Dual Diversity over Correlated Ricean Fading Channels

Petros S. Bithas, Nikos C. Sagias, and P. Takis Mathiopoulos

Abstract: **The performance of dual diversity receivers operating over correlated Ricean fading channels is analyzed. Using a previously derived rapidly converging infinite series representation for the bivariate Ricean probability density function, analytical expressions for the statistics of dual-branch selection combining, maximal-ratio combining, and equal-gain combining output signal-to-noise ratio (SNR) are derived. These expressions are employed to obtain novel analytical formulae for the average output SNR, amount of fading, average bit error probability, and outage probability. The proposed mathematical analysis is used to study various novel performance evaluation results with parameters of interest the fading severity, average input SNRs, and the correlation coefficient. The series convergence rate is also examined verifying the fast convergence of the analytical expressions. The accuracy of most of the theoretical performance evaluation results are validated by means of computer simulations.**

Index Terms: **Correlated fading, equal-gain combining (EGC), maximal-ratio combining (MRC), mobile satellite communications, Ricean distribution, selection combining (SC).**

I. INTRODUCTION

Wireless communication systems are subject to severe multipath fading that can seriously degrade their performance. One of the simplest and yet most efficient techniques to improve their performance is diversity. There are several diversity reception methods employed in digital communication receivers including maximal-ratio combining (MRC), equal-gain combining (EGC), and selection combining (SC) [1]. MRC is the optimal combining scheme, but comes at the expense of increased complexity. EGC provides an intermediate solution for improved performance and low implementation complexity, while SC is the least complicated, since only the selectively chosen single branch is processed. Consequently, SC gives much poorer performance in fading channels than MRC and EGC when the number of branches is large. The performance of these diversity techniques depends on the characteristics of the multipath fading envelopes.

There are different models describing the statistical behavior of the multipath fading envelopes depending on the nature of the radio propagation environment. The Ricean distribution is oftenly used to model propagation paths consisting of one strong direct line-of-sight (LOS) component and many random weaker components and is typically observed in microcellular, urban land mobile communications, and mobile satellite radio links [1]–[3]. Especially for satellite mobile communications, the Ricean distribution can be used to accurately characterize the satellite channel for the single-state [4], the clear state [5], and the multi-state model [6]. However, despite the obvious practical importance of studying the performance of dual diversity receivers operating over correlated Ricean fading channels, this research topic has not been adequately investigated. Reasons for this include the complicated form of the bivariate Ricean probability density function (PDF) and the absence of alternative expressions for the multivariate distribution.

Past work concerning the performance of dual diversity receivers operating over correlated fading channels can be found in [7]–[14]. In [7], the average output signal-to-noise ratio (SNR), the amount of fading (AoF), and the outage probability (OP) have been investigated in correlated lognormal fading. In [8], Karagiannidis *et al.* have derived a convergent infinite sum expression for the characteristic function of two correlated Nakagami-m variables, which has been applied to EGC diversity receivers. In [9], useful expressions for the OP and average bit error probability (ABEP) have been presented for dual selection diversity systems with correlated Rayleigh and Nakagami-m fading. In [10], considering correlated Weibull fading channels, analytical expressions for several performance criteria, such as average output SNR, AoF, ABEP, and OP, have been derived in closed form, while in [11], the exact OP using dual EGC is analytically derived for correlated Nakagami-m fading. As far as the Ricean fading channel is concerned, a study for dual branch EGC in slow, correlated, Ricean time selective fading has been presented in [12] for the special case of non-coherent detection of orthogonal binary frequency shift keying (BFSK). Moreover, a performance analysis limited to the noncoherent reception of orthogonal M-ary FSK with postdetection EGC over correlated fading channels has been presented in [13]. In [14], the cumulative distribution function (CDF) of the SC output SNR in equally correlated Rayleigh, Ricean, and Nakagami-m fading channels has been derived.

Recently in [15], infinite series representations have been derived for the Ricean PDF, CDF, covariance, and characteristic function of two correlated Ricean random variables (RVs). It was also depicted that these infinite series expressions converge rapidly and some limited performance results for the OP of SC receivers have been derived. Moreover in [16], capitalizing on [17], another form of infinite series representation for the joint CDF of two Ricean correlated RVs has been presented. Motivated by the previously reported approaches, in this paper we extend the work of [15] by presenting a detailed analysis of the performance of SC, EGC, and MRC receivers operating over correlated Ricean fading channels.

The organization of the paper is as follows. After this introduction, in Section II a brief description of the system and channel model is presented. Based upon this model, novel infinity series representations of the joint Ricean PDF, CDF, mo-

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ments generating function (MGF), and moments output SNR are derived. In Section III, important performance criteria of dual-branch SC, EGC, and MRC diversity receivers are studied. In Section IV, various numerical performance evaluation results are presented and discussed. Finally, concluding remarks are given in Section V.

