On the Statistics of the Error Propagation Effect of Binary Differential Phase-Shift Keying

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***Abstract*—We study a traditional problem related to binary differentially coherent phase-shift keying (DPSK) modulation, where an error in a symbol tends to cause an error in the next symbol; this phenomenon is referred to as error propagation. We derive the double bit-error rate (DBER) of two successive DPSK symbols over additive white Gaussian noise and Rician fading. Our findings show that the DBER decreases as the signal-to-noise ratio increases. A tight closed-form upper bound for the average DBER over a slowly varying Rician channel is also presented to simplify the numerical evaluation.**

***Index Terms*—Bit-error rate (BER), complex Gaussian, differential phase-shift keying (DPSK), double bit error rate, error propagation, Rician fading.**

I. Introduction

Differentially coherent phase-shift keying (DPSK) is one of the oldest and well-studied modulation techniques. Its benefit is that it eliminates the requirement for signal phase recovery at the expense of slightly decreased bit-error rate (BER) performance as compared to the non-differential version. DPSK is based on the information that can be extracted at the receiver by processing the phase difference of two successive symbols, after performing a bit level transmitter differential encoding.

The analytical derivation of DPSK BER performance over additive white Gaussian noise (AWGN) and/or fading is documented in many excellent books, e.g., [1]–[3]. Moreover, the performance of DPSK has been well-investigated in numerous papers under different systems, conditions, and fading channels. A problematic issue in DPSK is the error propagation effect [1, Sec. 4.5-5]. Since, for the detection of a DPSK symbol, the demodulator uses its previous one, an error in the previous symbol tends also to cause an error in the next symbol with high probability. Hence, there is a strong correlation between the probability of error of two successive symbols over AWGN, resulting in errors tending to appear in pairs.

In [4], an expression for the double symbol error rates of M-ary DPSK systems over the Rayleigh channel is presented. However, the derived expression is limited to Rayleigh fading and is in the form of a quadruple integral with complicated integrands. In an effort to further quantify the correlation of the DPSK errors between two successive binary symbols, we derive the corresponding BER over AWGN and Rician fading. Using the detector decision statistics and by decorrelating a set of complex Gaussian random variables (rv)s, we analytically derive the probabilities of the erroneous-erroneous (double BER (DBER) in short), erroneous-correct, and erroneous-correct pairs of successive bit errors. The analysis can be useful in various DPSK performance evaluation problems, such as relayed communications [5, Sec. 9] as well as in designs of error control coding systems [4].

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II. Channel Model and DPSK Detection

The transmitter generates the kth binary DPSK symbol (of energy per bit $E_b = \mathcal{E}\langle s_k^2\rangle$, with $\mathcal{E}\langle\cdot\rangle$ denoting expectation) as

$$s_k = q_k\, s_{k-1}, \quad (1)$$

where $q_k \in \{\pm 1\}$ is the kth information-bearing symbol. The transmitted symbol is affected by flat Rician fading, with the complex channel gain h distributed as $h \sim \mathcal{CN}(\mu_h, \sigma_h^2)$, with $\mu_h = \mathcal{E}\langle h\rangle = |\mu_h| \exp(\jmath\, \tau)$ and $\sigma_h^2 = \mathcal{E}\langle |h-\mu_h|^2\rangle$. The joint probability density function (pdf) of the envelope $|h|$ and phase $\angle h$ of h is given by [6, eq. (6.2)]

$$f_{|h|,\angle h}(r,\phi) = \frac{\exp(-K)}{\pi}\,(1+K)\, r\, \exp\Big[-(1+K)\, r^2\Big] \times \exp\Big[2\sqrt{K\,(1+K)}\, r\, \cos(\phi-\tau)\Big], \quad (2)$$

where K is the Rician factor, $\mu_h = \sqrt{K/(K+1)}\,\exp(\jmath\,\tau)$, $\sigma_h^2 = 1/(1+K)$, and $\mathcal{E}\langle |h|^2\rangle = 1$.

