New Results for the Multivariate Nakagami-m Fading Model with Arbitrary Correlation Matrix and Applications

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***Abstract*—New results for the multichannel Nakagami-m fading model with an arbitrary correlation matrix are presented in this paper. By using an efficient tridiagonalization method based on Householder matrices, the inverse of the Gaussian correlation matrix is transformed to tridiagonal, managing to derive a closed-form union upper bound for the joint Nakagami-m probability density function and an exact analytical expression for the moment generating function of the sum of identically distributed gamma random variables. Our analysis considers an arbitrary correlation structure, which includes as special cases the exponential, constant, circular, and linear correlation ones. Based on the proposed mathematical analysis, we obtain a tight union upper bound for the outage probability of multibranch selection diversity receivers as well as exact analytical expressions for the outage and the average error probability of multibranch maximal-ratio diversity receivers. Our analysis is verified by comparing numerically evaluated with extensive computer simulation performance evaluation results, showing the usefulness of the proposed approach.**

***Index Terms*—Bit error rate (BER), correlated fading, correlation models, diversity, Householder matrix, maximal-ratio combining (MRC), multichannel receivers, multivariate analysis, Nakagami-m fading, outage probability, selection combining (SC).**

I. INTRODUCTION

THE THEORY of multivariate stochastic processes can be used as an essential mathematical tool for modeling and analyzing realistic wireless communications channels with correlated fading. Such fading channels are usually met in digital contemporary communications systems which employ diversity receivers with not sufficiently separated antennas and where space or polarization diversity is applied (e.g. antenna arrays, handheld mobile terminals, and indoor base-stations). In those applications, the correlation between channels results in a degradation of the diversity gain obtained [1]–[3]. A versatile statistical distribution that accurately models a variety of fading environments is the Nakagami-m distribution [4]. It describes multipath scattering with relatively large delay-time spreads and with different clusters of reflected waves, providing greater flexibility in matching experimental data than the Rayleigh, Ricean, or lognormal distributions [5]. Also, it includes as special cases the Rayleigh and the one-sided Gaussian distributions.

In past, numerous papers have been published in the open technical literature dealing with multivariate fading channel models and/or systems performance analysis (see [1]–[41] and references therein). A very generic expression for the multivariate gamma-type distribution with an arbitrary covariance matrix being in the form of a multiple series of generalized Laguerre polynomials was presented back in 1951 [9]. That expression has been used for the derivation of the outage probability (OP) of selection combining (SC) receivers over arbitrarily correlated Nakagami-m fading channels [10]. However, the associated multivariate probability density function (PDF) used in [10] for deriving the OP becomes fairly complicated with poor convergence properties, when the statistics of more than two random variables (RVs) needs to be considered [39]. Hence, simpler formulas have been introduced considering specific structures of the correlation matrix. For example in [11], Mallik has presented exact closed-form PDF expressions for the multivariate Rayleigh distribution with exponential and constant correlation matrices. In a parallel and independent work, Karagiannidis *et al.* [12] have introduced the multivariate Nakagami-m PDF with exponential correlation and identically distributed (i.d.) fading channels. An infinite series approach for its corresponding cumulative distribution function (CDF) and a bound of the error resulting from the truncation of the infinite series have been also included. By approximating the correlation matrix with a Green's matrix, the same authors have generalized [12], presenting approximate expressions for the arbitrarily correlated Nakagami-m distribution [13]. In [14], a PDF-

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based approach for the performance analysis of maximal-ratio combining (MRC) and postdetection equal-gain combining (EGC) receivers with arbitrary channel parameters has been presented. By assuming correlated Nakagami-m fading with positive integer-order values for the fading parameter, the performance of SC, hybrid-selection/MRC (H-S/MRC), and threshold-based H-S/MRC has been analyzed in [20], [21]. In [20], Green's matrix approximations have been used for studying arbitrary correlation structures, while in [21], a more general model than the equal correlation one has been considered in the performance analysis of generalized SC (GSC) receivers. Recently, based on [9], the performance analysis of SC receivers over arbitrary correlated generalized gamma fading channels has been presented [22]. Although the derived formulas are very generic, they are computationally burden, since they are in the form of a multiple series of generalized Laguerre polynomials.

In this paper, a new statistical approach for the multivariate Nakagami-m fading channel model with arbitrary correlations is presented. By using a computationally efficient tridiagonalization method based on Householder matrices, a set of Gaussian RVs is transformed to another, with the latter one having a tridiagonal inverse correlation matrix (see Fig. 1). Next, correlated Nakagami-m RVs are generated having a similar to the desired power correlation matrix, $\mathbf{\Sigma}'$, for which the exact joint PDF is obtained. By applying a standard RVs transformation method, a closed-form union upper bound for the PDF of the multivariate Nakagami-m distribution with a power correlation matrix, $\mathbf{\Sigma}$, as well as an exact analytical expression for the moment generating function (MGF) of the sum of i.d. gamma RVs are derived. Our analysis is not only limited to arbitrary average power and fading parameters, but also considers arbitrary correlation structures, including the exponential, constant, circular, and linear correlation ones as special cases. Based on the proposed mathematical analysis, we provide a significant theoretical tool that can be efficiently used for the performance analysis of wireless communications systems operating over i.d. and arbitrarily correlated Nakagami-m fading channels. More specifically, a tight union upper bound for the OP of multibranch SC receivers is derived, while exact analytical expressions for the OP and average symbol error probability (ASEP) of multibranch MRC receivers are obtained. The usefulness of the proposed analysis is verified by comparing numerically evaluated with extensive computer simulation performance evaluation results.