II. SYSTEM AND CHANNEL MODEL

Let us consider a dual-branch diversity receiver operating over a correlated Ricean fading channel. The baseband received signal in the ℓ-th ($\ell = 1, 2$) antenna is $z_\ell = s\, h_\ell + n_\ell$, where s is the transmitted complex symbol of energy $E_s = \mathbb{E}\langle |s|^2 \rangle$ with $\mathbb{E}\langle\cdot\rangle$ denoting expectation and $|\cdot|$ absolute value, n_ℓ is the complex additive white Gaussian noise (AWGN) with single sided power spectral density N_0 identical to all branches, and h_ℓ is the channel complex gain. The n_ℓ's are assumed to be uncorrelated and by considering slowly varying fading the h_ℓ's are assumed to be known at the receiver [1]. The fading envelopes $R_1 = |h_1|$ and $R_2 = |h_2|$ are modeled as correlated Ricean RVs and the instantaneous SNR per symbol at the ℓ-th input branch is $X_\ell = R_\ell^2\, E_s/(2\, N_0)$. The joint PDF of X_1 and X_2 is given by [18]

$$f_{X_1,X_2}(x_1,x_2) = \frac{(1+K)^2}{2\pi\overline{\gamma}^2\,(1-\rho^2)} \times \exp\left[-\frac{2K}{1+\rho} - \frac{(1+K)\,(x_1+x_2)}{(1-\rho^2)\,\overline{\gamma}}\right] \times \int_0^{2\pi} \exp\left[2\frac{\rho\,(1+K)\sqrt{x_1\,x_2}\cos\theta}{(1-\rho^2)\,\overline{\gamma}}\right] \times I_0\left[\sqrt{\frac{4K(1+K)\left(x_1+x_2+2\sqrt{x_1x_2}\cos\theta\right)}{\overline{\gamma}\,(1+\rho)^2}}\right] d\theta \tag{1}$$

where $\overline{\gamma}$ is the average SNR per symbol at both input branches, i.e., $\overline{\gamma} = \Omega E_s/(2N_0)$, $\Omega = \mathbb{E}\langle R_1^2\rangle = \mathbb{E}\langle R_2^2\rangle$, K is the Ricean factor defined as the ratio of the specular signal power to the scattered power, and $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind [19, eq. (8.406)]. By using different values for K, the Ricean distribution spans the range from Rayleigh fading, i.e., $K = 0$, to no fading, i.e., $K \to \infty$. The Ricean distribution can be closely approximated by the Nakagami-m using a mapping between K and m [1, eq. (2.25)]. In (1), ρ denotes the Ricean correlation coefficient between R_1 and R_2. To the best of our knowledge, the relation between the envelopes correlation coefficients ρ and ρ_{ray}, which is the correlation coefficient between two correlated Rayleigh RVs is not available.[1] Such an expression is presented in the appendix of this paper.

Since a PDF in the form of (1) is very difficult, if not impossible, to be used for the performance analysis of dual-branch diversity receivers, an alternative approach would be to employ an infinite series representation for this PDF, such as the one presented in [15]. Hence, using the infinite series representation for the $I_0(\cdot)$ [19, eq. (8.445)], a term of the form $[x_1 + x_2 + 2\sqrt{x_1x_2}\cos(\theta)]^i$ appears. Using the multinomial identity this term can be simplified and after some mathematical manipulations the joint PDF of X_1 and X_2 can be expressed in sums, i.e., without the integrals, as

$$f_{X_1,X_2}(x_1,x_2) = \sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty} \mathcal{A}\exp\left[-\beta_1(x_1+x_2)\right] \times \left(\mathcal{B}\, x_1^{\beta_2-1} x_2^{\beta_3-1} + \mathcal{C}\,\overline{\gamma}^{-1}\, x_1^{\beta_2-1/2}\, x_2^{\beta_3-1/2}\right) \tag{2}$$

with

$$\mathcal{A} = \frac{2^{v_3+2h-1}(1+K)^{1+\beta_4}\rho^{2h}\left[K/(1+\rho)^2\right]^i}{\sqrt{\pi}\,\overline{\gamma}^{1+\beta_4}\,(1-\rho^2)^{1+2h}\, v_1!\,v_2!\,v_3!\,i!}\exp\left(\frac{-2K}{1+\rho}\right),$$

$$\mathcal{B} = \frac{\left[1+(-1)^{v_3}\right]\Gamma\left[h+(1+v_3)/2\right]}{\Gamma\left(h+1+v_3/2\right)\Gamma\left(1+2h\right)},$$

$$\mathcal{C} = \frac{\left[-1+(-1)^{v_3}\right]2\rho(1+K)\,\Gamma\left(1+h+v_3/2\right)}{(\rho^2-1)\,\Gamma(2+2h)\Gamma\left[h+(3+v_3)/2\right]},$$

$$\beta_1 = \frac{(1+K)}{(1-\rho^2)\,\overline{\gamma}},\ \beta_2 = v_1 + \frac{v_3}{2} + h + 1,$$

$$\beta_3 = v_2 + \frac{v_3}{2} + h + 1,\ \text{and}\ \beta_4 = i + 2h + 1$$

where $\Gamma(\cdot)$ is the Gamma function [19, eq. (8.310/1)].

By substituting (2) in the definition of the joint MGF of X_1 and X_2 [21, eq. (5.62)]

$$\mathcal{M}_{X_1,X_2}(s_1,s_2) = \mathbb{E}\langle \exp\left(-s_1 X_1 - s_2 X_2\right)\rangle \tag{3}$$

and using [19, eq. (3.381/4)], $\mathcal{M}_{X_1,X_2}(s_1,s_2)$ can be expressed as

$$\mathcal{M}_{X_1,X_2}(s_1,s_2) = \sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty} \frac{\mathcal{A}\,(\beta_1-s_1)^{-\beta_2}}{(\beta_1-s_2)^{\beta_3}} \times \left[\mathcal{B}\,\Gamma(\beta_2)\,\Gamma(\beta_3) + \frac{\mathcal{C}\,\Gamma\left(1/2+\beta_2\right)\Gamma\left(1/2+\beta_3\right)}{\overline{\gamma}\sqrt{(\beta_1-s_1)\,(\beta_1-s_2)}}\right]. \tag{4}$$