The received signal is filtered and sampled and the complex baseband signal corresponding to the kth symbol is given by

$$r_k = \sqrt{E_b}\, h_k\, s_k + n_k, \quad (3)$$

where h_k is the kth complex channel gain sample and $n_k \sim \mathcal{CN}(0, N_0)$ is the AWGN, with N_0 being the single-sided noise power spectral density. From (1) and (3), we have $r_k = r_{k-1}\, q_k + n_k - q_k\, n_{k-1}$, based on which the optimum detector is simply a minimum distance detector between r_k and $q_k\, r_{k-1}$, that can be further simplified to a sign detector, where $\hat{q}_k = +1$ and -1 for $\beta_k > 0$ and $\beta_k \le 0$, respectively, with $\beta_k = \Re\{r_k\, r_{k-1}^*\}$, where $\Re(\cdot)$ denotes the real part operator and $(\cdot)^*$ the complex conjugate.

III. Derivation of Joint Probabilities

The error propagation effect of DPSK requires the knowledge of two successive decision statistics, i.e., of rvs β_k and β_{k-1}. It can be easily shown that $\mathcal{E}\langle\beta_k\rangle = \mathcal{E}\langle\beta_{k-1}\rangle = 0$, $\mathcal{E}\langle\beta_k^2\rangle = \mathcal{E}\langle\beta_{k-1}^2\rangle = \{[K^2 + 1 + (1+K)\,\rho_{k,k-1}]E_b^2/N_0^2 +$

$2\,E_b/N_0+1\}N_0^2/2$, and $\mathcal{E}\langle\beta_k\,\beta_{k-1}\rangle=0$, with $\rho_{k,k-1}$ being the time correlation between the kth and $(k-1)$th envelopes.

A. Double Bit-Error Rate

Based on the sign detector and assuming that $q_k=q_{k-1}=q_{k-2}=1$, the joint error probability P_{ee} of the $(k-1)$th and kth symbols (or DBER) can be expressed as

$$P_{ee}=\Pr\{\beta_k\le 0,\,\beta_{k-1}\le 0|\ q_k=q_{k-1}=q_{k-2}=1\}. \quad (4)$$

Using the squared-norm identity of the sum of two complex numbers, the DBER can be rewritten as

$$P_{ee}=\Pr\Big\{|r_k+r_{k-1}|^2\le|r_k-r_{k-1}|^2,\,|r_{k-1}+r_{k-2}|^2\le |r_{k-1}-r_{k-2}|^2\Big\}=\Pr\Big\{|x_1|^2\le|x_3|^2,\,|x_2|^2\le|x_4|^2\Big\}, \quad (5)$$

where

$$\mathbf{x}=[x_1,\ x_2,\ x_3,\ x_4]^T =\begin{bmatrix} r_k+r_{k-1}\\ r_{k-1}+r_{k-2}\\ r_k-r_{k-1}\\ r_{k-1}-r_{k-2}\end{bmatrix}=\begin{bmatrix}\sqrt{E_b}\,(h_k+h_{k-1})+n_k+n_{k-1}\\ \sqrt{E_b}\,(h_{k-1}+h_{k-2})+n_{k-1}+n_{k-2}\\ \sqrt{E_b}\,(h_k-h_{k-1})+n_k-n_{k-1}\\ \sqrt{E_b}\,(h_{k-1}-h_{k-2})+n_{k-1}-n_{k-2}\end{bmatrix}, \quad (6)$$

with $(\cdot)^T$ denoting the transpose operator.

Conditioned on h_k, h_{k-1}, and h_{k-2}, x_1, x_2, x_3, and x_4 are jointly complex Gaussian rvs. Specifically, x_1 and x_3 are a set of independent complex Gaussian rvs, distributed as $\mathcal{CN}(\sqrt{E_b}\,(h_k+h_{k-1}),2\,N_0)$ and $\mathcal{CN}(\sqrt{E_b}\,(h_k-h_{k-1}),2\,N_0)$, respectively, while x_2 and x_4 are another set of independent complex Gaussian rvs, distributed as $\mathcal{CN}(\sqrt{E_b}\,(h_{k-1}+h_{k-2}),2\,N_0)$ and $\mathcal{CN}(\sqrt{E_b}\,(h_{k-1}-h_{k-2}),2\,N_0)$, respectively. Hence, the random vector $\mathbf{x}\sim\mathcal{CN}(\boldsymbol{\mu}_\mathbf{x},\mathbf{K}_\mathbf{x})$ has a mean vector

$$\boldsymbol{\mu}_\mathbf{x} =\sqrt{E_b}\big[h_k+h_{k-1},\ h_{k-1}+h_{k-2},\ h_k-h_{k-1},\ h_{k-1}-h_{k-2}\big]^T \quad (7a)$$

and covariance matrix

$$\mathbf{K}_\mathbf{x}=N_0\begin{bmatrix}2&1&0&1\\1&2&-1&0\\0&-1&2&-1\\1&0&-1&2\end{bmatrix}. \quad (7b)$$

