Unified Ergodic Capacity Expressions for AF Dual-Hop Systems With Hardware Impairments

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***Abstract*—This letter presents novel analytical expressions for the ergodic capacity (EC) of dual-hop amplify-and-forward (AF) relaying systems subject to hardware impairments operating over generalized fading channels. Our expressions are quite generic, providing that the moment-generating function (MGF) of the signal-to-noise ratio (SNR) or the MGF of the inverse SNR of each hop is readily available. The proposed analytical framework is further applied to assess the EC performance of two system setups: 1) a variable-gain relaying system equipped with multiple antennas at the relay and at the destination, assuming an interference-limited relay and a noise-limited destination and 2) a multi-source multi-destination AF system equipped with variable- or fixed-gain relays. The correctness of the proposed mathematical analysis is verified by Monte Carlo simulations results.**

***Index Terms*—Amplify-and-forward, capacity, fading channels, hardware impairments, relays.**

I. Introduction

Relaying communication systems provide increased coverage and achieve higher throughput comparing to their single-hop counterparts [1]. However, the performance of practical relaying systems is degraded by hardware impairments, e.g. power amplifier nonlinearities, I/Q imbalance, phase noise and quantization errors [2]. The performance of relaying systems with hardware impairments has been addressed in several research works using metrics such as the outage probability, the symbol error rate and the ergodic capacity (EC), e.g. see [2]–[6]. Despite that EC is an important performance metric, relatively few studies evaluate the capacity of relaying systems with hardware impairments. The main obstacle is the fact that no closed-form expressions for the statistics of their end-to-end (e2e) signal-to-noise ratio (SNR), including the probability density function (PDF) and the moments-generating function (MGF), are readily available. As such, previously published works present tight upper or lower bounds and asymptotic expressions on the EC, instead. For example, in [2], closed-form bounds for the EC of dual-hop relaying systems over Rayleigh and Nakagami fading channels with integer fading parameters, that are becoming asymptotically tight at high signal-to-noise ratio (SNR), have been derived. In [5] and [6] the EC of two-way relaying and cell free massive multiple-input multiple-output (MIMO) systems with hardware impairments is investigated, respectively. In both works, closed-form EC expressions are presented assuming a large (infinite) number of antennas. Furthermore, in [7] a unified MGF-based approach for the EC of relaying systems operating over generalized fading channels has been presented assuming, however, ideal hardware. To the best of our knowledge, such an analytical framework for relaying systems in the presence of hardware impairments is not available in the open technical literature.

Motivated by the above, the contributions of this letter are summarized as follows. First, novel unified analytical expressions for the EC of dual-hop amplify-and-forward (AF) relaying systems with hardware impairments are presented for fixed-gain (FG) or variable-gain (VG) relays. These expressions are valid for arbitrary fading channel models, providing that the MGF of the inverse instantaneous SNR of each hop is readily available. Then, simple closed-form expressions for Rayleigh and Nakagami-0.5 fading are further deduced for FG and VG relays, respectively. The proposed expressions are applied for the EC analysis of two generic system scenarios, that are: *i*) A dual-hop VG MIMO relay system. assuming an interference-limited relay and a noise-limited destination. *ii*) A multi-source multi-destination system with FG or VG relays. All analytical results are substantiated with semi-analytical Monte-Carlo simulations.

II. Ergodic Capacity Analysis

The EC of a dual-hop relaying system is given by

$$C = \frac{1}{2\ln(2)}\mathbb{E}\left\langle \ln\left(1+\gamma_{\rm e2e}\right)\right\rangle, \tag{1}$$

where $\gamma_{\rm e2e}$ is the e2e SNR,[1] while the factor 1/2 takes into account the fact that the e2e transmission is completed in two time slots. For fixed- and variable-gain relays the e2e SNR of a dual-hop system is given by [2, eqs. (13) and (14)]

$$\gamma_{\rm e2e-fg} = \frac{\gamma_1\,\gamma_2}{d\,\gamma_1\,\gamma_2 + c_2\,\gamma_2 + c} \tag{2a}$$

and

$$\gamma_{\rm e2e-vg} = \frac{\gamma_1\,\gamma_2}{d\,\gamma_1\,\gamma_2 + c_1\,\gamma_1 + c_2\,\gamma_2 + 1}, \tag{2b}$$

respectively, where γ_i is the SNR of the ith hop, $i \in \{1,2\}$, $d = \kappa_1^2\kappa_2^2 + {\kappa_1}^2 + {\kappa_2}^2$, with κ_i denoting the level of