An expression for the joint moments of X_1 and X_2, defined as $\mu_{X_1,X_2}(k,\lambda) = \mathbb{E}\langle X_1^k X_2^\lambda\rangle$ [21, eq. (5.38)], can be derived, by substituting (2) in this definition and using again [19, eq. (3.381/4)], as

$$\mu_{X_1,X_2}(k,\lambda) = \sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty} \mathcal{A}\left\{\frac{\mathcal{B}\beta_1\Gamma\left(\lambda+\beta_2\right)\Gamma\left(k+\beta_3\right)}{\beta_1^{(2+\lambda+k+\beta_4)}} + \frac{\mathcal{C}\Gamma\left(1/2+\lambda+\beta_2\right)\Gamma\left(1/2+k+\beta_3\right)}{\overline{\gamma}\,\beta_1^{(2+\lambda+k+\beta_4)}}\right\}. \tag{5}$$

The joint CDF of X_1 and X_2 can be obtained using $F_{X_1,X_2}(x_1,x_2) = \int_0^{x_1}\int_0^{x_2} f_{X_1,X_2}(x_1,x_2)\,dx_1dx_2$ [21, eq. (6.6)]. Substituting (2) in the above equation and interchanging the order of summations and integrations, some integrals

[1]It is noted that the relation between the power correlation coefficient of Ricean correlated RVs and the correlation coefficient of their underlying complex Gaussian RVs has been presented in [1, Appendix 9C] and [20].

of the form $\int_0^{\xi} y^a \exp\left(-\Xi\, y^2\right) dy$ appear, where a, Ξ, and ξ are real constants. These integrals can be efficiently solved after applying the transformation $t = \Xi\, y^2$ and using the definition of the lower incomplete Gamma function [19, eq.(8.350/1)], $\gamma(\alpha, x) = \int_0^x t^{\alpha-1}\exp(-t)dt$. Following this and after some straightforward mathematical simplifications, the joint CDF of X_1 and X_2 can be expressed in sums as

$$F_{X_1,X_2}(x_1,x_2) = \sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty} \mathcal{A}\left\{\frac{\mathcal{B}\gamma\left(\beta_2,\beta_1 x_1\right)\gamma\left(\beta_3,\beta_1 x_2\right)}{\left(\beta_1\right)^{\beta_4+1}} + \frac{\mathcal{C}\,\gamma\left(\beta_2+1/2,\beta_1 x_1\right)\gamma\left(\beta_3+1/2,\beta_1 x_2\right)}{\overline{\gamma}\,\left(\beta_1\right)^{\beta_4+2}}\right\}. \tag{6}$$

III. PERFORMANCE ANALYSIS

In this section, capitalizing on the previously derived formulae, expressions for various performance measures of the three diversity receivers under consideration will be obtained.

A. SC Receivers

A.1 Average Output SNR and AoF

By denoting the instantaneous SNR at the output of the SC receiver as $X_{sc} = \max(X_1, X_2)$ [21, eq. (6.54)], the CDF of X_{sc} is expressed as $F_{sc}(x) = F_{X_1,X_2}(x,x)$ [1]. By differentiating this CDF, the PDF of X_{sc}, $f_{sc}(x)$, can be easily derived as

$$\begin{aligned}f_{sc}(x) = &\sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty} \mathcal{A}\,\beta_1^{-\beta_2}\,\exp(-\beta_1\, x)\\ &\times\left\{\mathcal{B}\left[x^{\beta_3-1}\,\gamma\left(\beta_2,\beta_1\, x\right)+\beta_1^{v_1-v_2}\,x^{\beta_2-1}\,\gamma\left(\beta_3,\beta_1\, x\right)\right]\right.\\ &\quad+\frac{1}{\sqrt{\beta_1}\,\overline{\gamma}}\,\mathcal{C}\,\left[x^{\beta_3-1/2}\,\gamma\left(\beta_2+1/2,\beta_1\, x\right)\right.\\ &\quad\left.\left.+\beta_1^{v_1-v_2}\,x^{\beta_2-1/2}\,\gamma\left(\beta_3+1/2,\beta_1\, x\right)\right]\right\}.\end{aligned} \tag{7}$$

By substituting (7) in the definition of the n-th moment of X_{sc} and interchanging the order of summation and integration, integrals of the following form need to be solved

$$I = \int_0^{\infty} y^a\,\exp(-\xi\, y)\,\gamma(u, \Xi\, y)\,dy \tag{8}$$

where a, ξ, u, and Ξ are positive constants. Representing the lower incomplete Gamma function as [19, eq. (8.351)]

$$\gamma(u,\Xi\, y) = \frac{(\Xi\, y)^u}{u}\;{}_1F_1(u; u+1; -\Xi\, y) \tag{9}$$

the integral in (8) can be solved using [19, eq. (7.621/4)] as

$$I = \frac{\Xi^u}{\xi^{\alpha+u+1}\,u}\Gamma\left(\alpha+u+1\right){}_2F_1\left(u,\alpha+u+1;u+1;-\frac{\Xi}{\xi}\right) \tag{10}$$

where ${}_2F_1\left(\cdot,\cdot;\cdot;\cdot\right)$ is the Gauss hypergeometric function [19, eq. (9.100)]. Hence, using (10), the n-th moment of X_{sc}, $\mu_{sc}(n)$, can be expressed as