Since the determinant of $\mathbf{K}_\mathbf{x}$ is zero, the elements of $\mathbf{x}$ are linearly dependent, and x_4 can be written as $x_4=x_1-x_2-x_3$. In order to construct an appropriate basis, we denote the rv $\hat{x}_0$ as

$$\hat{x}_0=\hat{x}_2+\hat{x}_4=\hat{x}_1-\hat{x}_3, \quad (8)$$

with $\hat{x}_1=x_1-\sqrt{E_b}\,(h_k+h_{k-1})$, $\hat{x}_2=x_2-\sqrt{E_b}\,(h_{k-1}+h_{k-2})$, $\hat{x}_3=x_3-\sqrt{E_b}\,(h_k-h_{k-1})$, and $\hat{x}_4=x_4-\sqrt{E_b}\,(h_{k-1}-h_{k-2})$ being the zero-mean versions of x_1, x_2, x_3, and x_4, respectively, and thus,

$$\mathbf{x}=\begin{bmatrix}\sqrt{E_b}\,(h_k+h_{k-1})\\ \sqrt{E_b}\,(h_{k-1}+h_{k-2})\\ \sqrt{E_b}\,(h_k-h_{k-1})\\ \sqrt{E_b}\,(h_{k-1}-h_{k-2})\end{bmatrix}+\begin{bmatrix}\hat{x}_0+\hat{x}_3\\ \hat{x}_0-\hat{x}_4\\ \hat{x}_3\\ \hat{x}_4\end{bmatrix}. \quad (9)$$

The random vector $\hat{\mathbf{x}}=[\hat{x}_0,\ \hat{x}_3,\ \hat{x}_4]^T$ of the basis elements is distributed as $\mathcal{CN}(\mathbf{0}_3,\mathbf{K}_{\hat{\mathbf{x}}})$, where the mean is given by $\mathbf{0}_3=[0,\ 0,\ 0]^T$, and the covariance matrix by

$$\mathbf{K}_{\hat{\mathbf{x}}}=N_0\begin{bmatrix}4&-2&2\\-2&2&-1\\2&-1&2\end{bmatrix}. \quad (10)$$

We now perform a standard decomposition to transform correlated complex Gaussian rvs to independent and identically distributed (iid) complex Gaussians [7]. We split $\mathbf{K}_{\hat{\mathbf{x}}}=\mathbf{A}^T\mathbf{A}$ as a product of a lower triangular matrix and its transpose, resulting in

$$\mathbf{K}_{\hat{\mathbf{x}}}=\underbrace{\sqrt{N_0}\begin{bmatrix}2&0&0\\-1&1&0\\1&0&1\end{bmatrix}}_{\mathbf{A}^T}\underbrace{\sqrt{N_0}\begin{bmatrix}2&-1&1\\0&1&0\\0&0&1\end{bmatrix}}_{\mathbf{A}}. \quad (11)$$

The random vector $\hat{\mathbf{x}}$ can then be expressed as

$$\hat{\mathbf{x}}=\mathbf{A}^T\,\mathbf{y}, \quad (12)$$

where $\mathbf{y}=[y_1,\ y_2,\ y_3]^T$ is a zero-mean complex Gaussian random vector having iid $\mathcal{CN}(0,1)$ elements, i.e., $\mathbf{y}\sim\mathcal{CN}(\mathbf{0}_3,\mathbf{I}_3)$, with $\mathbf{I}_3$ being the 3×3 identity matrix.