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[1]Mathematical Notations: $\mathbb{E}\langle\cdot\rangle$ denotes expectation, $(\cdot)^\star$ denotes complex conjugate, $(\cdot)^\dagger$ denotes hermitian transpose, $\|\cdot\|$ denotes euclidian norm, $\mathbf{I}$ is the unity matrix, $\det(\cdot)$ denotes matrix determinant, $\mathcal{M}_X(\cdot)$ is the MGF of the random variable X, $K_a(\cdot)$ is the modified Bessel function of the second kind and order a [8, eq. (8.407/1)], $W_{\mu,\nu}(\cdot)$ is the Whittaker function [8, eq. (9.220/4)], ${}_pF_q(\cdot)$ is the generalized hypergeometric function [8, eq. (9.14/1)], $\mathrm{erfi}(\cdot)$ is the imaginary error function [8, p. 107], and $H^{\,m_1,n_1:m_2,n_2:m_3,n_3}_{\,p_1,q_1:p_2,q_2:p_3,q_3}$ is the bivariate Fox H-function [9, eqs. (2.56)–(2.60)].

impairments in ith hop, $c_i = 1 + \kappa_i^2$, and c is a constant related to the amplifier gain. Note that, based on the PDF of γ_1 and γ_2, the computation of the EC with the help of (1) requires the numerical evaluation of a two-fold integral. However, this integral converges rather slowly because of the presence of the logarithmic kernel.

In what follows, we first show that the EC of AF relaying systems can be expressed as the difference of the EC of two generic diversity receivers with independent branches. Using this interesting information theoretical result, simple to evaluate integral expressions for the EC are presented, assuming arbitrarily distributed γ_i's. The proposed expressions utilize an exponentially decayed kernel that guarantees rapid convergence and efficient numerical evaluation.

A. Fixed-Gain Relays

An exact, single integral expression for the EC of dual-hop systems with FG relays can be deduced as follows:

Proposition 1: The EC of dual-hop systems with FG relays can be expressed in terms of the MGFs of γ_1 and $1/\gamma_2$ as

$$C_{\text{fg}} = \frac{1}{\ln(2)} \int_0^\infty \frac{\exp(-t^2)}{t} \mathcal{M}_{\frac{1}{\gamma_2}}\left(\frac{c}{c_2} t^2\right) \times \left[\mathcal{M}_{\gamma_1}\left(\frac{d}{c_2} t^2\right) - \mathcal{M}_{\gamma_1}\left(\frac{1+d}{c_2} t^2\right)\right] \mathrm{d}t. \quad (3)$$

Proof: Substituting (2a) to (1) and after performing some algebraic manipulations, the EC can be expressed as

$$C_{\text{fg}} = \frac{1}{2\ln(2)} \left[\mathbb{E}\left\langle \ln(1+Z_1)\right\rangle - \mathbb{E}\left\langle \ln(1+Z_2)\right\rangle\right] \quad (4)$$

where $Z_1 \triangleq \gamma_1 (1+d)/c_2 + (c/c_2)/\gamma_2$ and $Z_2 \triangleq \gamma_1 (d/c_2) + (c/c_2)/\gamma_2$. The expectations $\mathbb{E}\left\langle \ln(1+Z_i)\right\rangle$, $i \in \{1,2\}$, can be expressed in terms of the MGF of Z_i as [10, eq. (5)]

$$\mathbb{E}\left\langle \ln(1+Z_i)\right\rangle = \int_0^\infty \exp(-s) \frac{1-\mathcal{M}_{Z_i}(s)}{s} \mathrm{d}s. \quad (5)$$