$$\begin{aligned}\mu_{sc}\left(n\right) = &\sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty}\frac{\mathcal{A}\,\overline{\gamma}}{\beta_1^{\beta_5}}\left\{\mathcal{B}\,\Gamma\left(\beta_5\right)\right.\\ &\times\left[\frac{{}_2F_1\left(\beta_2,\beta_5;\beta_2+1;-1\right)}{\beta_2}+\frac{{}_2F_1\left(\beta_3,\beta_5;\beta_3+1;-1\right)}{\beta_3}\right]\\ &+\frac{\Gamma\left(1+\beta_5\right)}{\mathcal{C}^{-1}\overline{\gamma}\,\beta_1}\left[\frac{{}_2F_1\left(\beta_2+1/2,\beta_5+1;\beta_2+3/2;-1\right)}{\beta_2+1/2}\right.\\ &\qquad\left.\left.+\frac{{}_2F_1\left(\beta_3+1/2,\beta_5+1;\beta_3+3/2;-1\right)}{\beta_3+1/2}\right]\right\}\end{aligned} \tag{11}$$

where $\beta_5 = i + 2h + n + 2$. From the above equation, the average output SNR, $\overline{\gamma}_{sc}$, can be obtained by setting $n = 1$. Furthermore, the AoF, A_F, can be easily obtained since

$$A_F = \frac{\mathrm{var}\left(\mathrm{X_{sc}}\right)}{\overline{\gamma}_{sc}^2} = \frac{\mu_{sc}\left(2\right)}{\overline{\gamma}_{sc}^2} - 1. \tag{12}$$

A.2 ABEP Performance

The MGF of the SC output SNR is defined as $\mathcal{M}_{sc}(s) = \mathbb{E}\langle\exp\left(-s\, X_{sc}\right)\rangle$ which by substituting (7) results integrals of the form (8). Thus, following a similar procedure used for deriving (11), the MGF of X_{sc} can be expressed as

$$\begin{aligned}\mathcal{M}_{sc}(s) = &\sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty}\frac{\mathcal{A}\,\Gamma(\beta_4+1)}{(\beta_1-s)^{\beta_4+1}}\\ &\times\left\{\mathcal{B}\left[\frac{{}_2F_1\left[\beta_2,\beta_4+1;\beta_2+1;-\beta_1/(\beta_1-s)\right]}{\beta_2}\right.\right.\\ &\left.+\frac{{}_2F_1\left[\beta_3,\beta_4+1;\beta_3+1;-\beta_1/(\beta_1-s)\right]}{\beta_1^{v_1-v_2}\beta_3}\right]+\frac{\mathcal{C}\,\left(\beta_4+1\right)}{\overline{\gamma}(\beta_1-s)}\\ &\times\left[\frac{{}_2F_1\left[\beta_2+1/2,\beta_4+2;\beta_2+3/2;\beta_1/(s-\beta_1)\right]}{\beta_2+1/2}\right.\\ &\left.\left.+\frac{{}_2F_1\left[\beta_3+1/2,\beta_4+2;\beta_3+3/2;\beta_1/(s-\beta_1)\right]}{\beta_1^{v_1-v_2}\left(\beta_3+1/2\right)}\right]\right\}.\end{aligned} \tag{13}$$

By using (13) and following the MGF-based approach [1], the ABEP can be readily evaluated for a variety of modulation schemes as well as for arbitrary values of the fading severity parameter K and average input SNR as follows:

- Using numerical integration of an integral involving (13), for M-ary phase shift keying (PSK), binary PSK (BPSK), M-ary quadrature amplitude modulation (QAM), and M-ary differential PSK (DPSK), since integrals with finite limits are obtained.
- Directly for non-coherent BFSK and differential binary PSK.

B. EGC Receivers

B.1 ABEP, Average Output SNR, and AoF

Extending [22] for the correlated Ricean fading, the conditional SNR per symbol at the output of the dual-branch EGC combiner is given by [1, eq. (9.46)], $X_{egc} = \frac{1}{2}\left(\sqrt{X_1}+\sqrt{X_2}\right)^2$. By definition, the n-th moment of the EGC output SNR is given by

$$\begin{aligned}\mathbb{E}\left\langle X_{egc}^n\right\rangle =&\mathbb{E}\left\langle\left[\frac{1}{2}\left(\sqrt{X_1}+\sqrt{X_2}\right)^2\right]^n\right\rangle \\ =&\left(\frac{1}{2}\right)^n \mathbb{E}\left\langle\left(\sqrt{X_1}+\sqrt{X_2}\right)^{2n}\right\rangle.\end{aligned} \tag{14}$$

Using the binomial theorem [19, eq. (1.111)], the n-th moment of X_{egc} can be expressed as

$$\mu_{egc}(n)=\left(\frac{1}{2}\right)^n\sum_{k=0}^{2n}\binom{2n}{k}\mathbb{E}\left\langle X_1^{k/2}X_2^{(2n-k)/2}\right\rangle. \tag{15}$$

By substituting (5) in (15), the moments of the EGC output SNR can be derived as

$$\begin{aligned}\mu_{egc}(n)=&\sum_{k=0}^{2n}\sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty}\binom{2n}{k}\mathcal{A}\,\beta_1^{-(1+\beta_5)}2^{-n}\\ &\times\left[\mathcal{B}\,\beta_1\,\Gamma\left(\frac{2n-k}{2}+\beta_2\right)\Gamma\left(\frac{k}{2}+\beta_3\right)\right.\\ &\left.+\frac{\mathcal{C}}{\overline{\gamma}}\Gamma\left(\frac{1+2n-k}{2}+\beta_2\right)\Gamma\left(\frac{1+k}{2}+\beta_3\right)\right].\end{aligned} \tag{16}$$

Since direct evaluation of the MGF output SNR for the EGC receiver is a very difficult task, an alternative method to approximate it and consequently evaluate the ABEP must be used. Such method is the so-called Padé approximants [23], which has been used in the past to study the performance of EGC [24] and generalized selection combining (GSC) [25] diversity receivers and as well as to approximate PDFs [26]. Its main advantage is that due to the form of the produced rational approximation, the ABEP can be calculated directly using simple expressions. Hence, the MGF can be represented as a formal power series (e.g., Taylor), using (16), as

$$\mathcal{M}_{egc}(s)=\sum_{n=0}^{\infty}\frac{\mu_{egc}(n)}{n!}\,s^n. \tag{17}$$