Substituting (9) in (5), the DBER is

$$P_{ee}=\Pr\Bigg\{\Big|\sqrt{E_b}\,(h_k+h_{k-1})+\hat{x}_0+\hat{x}_3\Big|^2 \le\Big|\sqrt{E_b}\,(h_k-h_{k-1})+\hat{x}_3\Big|^2,\ \ \Big|\sqrt{E_b}(h_{k-1}-h_{k-2})+\hat{x}_4\Big|^2 \ge\Big|\sqrt{E_b}\,(h_{k-1}+h_{k-2})+\hat{x}_0-\hat{x}_4\Big|^2\Bigg\}, \quad (13)$$

where, by applying (12) and after some algebra, we obtain

$$P_{ee}=\Pr\Bigg\{\Re\bigg\{\frac{E_b}{N_0}\,h_k\,h_{k-1}^*+y_1^*\,y_2 +\sqrt{\frac{E_b}{N_0}}(h_k\,y_1^*+h_{k-1}\,y_2^*)\bigg\}\le 0,\,\Re\bigg\{\frac{E_b}{N_0}\,h_{k-1}\,h_{k-2}^* +y_1^*\,y_3+\sqrt{\frac{E_b}{N_0}}(y_1\,h_{k-2}^*-y_3\,h_{k-1}^*)\bigg\}\le 0\Bigg\}. \quad (14)$$

Conditioned on y_1, the joint probability of the two events can be expressed as a product of two probabilities in the form

$$P_{ee|y_1}=\Pr\Bigg\{\Re\bigg\{\bigg(y_1+\sqrt{\frac{E_b}{N_0}}\,h_{k-1}\bigg)y_2^*\bigg\}\le-\Re\bigg\{\frac{E_b}{N_0}\,h_k\,h_{k-1}^* +\sqrt{\frac{E_b}{N_0}}\,h_k y_1^*\bigg\}\Bigg\}\Pr\Bigg\{\Re\bigg\{\bigg(y_1+\sqrt{\frac{E_b}{N_0}}\,h_{k-1}\bigg)y_3^*\bigg\} \ge\Re\bigg\{\frac{E_b}{N_0}\,h_{k-1}\,h_{k-2}^*+\sqrt{\frac{E_b}{N_0}}\,h_{k-2}\,y_1^*\bigg\}\Bigg\}. \quad (15)$$

Using the statistics of y_2 and y_3, we can express (15) as

$$P_{ee|y_1}=Q\bigg(\frac{\zeta_k}{\eta_{k-1}}\bigg)\,Q\bigg(\frac{\zeta_{k-1}}{\eta_{k-1}}\bigg), \quad (16)$$

where $\zeta_k=\Re\{h_k\,h_{k-1}^*\,E_b/N_0+h_k\,y_1^*\,\sqrt{E_b/N_0}\}$, $\eta_{k-1}=|y_1+h_{k-1}\,\sqrt{E_b/N_0}|/\sqrt{2}$, and $Q(\cdot)$ denotes the Gaussian Q-function.

Since y_1 is a $\mathcal{CN}(0,1)$ complex Gaussian rv, we can write it as $y_1 = v_1 + \jmath\, v_2$ with v_1 and v_2 being two iid $\mathcal{N}(0, 1/2)$ real Gaussian rvs. Averaging $P_{ee|y_1}$ in (16) over the statistics of v_1 and v_2, the DBER is given by

$$P_{ee} = \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\Big\{-(v_1^2 + v_2^2)\Big\} \times Q\left(\frac{\Re\Big\{\frac{E_b}{N_0}\, h_k\, h_{k-1}^* + \sqrt{\frac{E_b}{N_0}}\, h_k\,(v_1 - \jmath\, v_2)\Big\}}{\frac{1}{\sqrt{2}} \left| v_1 + \jmath\, v_2 + \sqrt{\frac{E_b}{N_0}}\, h_{k-1} \right|} \right) \times Q\left(\frac{\Re\Big\{\frac{E_b}{N_0}\, h_{k-1}\, h_{k-2}^* + \sqrt{\frac{E_b}{N_0}}\, h_{k-2}\,(v_1 - \jmath\, v_2)\Big\}}{\frac{1}{\sqrt{2}} \left| v_1 + \jmath\, v_2 + \sqrt{\frac{E_b}{N_0}}\, h_{k-1} \right|} \right) \mathrm{d}v_1 \mathrm{d}v_2. \tag{17}$$