Since γ_1 and $1/\gamma_2$ are independent random variables, the MGF of Z_1 and Z_2 will be $\mathcal{M}_{Z_1}(s) = \mathcal{M}_{\gamma_1}(s(1+d)/c_2)\,\mathcal{M}_{1/\gamma_2}(s\,c/c_2)$ and $\mathcal{M}_{Z_2}(s) = \mathcal{M}_{\gamma_1}(s\,d/c_2)\,\mathcal{M}_{1/\gamma_2}(s\,c/c_2)$, respectively. By substituting (5) in (4) and performing the change of variables $s = t^2$ yields (3), thus, completing the proof. ■

It should be stressed that Proposition 1 is valid for arbitrarily distributed γ_i's, provided that the MGFs of γ_1 and $1/\gamma_2$ are readily available. For Rayleigh fading, a closed-form EC expression is next derived.

Corollary 1: The EC of a dual-hop AF system with a FG relay operating over Rayleigh fading channels can be expressed in terms of the bivariate H-function[2] as

$$C_{\text{fg}-\text{Rayleigh}} = -\frac{A\,\pi}{2\sin(g)\ln(2)} \times H^{0,1:2,0:1,1}_{1,0:0,2:2,2}\left[\begin{matrix}\sqrt{F}\\ B\end{matrix}\,\middle|\,\begin{matrix}(-1/2;1,1)\\ -\end{matrix}\,\middle|\,\begin{matrix}-\\ (-\frac{1}{2},1),(\frac{1}{2},1)\end{matrix}\,\middle|\,\begin{matrix}(0,1),(-\frac{g}{\pi},\frac{g}{\pi})\\ (0,1),(-\frac{g}{\pi},\frac{g}{\pi})\end{matrix}\right], \quad (6)$$

where $B = c/(\overline{\gamma}_2\, c_2)$ $A = \sqrt{B}\,\overline{\gamma}_1/c_2$, $F = d\,(1+d)\,\overline{\gamma}_1^2/c_2^2$, $G = (1+2\,d)\,\overline{\gamma}_1/c_2$, $g = \arccos(G/(2\sqrt{F}))$, and $\overline{\gamma}_i$ is the average SNR of the ith hop.

Proof: For Rayleigh fading channels, γ_1 and γ_2 follow an exponential distribution. Using [8, eq. (3.310)] and [8, eq. (3.471/9)], the required MGFs of γ_1 and $1/\gamma_2$ can be obtained in closed-form. By substituting these expressions into (3) and performing the change of variables $t = \sqrt{s}$, the evaluation of C_{fg} requires a solution for the following integral

$$\mathcal{I} = A \int_0^\infty \frac{\exp(-s)\sqrt{s}\,K_1(2\sqrt{B\,s})}{1 + s\,G + s^2\,F} \mathrm{d}s. \quad (7)$$

In order to obtain a closed form for $\mathcal{I}$, the exponential, the Bessel function and the fraction are first expressed in terms of H-functions using the identities [12, eq. (8.4.3.1), (8.4.23.1) and (8.4.2.14)], respectively. Observing that $\mathcal{I}$ is the Mellin transform of the product of three H-functions and following [9, Ch. 2] $\mathcal{I}$, can be expressed in terms of the bivariate H-function after performing some mathematical manipulations as (6), thus completing the proof. □

B. Variable-Gain Relays

A single integral expression for the EC of dual-hop systems with VG relays, which becomes tight at medium and high SNR, can be deduced as follows:

Proposition 2: A tight approximation for the EC of a dual-hop AF system with a VG relay can be expressed in terms of the MGFs of $1/\gamma_1$ and $1/\gamma_2$ as

$$C_{\text{vg}} \approx \frac{1}{\ln(2)} \left\{ \ln\left(\sqrt{\frac{1+d}{d}}\right) + \int_0^\infty \frac{e^{-t^2}}{t} \left[\mathcal{M}_{\frac{1}{\gamma_1}}\left(\frac{c_2 t^2}{d}\right) \times \mathcal{M}_{\frac{1}{\gamma_2}}\left(\frac{c_1 t^2}{d}\right) - \mathcal{M}_{\frac{1}{\gamma_1}}\left(\frac{c_2 t^2}{1+d}\right) \mathcal{M}_{\frac{1}{\gamma_2}}\left(\frac{c_1 t^2}{1+d}\right)\right] \mathrm{d}t \right\}. \quad (8)$$