Although $\mu_{egc}(n)$ can be evaluated in closed form, the above infinite series does not always converge. However, using Padé approximants only a finite number of terms W can be used, thus truncating the series in (17). In our analysis, $\mathcal{M}_{egc}(s)$ is approximated using sub-diagonals ($R_{[A/A+1]}(s)$) Padé approximants ($B=A+1$), since it is only for such order of approximants that the convergence rate and the uniqueness can be assured [23], [24]. By obtaining accurate approximation expressions for the MGF of EGC output SNR and using the MGF-based approach, the ABEP of EGC can be derived.

C. MRC Receivers

C.1 ABEP Performance

The MGF of the instantaneous SNR at the output of a MRC receiver, X_{mrc}, can be obtained, using (4), as $\mathcal{M}_{mrc}(s)=\mathcal{M}_{X_1,X_2}(s,s)$. Hence, similarly to the SC receivers, the ABEP can be calculated using the $\mathcal{M}_{mrc}(s)$ in a straightforward manner.

C.2 Outage Probability

The CDF of X_{mrc}, $F_{mrc}(x)$, can be derived as

$$F_{mrc}(x)=\mathcal{L}^{-1}\left\{\frac{\mathcal{M}_{mrc}(s)}{s};x\right\}_{s=0} \tag{18}$$

where $\mathcal{L}^{-1}\{\cdot;\cdot\}$ denotes inverse Laplace transformation. Hence, after some straightforward mathematical manipulations $F_{mrc}(x)$ can be obtained as

$$\begin{aligned}F_{mrc}(x)=&\sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty}\mathcal{A}\left[\frac{\mathcal{B}\Gamma(\beta_2)\,\Gamma(\beta_3)\,\gamma\left(1+\beta_4,x\beta_1\right)}{\beta_1^{1+\beta_4}\,\Gamma\left(1+\beta_4\right)}\right.\\ &\left.+\frac{\mathcal{C}\,\Gamma\left(1/2+\beta_2\right)\Gamma\left(1/2+\beta_3\right)\gamma\left(2+\beta_4,x\beta_1\right)}{\beta_1^{1+\beta_4}\,\overline{\gamma}\,\beta_1\,\Gamma\left(2+\beta_4\right)}\right].\end{aligned} \tag{19}$$

Using (19), the OP can be obtained as $P_{out}(x_{th})=F_{mrc}(x_{th})$.

C.3 Average Output SNR and AoF

The PDF of X_{mrc} can be obtained by differentiating (19) as

$$\begin{aligned}f_{mrc}(x)=&\sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty}\frac{\mathcal{A}\,K^i\,x^{\beta_4}\exp\left(-x\beta_1\right)}{(1+\rho)^{2i}}\\ &\times\left[\frac{\mathcal{B}\Gamma(\beta_2)\,\Gamma(\beta_3)}{\Gamma\left(1+\beta_4\right)}+\frac{\mathcal{C}\Gamma\left(1/2+\beta_2\right)\Gamma\left(1/2+\beta_3\right)x}{\overline{\gamma}\,\Gamma\left(2+\beta_4\right)}\right].\end{aligned} \tag{20}$$

Hence, using (20) the n-th moment of X_{mrc} can be expressed as

$$\begin{aligned}\mu_{mrc}(n)=&\sum_{\substack{i,h=0\\ v_1+v_2+v_3=i}}^{\infty}\frac{\mathcal{A}\beta_1^{-(2+n+\beta_4)}}{2^n\Gamma\left(1+\beta_4\right)\Gamma\left(2+\beta_4\right)}\\ &\times\left[\mathcal{B}\Gamma(\beta_2)\,\Gamma(\beta_3)\,\beta_1\,\Gamma\left(2+\beta_4\right)\Gamma\left(\beta_5\right)\right.\\ &\left.+\frac{\mathcal{C}}{\overline{\gamma}}\Gamma\left(\frac{1}{2}+\beta_2\right)\Gamma\left(\frac{1}{2}+\beta_3\right)\Gamma\left(1+\beta_4\right)\Gamma\left(1+\beta_5\right)\right].\end{aligned} \tag{21}$$

Setting $n=1$ in (21), the average output SNR, $\overline{\gamma}_{mrc}$, can be obtained, whereas the AoF can be also easily derived using (12) and (21).

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, using the previously derived analytical expressions, we present representative numerical performance evaluation results, such as AoF, ABEP, and OP for the considered SC, EGC, and MRC receivers. We have investigated these performances under a wide range of channel conditions, e.g., $0<\rho<1$, $0<K<10\,\text{dB}$, and $-5\,\text{dB}<\overline{\gamma}<20\,\text{dB}$, thus considering both terrestrial and satellite communication scenarios. It should be noted that for all considered ranges of values quick convergence of the series has been observed.