By applying the pair of transformations $\rho \cos(\theta) = v_1 + \Re\{h_{k-1}\}\sqrt{E_b/N_0}$ and $\rho \sin(\theta) = v_2 + \Im\{h_{k-1}\}\sqrt{E_b/N_0}$, with $\Im\{\cdot\}$ denoting the imaginary part operator, (17) can be written as

$$P_{ee} = \frac{1}{\pi} \int_{-\pi}^{\pi} \exp\left\{-\frac{E_b}{N_0}\left[\Re\{h_{k-1}\}\sin(\theta) - \Im\{h_{k-1}\}\cos(\theta)\right]^2\right\} \times Q\left(\sqrt{\frac{2\,E_b}{N_0}}\,[\Re\{h_k\}\cos(\theta) + \Im\{h_k\}\sin(\theta)]\right) \times Q\left(\sqrt{\frac{2\,E_b}{N_0}}\,\left[\Re\{h_{k-2}\}\cos(\theta) + \Im\{h_{k-2}\}\sin(\theta)\right]\right) \times \int_{0}^{\infty} \rho\, \exp\left\{ -\left[\rho - \sqrt{\frac{E_b}{N_0}}\,[\Re\{h_{k-1}\}\cos(\theta) + \Im\{h_{k-1}\}\sin(\theta)]\right]^2\right\} \mathrm{d}\rho\, \mathrm{d}\theta, \tag{18}$$

The integral with respect to ρ in (18) can be obtained in closed form with the help of [8, eq. (3.462/5)]. Moreover, the real and imaginary parts of the complex channel gain h_k can be written as $\Re\{h_k\} = |h_k|\cos(\phi_k)$ and $\Im\{h_k\} = |h_k|\sin(\phi_k)$ (similar expressions also for h_{k-1} and h_{k-2}), where $\phi_k = \mathrm{atan}(\Im\{h_k\}/\Re\{h_k\})$ is the channel phase of the kth sample. Using these results, the DBER in (18) is expressed as

$$P_{ee} = \frac{1}{\pi} \int_{0}^{\pi} Q\left(\sqrt{\frac{2\,E_b}{N_0}}\,|h_k|\,\cos(\theta - \phi_k)\right) \times Q\left(\sqrt{\frac{2\,E_b}{N_0}}\,|h_{k-2}|\,\cos(\theta - \phi_{k-2})\right) \times \left[\exp\left(-|h_{k-1}|^2\,\frac{E_b}{N_0}\right) + \sqrt{2\,\pi}\,\sqrt{\frac{2\,E_b}{N_0}}\,|h_{k-1}| \times \cos(\theta - \phi_{k-1})\,\exp\left(-|h_{k-1}|^2\,\frac{E_b}{N_0}\,\sin^2(\theta - \phi_{k-1})\right) \times Q\left(-\sqrt{\frac{2\,E_b}{N_0}}\,|h_{k-1}|\,\cos(\theta - \phi_{k-1})\right)\right] \mathrm{d}\theta. \tag{19}$$

B. Pairs of Erroneous-Correct Joint Probabilities

Proceeding along the same lines as the derivation of (19), we can also obtain the joint probability of successive correct and erroneous symbols as well as the joint probability of two successive correct symbols. The probability of a correct decision is accompanied by an inverted inequality in (4), and hence, in (15). Therefore, a general form of (16) is

$$P_{qq|y_1} = Q\left(\pm\frac{\zeta_k}{\eta_{k-1}}\right) Q\left(\pm\frac{\zeta_{k-1}}{\eta_{k-1}}\right), \tag{20}$$

where each q can be substituted by e and c for erroneous and correct when + and - signs appear in the arguments of the Gaussian Q-functions, respectively. Following this formulation in (17)–(18), a general expression for all four pairs of erroneous and correct successive symbols is

$$P_{qq} = \frac{1}{\pi} \int_{0}^{\pi} Q\left(\pm\sqrt{\frac{2\,E_b}{N_0}}\,|h_k|\,\cos(\theta - \phi_k)\right) \times Q\left(\pm\sqrt{\frac{2\,E_b}{N_0}}\,|h_{k-2}|\,\cos(\theta - \phi_{k-2})\right) \times \left[\exp\left(-|h_{k-1}|^2\,\frac{E_b}{N_0}\right) + \sqrt{2\,\pi}\,\sqrt{\frac{2\,E_b}{N_0}}\,|h_{k-1}| \times \cos(\theta - \phi_{k-1})\,\exp\left(-|h_{k-1}|^2\,\frac{E_b}{N_0}\,\sin^2(\theta - \phi_{k-1})\right) \times Q\left(-\sqrt{\frac{2\,E_b}{N_0}}\,|h_{k-1}|\,\cos(\theta - \phi_{k-1})\right)\right] \mathrm{d}\theta. \tag{21}$$