Proof: For VG relays, it can be observed that the e2e SNR can be approximated as

$$\gamma_{\text{e2e}-\text{vg}} \approx \frac{\gamma_1\,\gamma_2}{d\,\gamma_1\,\gamma_2 + c_1\,\gamma_1 + c_2\,\gamma_2}. \quad (9)$$

This approximation has been employed in several works, e.g. [1], and it is tight at medium and high SNR values.

Substituting (9) to (1) and following a similar line of arguments as in the proof of Proposition 1, the EC of VG AF systems can be expressed as

$$C_{\text{vg}} \approx \frac{1}{2\ln(2)} \left[\ln\left(\frac{d+1}{d}\right) + \mathbb{E}\left\langle \ln\left(1 + \frac{c_1}{d+1}\frac{1}{\gamma_2} + \frac{c_2}{d+1}\frac{1}{\gamma_1}\right)\right\rangle - \mathbb{E}\left\langle \ln\left(1 + \frac{c_1}{d}\frac{1}{\gamma_2} + \frac{c_2}{d}\frac{1}{\gamma_1}\right)\right\rangle \right]. \quad (10)$$

The proof is completed by taking into account the independence of $1/\gamma_1$ and $1/\gamma_2$, employing (5) and performing some straightforward algebraic manipulations. ■

Next, a closed form expression for the EC of VG systems in Nakagami-0.5 fading will be presented. As pointed out in [13], this fading model corresponds to a worst case propagation scenario and thus, our result serves as a benchmark for the EC performance of realistic relaying systems.

[2]Note that the bivariate H-function can be efficiently numerically evaluated using the MATLAB code presented in [11].

Corollary 2: The EC of VG systems operating over Nakagami-0.5 fading channels can be expressed as

$$C_{\rm vg} \approx \frac{1}{\ln(2)}\left\{0.5\,\ln\left(\frac{d+1}{d}\right)+\mathcal{Q}\left[\sqrt{\frac{c_1}{2\,\overline{\gamma}_2\,(1+d)}}+\right.\right.$$
$$\left.\left.+\sqrt{\frac{c_2}{2\,\overline{\gamma}_1\,(1+d)}}\right]-\mathcal{Q}\left(\sqrt{\frac{c_1}{2\,\overline{\gamma}_2\,d}}+\sqrt{\frac{c_2}{2\,\overline{\gamma}_1\,d}}\right)\right\}, \tag{11}$$

where $\overline{\gamma}_i$ is the average SNR of the ith hop and

$$\mathcal{Q}(x) \triangleq \frac{\pi}{2}\,\mathrm{erfi}(x) - x^2\,{}_2F_2\left(1,1;\frac{3}{2},2;x^2\right). \tag{12}$$

Proof: Assuming Nakagami-0.5 fading channels and using [13, eq. (5)], yields $\mathcal{M}_{1/\gamma_i}(s) = \exp\left(-\sqrt{2\,s/\overline{\gamma}_i}\right)$. By substituting $\mathcal{M}_{1/\gamma_i}(s)$ into (8) and performing the change of variables $t=\sqrt{s}$, the evaluation of $C_{\rm vg}$ requires a solution for the following integral

$$\mathcal{Q}(x) = \int_0^\infty \frac{\exp(-s-x\sqrt{s})\,\sinh(x\sqrt{s})}{s}\,\mathrm{d}s. \tag{13}$$

Using [12, eqs. (8.4.3/1), (8.4.4/7), (2.24.1/1), (8.2.2/3) and (7.11.2/11)], $\mathcal{Q}(x)$ can be expressed as (12). ■

III. Applications to Relaying Systems

Based on the previous analysis, this section presents applications for the following two system scenarios.