Using (11) for SC, (16) for EGC, and (21) for MRC as well as (12), the AoF performances have been obtained. These

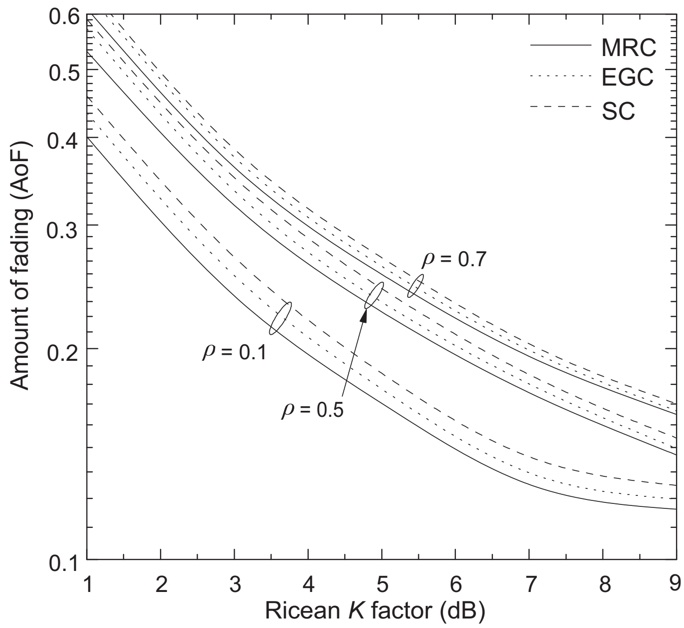

Fig. 1. AoF versus K for different values of ρ and for MRC, EGC, and SC.

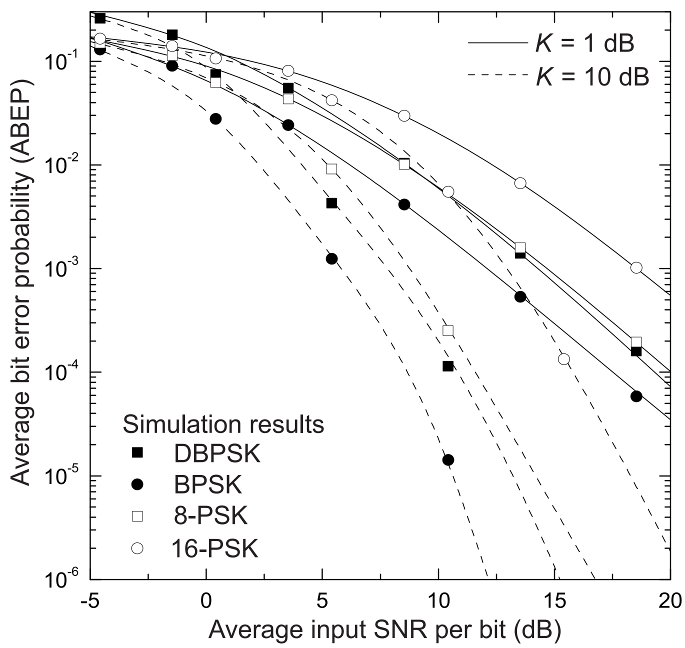

Fig. 2. EGC performance for DBPSK and M-ary PSK signals: ABEP versus $\overline{\gamma}_b$ for $K = 1$ and 10 dB.

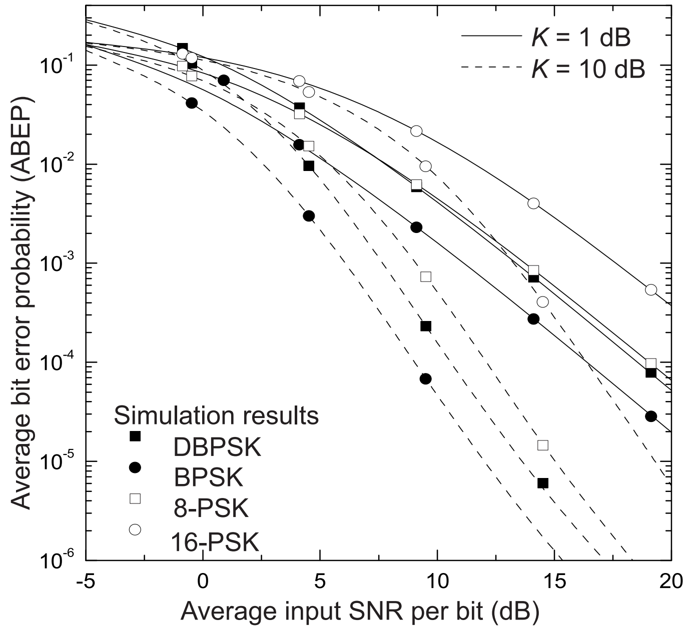

Fig. 3. MRC performance for DPSK and M-ary PSK signals: ABEP versus $\overline{\gamma}_b$ for $K = 1$ and 10 dB.

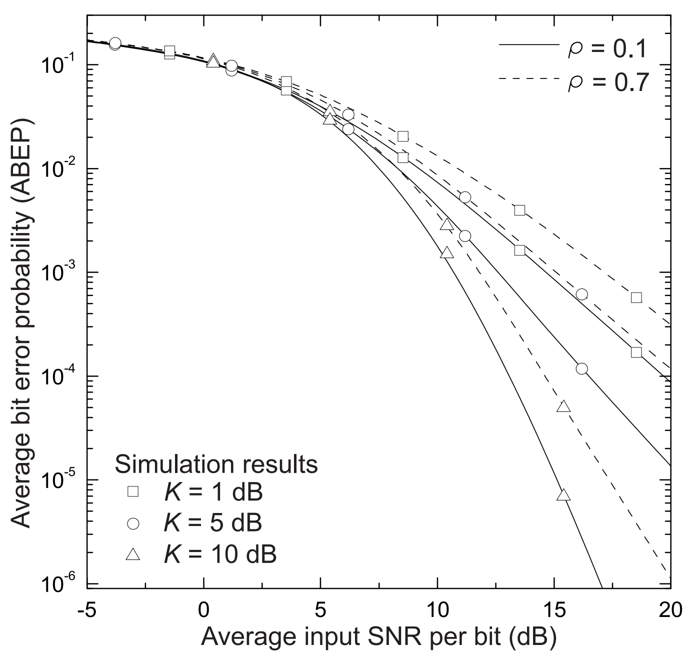

Fig. 4. SC performance for 16-QAM signals: ABEP versus $\overline{\gamma}_b$ for $K = 1, 5$ and 10 dB and $\rho = 0.1$ and 0.7.

results can be found in Fig.1 and are presented as functions of K for several values of ρ. They show that as ρ increases and/or K decreases the AoF, i.e., the severity of the fading, increases. Clearly, MRC provides the best performance and SC the worst. However, the difference in performances are not significantly large. Moreover, as the correlation coefficient increases the diversity gain of MRC, as compared to EGC and SC, decreases.