It should be mentioned that (21) is quite generic and can be used when the channel coherence time is of the order of the symbols duration, i.e., the channel changes from symbol to symbol (fast varying). Hence, (21) must be averaged over the trivariate channel pdf [6, Sec. 6] to derive its average value. It is obvious that the average joint probability of successive correct-erroneous symbols equals to the joint probability of successive erroneous-correct symbols ($P_{ce} = P_{ec}$), due to the symmetry of the correlation between kth and $(k-2)$th samples. Moreover, using (21), it can be concluded that $P_{ee}+P_{ec}+P_{ce}+P_{cc} = 1$, that is in agreement to the total probability law.

In a slowly varying channel, the symbol duration is much shorter as compared to the channel coherence time, i.e., $|h_k| = |h_{k-1}| = |h_{k-2}| \equiv r$ and $\phi_k = \phi_{k-1} = \phi_{k-2} \equiv \phi$. After changing variables as $z = \sqrt{2\,E_b/N_0}\, r\, \cos(\theta - \phi)$, (21) can be written as

$$P_{qq} \approx \frac{\exp(-\gamma)}{\pi} \int_{-\sqrt{2\,\gamma}\,\cos(\phi)}^{\sqrt{2\,\gamma}\,\cos(\phi)} \frac{Q(\pm z)\, Q(\pm z)}{\sqrt{2\,\gamma\,\cos^2(\phi) - z^2}} \times \left[1 + \sqrt{2\,\pi}\, z\, \exp\left(\frac{z^2}{2}\right) Q(-z)\right] \mathrm{d}z, \tag{22}$$

with $\gamma = r^2\, E_b/N_0$ being the instantaneous signal-to-noise ratio (SNR) per bit. Note that for AWGN, (22) can be used with $\gamma = E_b/N_0$ ($r = 1$) and $\phi = 0$, while for Rician fading, (22) must be averaged over the joint pdf in (2),

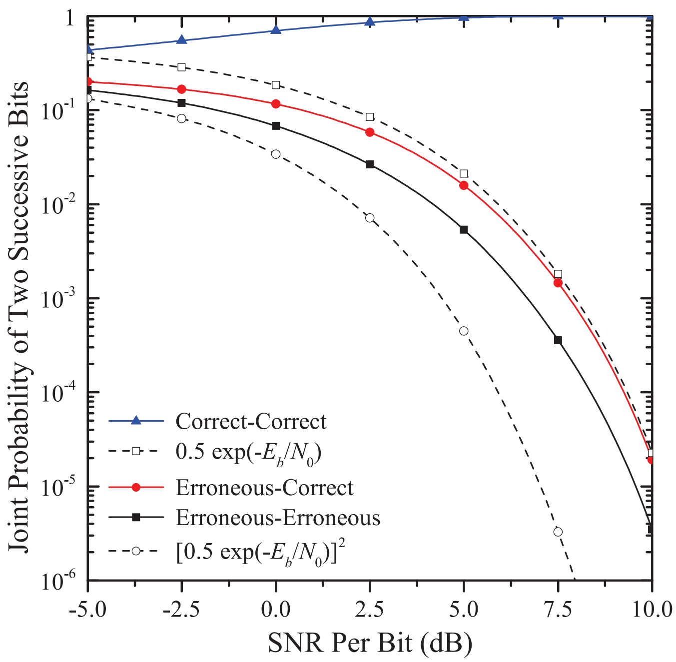

Fig. 1. Joint BER of two successive bits as a function of the SNR per bit for erroneous-erroneous, erroneous-correct, and correct-correct pairs for AWGN.