A. MIMO With Co-Channel Interference

Consider a dual-hop system consisting of a single-antenna source node, $\mathcal{S}$, an interference-limited relay node, $\mathcal{R}$, and a noise-limited destination node, $\mathcal{D}$, equipped with N_r and N_d antennas, respectively. Note that the considered system configuration typically stems from cell-edge or frequency division relaying [14]. During the first time slot, the signal at $\mathcal{R}$ can be expressed as [2, eq. (6)]

$$y_r = \mathbf{w}^\dagger\,\mathbf{h}_1\,(s+\eta_1) + \sum_{m=1}^{M}\mathbf{w}^\dagger\,\mathbf{g}_m\,s_{I,m}, \tag{14}$$

where s and $s_{I,m}$ ($m=1,2,\ldots,M$) are the source and mth interferer symbols, respectively, with $\mathbb{E}\langle s\,s^\star\rangle = P_s$ and $\mathbb{E}\langle s_{I,m}\,s_{I,m}^\star\rangle = P_{I,m}$, $\mathbf{h}_1$ and $\mathbf{g}_m$ are $N_r\times 1$ channel vectors for the $\mathcal{S}$ and mth interferer to $\mathcal{R}$ links, respectively, having independent and identically distributed complex circular Gaussian random variables $\mathcal{CN}(0,1)$ and $\mathbf{w} = \mathbf{h}_1/\|\mathbf{h}_1\|^2$ is a vector chosen according to the maximal-ratio combining (MRC) principle. Moreover, $\eta_1 \sim \mathcal{CN}(0,\kappa_1^2\,P_s)$ is the distortion noise from hardware impairments of the first hop. The signal at $\mathcal{D}$ is

$$y_d = \mathbf{u}^\dagger\,[\mathbf{H}_2\,\mathbf{v}\,(\mathcal{G}\,y_r+\eta_2)+\mathbf{n}], \tag{15}$$

where $\mathbf{H}_2$ is an $N_d\times N_r$ channel matrix for the $\mathcal{R}$ to $\mathcal{D}$ link, $\mathbf{n}$ is an $N_d\times 1$ additive white Gaussian noise (AWGN) vector with $\mathbb{E}\langle\mathbf{n}\,\mathbf{n}^\dagger\rangle = N_0\,\mathbf{I}$, $\mathcal{G}$ is the relay gain, $\eta_2\sim\mathcal{CN}(0,\kappa_2^2\,P_r)$ is the distortion noise from hardware impairments of the second hop, with P_r denoting the relay transmitting power, $\mathbf{u}$ and $\mathbf{v}$ are transmit pre-coding and receive filtering vectors

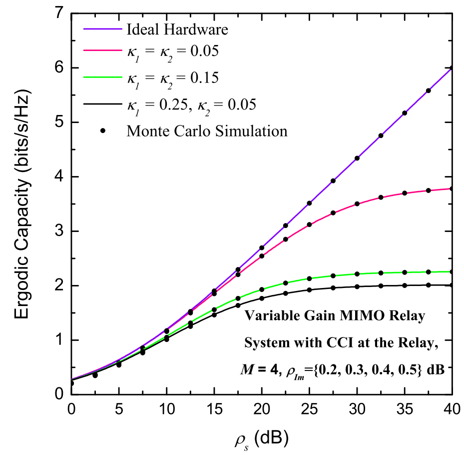

Fig. 1. EC of dual-hop MIMO AF systems with hardware impairments and co-channel interference as a function of $\rho_s=\rho_r$ for $N_t=N_r=2$, $M=4$.

at $\mathcal{R}$ and $\mathcal{D}$, respectively, which are selected as the eigenvector corresponding to the largest eigenvalue, $\lambda_{\max}$, of the Wishart matrix $\mathbf{H}_2^\dagger\,\mathbf{H}_2$. Assuming VG relays [2, eq. (12)]

$$\mathcal{G} = \sqrt{\frac{P_r}{P_s\,|\mathbf{w}^\dagger\,\mathbf{h}_1|^2(1+\kappa_1^2)+\sum_{m=1}^{M}|\mathbf{w}^\dagger\,\mathbf{g}_m|^2\,P_{I,m}}}. \tag{16}$$