In Figs. 2 and 3, the ABEP performance of dual-branch EGC (see Section III-B.1) and MRC (see Section III-C.1) receivers are plotted as a function of the average input SNR per bit, $\overline{\gamma}_b = \overline{\gamma}/\log_2 M$, for DBPSK and M-ary PSK (with Gray encoding), for $\rho = 0.5$ and several values of K. As expected, the ABEP improves as $\overline{\gamma}_b$ increases, while for a fixed value of $\overline{\gamma}_b$ it also improves as K increases. Similar behavior is observed in Figs. 4–6 for the SC (see Section III-A.2), MRC (see Section III-C.1), and EGC (see Section III-B.1), respectively. In these figures, the ABEP of a 16-QAM modulation scheme (with Gray encoding) is plotted again as a function of $\overline{\gamma}_b$ for several values of K and ρ. Note that as ρ increases, the ABEP decreases and MRC has slightly better ABEP as compared to the other two diversity reception techniques. In Fig. 7, using (19) the OP ver-

Table 1. Minimum number of terms ($i_{\min}$, $h_{\min}$) of (13) required for obtaining seven significant digits accuracy for the ABEP of DBPSK.

$\overline{\gamma}$ (dB)	$\rho = 0.2$				$\rho = 0.7$			
	K = 1 dB		K = 7 dB		K = 1 dB		K = 7 dB	
	$i_{\min}$	$h_{\min}$	$i_{\min}$	$h_{\min}$	$i_{\min}$	$h_{\min}$	$i_{\min}$	$h_{\min}$
-5	13	7	28	13	13	28	28	47
0	11	5	26	11	11	20	26	41
5	9	4	20	8	9	14	23	30
10	5	2	18	4	5	6	15	20
15	3	1	11	2	4	4	11	12
20	2	1	8	1	3	2	7	4

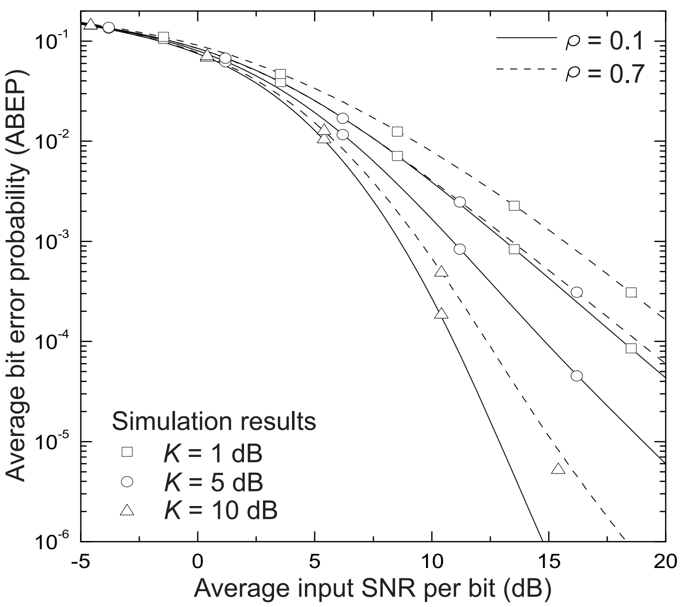

Fig. 5. MRC performance for 16-QAM signals: ABEP versus $\overline{\gamma}_b$ for $K = 1, 5$ and 10 dB and $\rho = 0.1$ and 0.7.

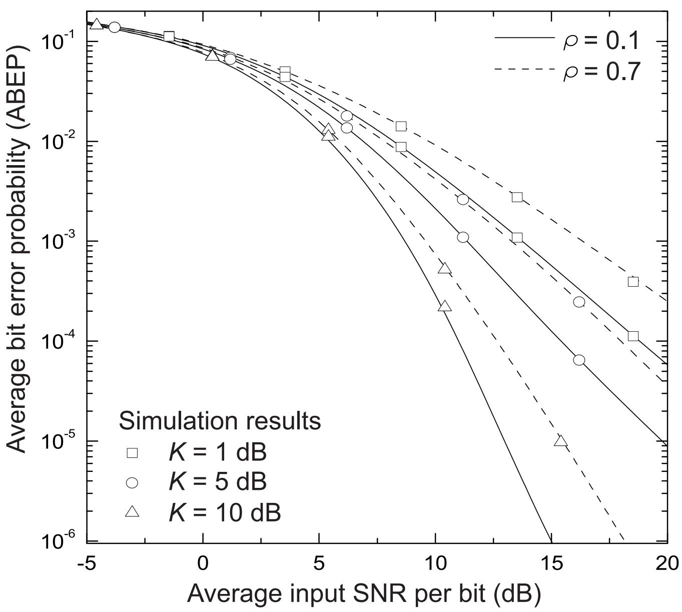

Fig. 6. EGC performance for 16-QAM signals: ABEP versus $\overline{\gamma}_b$ for $K = 1, 5$ and 10 dB and $\rho = 0.1$ and 0.7.

sus $\overline{\gamma}_b/x_{th}$ for several values of K and ρ is illustrated. Clearly, the OP deteriorates with increasing ρ, while it improves with increasing K. In order to verify the validity of the theoretically derived formulae, equivalent computer simulated results (represented by circles, squares, and triangles signs) are also included in all ABEP performance results presented in Figs. 2–6. The excellent agreement between simulated and analytical results verifies the correctness of the theoretical derivations.