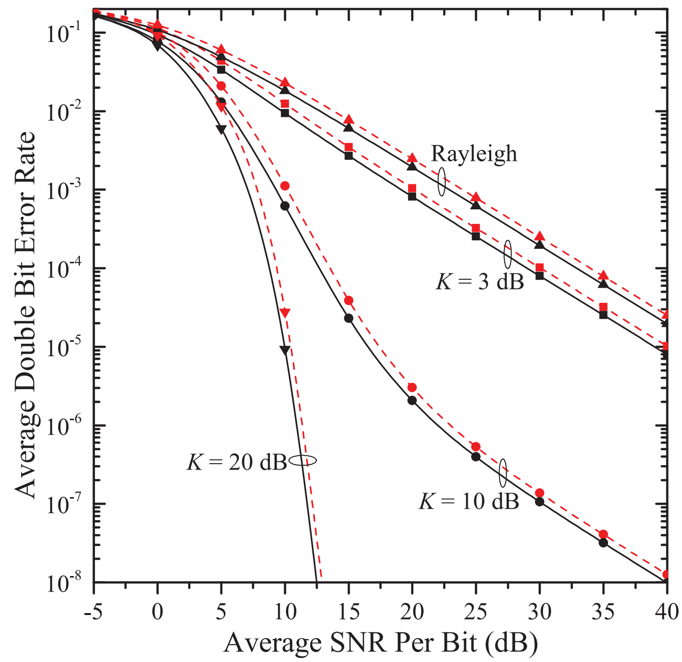

Fig. 2. Average DBER as a function of the average SNR per bit for slowly varying Rician fading.

C. Average Double Bit-Error Rate Bound

By considering a much shorter symbol duration as compared to the channel coherence time and in order to get some insights on the DBER, the inequalities $0.5 \geq Q[a\cos(x)] \geq Q(a)$ have been used, with $a > 0$ and $|x| \leq \pi/2$. By averaging (19) over (2) and after a lot of mathematical steps and approximations, a simple closed-form upper bound of the average DBER is obtained as

$$P_{ee} \leq \frac{1}{4}\frac{1+K}{\overline{\gamma}+1+K}\exp\left(-\frac{\overline{\gamma}\,K}{\overline{\gamma}+1+K}\right), \qquad (23)$$

with $\overline{\gamma} = \mathcal{E}\langle\gamma\rangle = E_b/N_0$ being the average SNR per bit. Note that for high values of $\overline{\gamma}$ and fixed K, (23) can be further simplified as $P_{ee} \leq (1+K)\exp(-K)/(4\,\overline{\gamma})$.

IV. Numerical Results

In Fig. 1, three curves for the joint probability of erroneous-erroneous, erroneous-correct, and correct-correct pairs of successive bits, computed using (22), are plotted as a function of the SNR per bit (E_b/N_0) under AWGN. From this figure it is clearly seen that the joint probabilities P_{ee} and P_{ec} of erroneous-erroneous and erroneous-correct successive bits, respectively, decrease as the SNR increases, while the correct-correct joint BER P_{cc} increases because $P_{ee} + 2\,P_{ec} + P_{cc} = 1$. In the same figure, two additional curves for the BER ($P_{dpsk} = 0.5\exp(-E_b/N_0)$) and the squared BER ($P_{dpsk}^2$) of DPSK over AWGN are also included for the purpose of comparison. It can be observed that, since for high SNR, we have $P_{dpsk}\,(1-P_{dpsk}) \simeq P_{dpsk}$, the joint probability of the erroneous-correct pair is tightly approximated by the BER of DPSK, i.e., $P_{ec} \simeq P_{ce} \simeq P_{dpsk}$. Moreover, from the comparison of the squared BER with the DBER, a 2.5 dB SNR difference is observed due to the propagation effect. It has to be noted here that since for AWGN and $|\ell - k| \neq 1$, $\Pr\{\beta_k \leq 0, \beta_\ell \leq 0\} = \Pr\{\beta_k \leq 0\}\Pr\{\beta_\ell \leq 0\}$, the error propagation effect is limited only to successive bit errors.

A slowly varying Rician fading channel is considered in Fig. 2, where solid curves for the average DBER, computed by averaging (22) over the channel statistics in (2), are plotted as a function of the average SNR per bit for a Rician factor $K = -\infty$, 3, 10, and 20 dB. As expected, the higher the value of K, the better the DBER performance obtained. In the same figure, dashed curves for the upper DBER bound are also plotted in order to verify the tightness of (23). It can be easily concluded that the curves for the upper bound are noticeably close to the corresponding ones for the exact average DBER for any value of the average SNR and the Rician factor.

As a final note it should be mentioned that all presented results in Figs. 1 and 2 have been verified by extensive computer simulations.

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