Using (14), (15) and (16), the e2e SINR of the considered system is of the form of (2b), with $\gamma_1 = \rho_s\|\mathbf{h}_1\|^2/\chi$, $\chi = \sum_{m=1}^{M}|\mathbf{h}_1^\dagger\,\mathbf{g}_m|^2\,\rho_{I,m}/\|\mathbf{h}_1\|^2$, $\gamma_2=\rho_r\,\lambda_{\max}$, $\rho_s = P_s/N_0$, $\rho_r = P_r/N_0$ and $\rho_{I,m} = P_{I,m}/N_0$. According to [15], $\|\mathbf{h}_1\|^2$ and χ follow a gamma distribution, with parameters N_r and $1/N_r$ and a hyper-exponential distribution, respectively. Employing [8, eq. (3.381/8), (3.381/4), (3.471/7)] and [15, eq. (B-2)], $\mathcal{M}_{1/\gamma_1}(s)$ can be obtained as

$$\mathcal{M}_{\frac{1}{\gamma_1}}(s) = N_r\,(s/\rho_s)^{\frac{N_r-1}{2}}\left(\prod_{m=1}^{M}\rho_{I,m}\right)^{-1}$$
$$\times\sum_{j=1}^{M}\frac{\rho_{I,j}^{\frac{N_r+1}{2}}\exp\left(s\,\frac{\rho_{I,j}}{2\,\rho_s}\right)}{\prod_{\substack{\ell=1\\ \ell\neq j}}^{M}\left(\frac{1}{\rho_{I,\ell}}-\frac{1}{\rho_{I,j}}\right)}\,W_{-(N_r+1)/2,-N_r/2}\left(\frac{\rho_{I,j}}{\rho_s}\,s\right). \tag{17}$$

A simple closed-form expression for $\mathcal{M}_{1/\gamma_2}(s)$, assuming arbitrary values of N_r and N_d, can be obtained by employing [16, eq. (8)] and [8, eq. (3.471/9)] as

$$\mathcal{M}_{\frac{1}{\gamma_2}}(s) = \frac{2}{D}\sum_{a=1}^{P}\sum_{b=b_1}^{b_2}\beta_{ab}\left(\frac{s}{a\rho_r}\right)^{\frac{1+b}{2}}K_{1+b}\left(2\sqrt{\frac{as}{\rho_r}}\right),$$

where $b_1 = Q-P$, $Q=\max\{N_r,N_d\}$, $P=\min\{N_r,N_d\}$, $b_2 = (Q+P)a-2a^2$, $D=\prod_{k=1}^{P}(P-k)!(Q-k)!$, β_{ab} is the coefficient of the term $e^{-ax}x^b$ in the expansion of $\frac{d}{dx}\det[\mathbf{S}(x)]$, with $\mathbf{S}(x)$ being a $P\times P$ Hankel matrix having elements given by $s_{ab}(x) = t^{-1}x^t\,{}_1F_1(t,1+t,-x)$, $t=Q-P+a+b-1$. Fig. 1 depicts the EC of the considered system as a function of $\rho_s=\rho_r$, for $N_t=N_r=2$ and various values of κ. For comparison purposes, the case of ideal hardware, i.e. $\kappa=0$, has been also included. As it is evident,

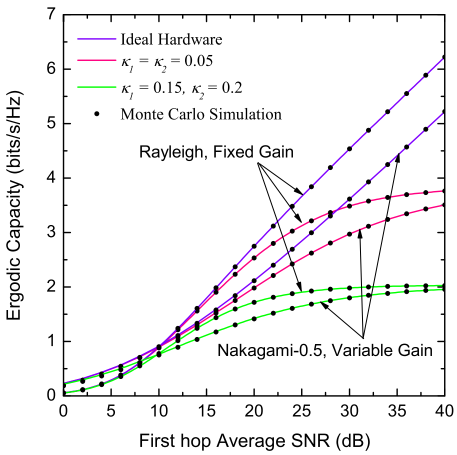

Fig. 2. EC of multi-source multi-destination AF systems with hardware impairments as a function of the first hop average SNR.