Finally, the rate of convergence of the infinite series expressions has also been investigated. In Table 1, the minimum values for the i and h terms, $i_{\min}$ and $h_{\min}$, which guarantee seven significant figure accuracy (i.e., $\leq 10^{-7}$) are presented for the ABEP of DBPSK signals (see (13)) versus $\overline{\gamma}_b$ for different values of ρ and K. It is noted that by increasing K and/or ρ, larger val- ues for the i or h terms are required, respectively. It is also clear that only a relatively small number of terms is necessary to achieve an excellent accuracy and compared to the Nakagami-m channel, the required number of terms is significantly smaller for a similar target accuracy [27], [28]. Our research has also shown that almost identical results, in terms of the rate of convergence, were also obtained by using other modulation formats, such as M-ary QAM and M-ary PSK.

V. CONCLUSIONS

In this paper, an analytical performance study of dual-branch diversity receivers operating over correlated Ricean fading channels has been presented. Based on an infinite series expression of the bivariate Ricean PDF, analytical formulae for the CDF, MGF, and the moments of dual-branch SC, EGC, and MRC output SNR were derived. Using these expressions, novel analytical formulae for the average output SNR, AoF, ABEP, and OP have been obtained in infinite series form. The proposed formulae were used to obtain various novel performance evaluation results having as variables fading severity, average input SNR, and Ricean correlation coefficient. The accuracy of most of the theoretical results has been verified by means of computer simulation.

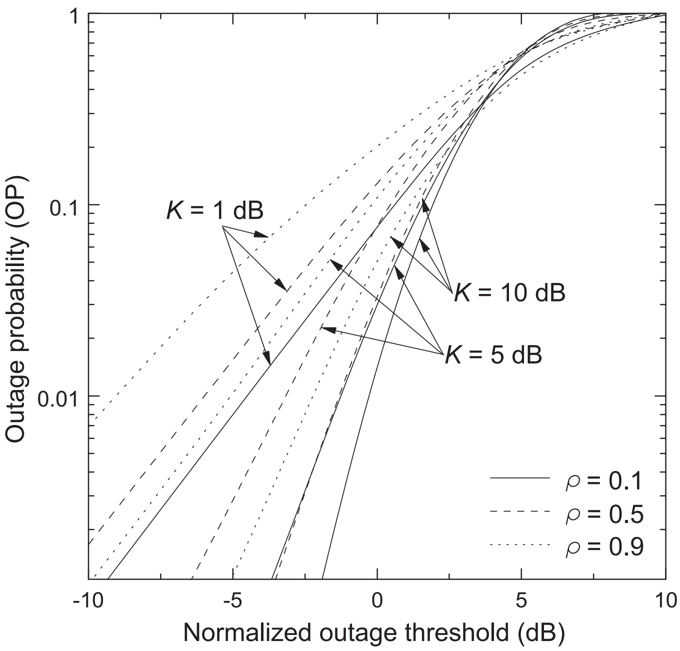

Fig. 7. MRC performance: OP versus $\overline{\gamma}_b/x_{th}$ for several values of K and ρ.

APPENDIX: RELATION BETWEEN RAYLEIGH AND RICEAN CORRELATION COEFFICIENTS

In this appendix, a closed-form expression relating the Ricean, ρ, and the Rayleigh, ρ_{ray}, correlation coefficients of the envelopes is derived. These correlation coefficients are significantly different due to the different statistical behavior of their multipath fading envelopes [29]. The Ricean complex channel fading h_1 and h_2 are related to the complex Gaussian random variables g_1 and g_2 by

$$h_1 = g_1 + A \text{ and } h_2 = g_2 + A \tag{A-1}$$

where A denotes the power ratio of the LOS component to the average power of the scattered component.

Substituting (A-1) in the definition of ρ, i.e.,

$$\rho = \frac{\mathbb{E}\left\langle \left(R_1 - \overline{R}_1\right)\left(R_2 - \overline{R}_2\right)\right\rangle}{\sqrt{\mathbb{E}\left\langle R_1^2\right\rangle - \overline{R}_1^2}\sqrt{\mathbb{E}\left\langle R_2^2\right\rangle - \overline{R}_2^2}} \tag{A-2}$$

joint moments of the form $\mathbb{E}\langle R_1^n\, R_2^m\rangle$ appear. These joint moments can be solved, with the aid of [30], as

$$\mathbb{E}\langle R_1^n\, R_2^m\rangle = (1-\rho_{ray})^{1+(n+m)/2}\,\Omega_1^{n/2}\,\Omega_2^{m/2}\Gamma\left(1+\frac{n}{2}\right) \times \Gamma\left(1+\frac{m}{2}\right){}_2F_1\left(1+\frac{n}{2}, 1+\frac{m}{2}; 1; \rho_{ray}\right). \tag{A-3}$$

Hence, after some straightforward mathematical manipulations the relation between ρ and ρ_{ray} can be expressed in the following compact form as

$$\rho = \frac{\pi}{4-\pi}\left[(1-\rho_{ray})^2\,{}_2F_1\left(\frac{3}{2}, \frac{3}{2}; 1; \rho_{ray}\right) - 1\right]. \tag{A-4}$$

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