmetric.

In Figs. 7 and 8, the FF curves are plotted for id Nakagami-m fading channels. In Fig. 7, η is plotted as a function of $\overline{\gamma}$ for an MRC receiver with $L = 4$ branches and several values of the Nakagami-m fading parameter. Additionally to the above mentioned findings, it can be observed that for fixed $\overline{\gamma}$ and L, η improves as m increases. In the same figure, curves for $L = 1$ are included for comparison purposes. In Fig. 8, curves for η are illustrated as a function of m for dual-branch SSC, SC, and MRC, as well as quadruple-branch SC and MRC receivers for $\overline{\gamma} = 10$ dB and id Nakagami-m fading. For the SSC receiver, the optimum switching threshold has been set using (38) with (26). Similarly to Figs. 3 and 4, the MRC provides the best performance.

6 Conclusions

In this paper, an analysis for the statistics of the output CC of several diversity receivers was presented under ORA with constant transmit power. The moments of the CC at the output of MRC, SC, EGC, GSC, and SSC receivers were derived in closed form, when they operate over not necessarily id Rayleigh and Nakagami-m fading channels. The FF was defined and obtained in closed form for each one of the above mentioned diversity receiver. The skewness and kurtosis of the distribution of the CC were also studied. The presented analysis can be further extended to other generic distributions and/or correlated fading channels.

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Author Biographies

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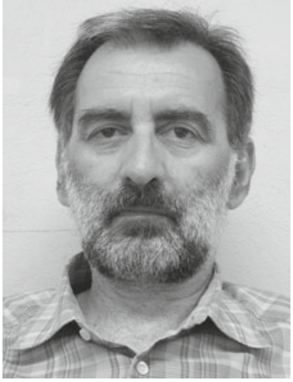

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