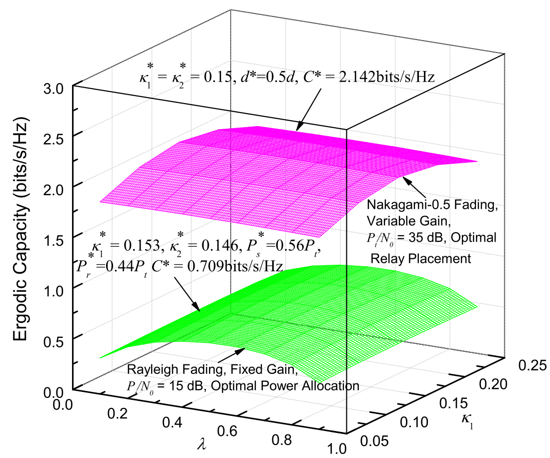

Fig. 3. EC of multi-source multi-destination AF systems with hardware impairments, assuming $\kappa_1 + \kappa_2 = 0.3$, $v = 2.5$.

analytical results obtained using Proposition 1 perfectly match Monte Carlo simulations for all cases.

B. Multi-Source Multi-Destination

Consider a multi-source multi-destination network with N source nodes $\mathcal{S}_i$, $i = 1, \ldots, N$, which can also act as relays, and D destination nodes $\mathcal{D}_j$, $j = 1, \ldots, D$. A similar system configuration with ideal hardware has been studied in [17], as a typical example of the so-called intelligent transportation systems (ITS) paradigm. It is assumed that the frequency bands of the $\mathcal{S}_i$'s comply with orthogonal frequency division and channel models are characterized using time-division duplex (TDD) [17]. In the first slot, $\mathcal{S}_i$ broadcasts its signal to $\mathcal{S}_k$, $k \neq i$. In the second phase, the signal from $\mathcal{S}_k$ is amplified and forwarded to a destination node, $\mathcal{D}_j$. Following a similar line of arguments as in [2] and [17], the e2e SNR at $\mathcal{D}_j$ is obtained as (2), for FG and VG relays, with $\gamma_1 = \gamma_{\mathcal{S}_i,\mathcal{S}_k}$ and $\gamma_2 = \gamma_{\mathcal{S}_k,\mathcal{D}_j}$. Fig. 2 depicts the EC of the $\mathcal{S}_i$ to $\mathcal{D}_j$ link as a function of the first hop SNR assuming FG Rayleigh or VG Nakagami-0.5 channels and, as it is evident, an excellent match with simulations is observed.

Next, the power allocation and relay placement problems are investigated. For power allocation it is assumed that $\overline{\gamma}_1 = \lambda\, P_t$ and $\overline{\gamma}_2 = (1-\lambda)\, P_t$, where P_t is the total transmit power and λ is the power allocation factor. For relay placement it is assumed that $\overline{\gamma}_1 = p_1 \lambda^{-v}$ and $\overline{\gamma}_2 = p_2 (1-\lambda)^{-v}$, where λ and $1-\lambda$ are the normalized distances of the source-to-relay and the relay-to-destination links, respectively, v is the path loss factor and p_i are parameters taking into account the impact of transmit power allocation, antenna gains, shadowing and noise power. For all cases, $\kappa_1 + \kappa_2 = 0.3$.

Fig. 3 depicts EC as a function of λ and κ_1, as well as the corresponding EC maximum, C^*. It is clear that for Nakagami-0.5 the optimal[3] EC is attained when $\kappa_1 = \kappa_2$, whereas, for Rayleigh, is found for $\kappa_1 = 0.153$, $\kappa_2 = 0.146$. Finally, it is noted that all analytical expressions are computationally very efficient, and thus, they are useful for the design and optimization of practical relaying systems.

[3]In Mathematica, the constrained optimization problems can be efficiently solved numerically by using the standard `FindMaximum` built in function. In order to solve the above mentioned problems, (3) and (11) have been used for Rayleigh and Nakagami-0.5 cases, respectively.

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