The remainder of this paper is organized as follows. Union bounds for the multivariate Nakagami-m PDF and CDF are provided in Section II. The performance of SC and MRC receivers is analyzed in Section III, while numerically evaluated and computer simulation performance evaluation results are presented and compared in Section IV. Section V concludes providing useful remarks.

II. Multivariate Nakagami-m Fade Statistics

In this section, first, arbitrarily correlated Nakagami-m RVs are generated from Gaussian RVs. Moreover, the most popular correlation models usually met in practical wireless communications systems are reported. Next, using a new statistical approach based on the Householder tridiagonalization

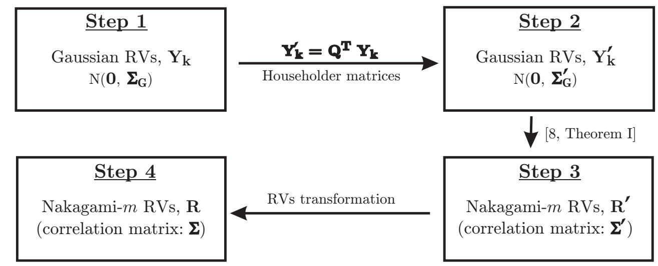

Fig. 1. The four-step procedure for generating Nakagami-m RVs with an arbitrary correlation matrix. ($\mathbf{\Sigma}$ and $\mathbf{\Sigma}'$ are the desired and its similar power correlation matrices, respectively.)

method, union upper bound expressions for the PDF and CDF of the multivariate Nakagami-m distribution with arbitrary correlations are derived.

A. Preliminaries and Correlation Models

Let $\mathbf{Y}_k = [Y_{k,1}\, Y_{k,2} \cdots Y_{k,L}]^{\mathrm{T}}$ ($k = 1, 2, \ldots, 2m$ and T denotes the transpose operand) be $2m$ L-dimensional real column vectors[1], which are i.d. and independent Gaussian RVs with zero mean $\mathbb{E}\langle Y_{k,\ell}\rangle = 0$ and variance $\mathbb{E}\langle Y_{k,\ell}^2\rangle = \sigma^2$ ($\ell = 1, 2, \ldots, L$ and $\mathbb{E}\langle\cdot\rangle$ denotes expectation). Their correlation matrix, $\mathbf{\Sigma_G} \in \Re^{L\times L}$, is symmetric and positive definite[2]. Also, let $R_\ell = \|\mathbf{X}_\ell\| = \sqrt{\sum_{k=1}^{2m} Y_{k,\ell}^2}$ be the Euclidean norm of the $2m$-dimensional column vector $\mathbf{X}_\ell = [Y_{1,\ell}\, Y_{2,\ell} \cdots Y_{2m,\ell}]^{\mathrm{T}}$ composed by the ℓth components of $\mathbf{Y}_k$'s. Clearly, R_ℓ's are correlated Nakagami-m RVs with marginal PDFs given by [4]

$$f_{R_\ell}(r) = \frac{2r^{2m-1}}{\Gamma(m)\,\Omega^m} \exp\left(-\frac{r^2}{\Omega}\right) \tag{1}$$

with $\Gamma(\cdot)$ being the Gamma function [42, eq. (8.310/1)], $\Omega = 2\sigma^2 = \mathbb{E}\langle R_\ell^2\rangle/m$ being a parameter related to the average fading power, and $m \geq 1/2$ being the fading parameter. Their power correlation matrix, $\mathbf{\Sigma} \in \Re^{L\times L}$, is given by $\mathbf{\Sigma}_{i,j} \equiv 1$ for $i = j$ $(i, j = 1, 2, \ldots, L)$ and $\mathbf{\Sigma}_{i,j} = \mathbf{\Sigma}_{j,i} \equiv \rho_{i,j}$ for $i \neq j$, with $0 \leq \rho_{i,j} < 1$ being the power correlation coefficient (i.e., between R_i^2 and R_j^2) [1, eq. (9.195)]. It can be easily proved that the correlation matrix of the underlying Gaussian processes, $\mathbf{\Sigma_G}$, is related to the power correlation matrix, $\mathbf{\Sigma}$, as $\mathbf{\Sigma_G} = \sqrt{\mathbf{\Sigma}}$ ($\sqrt{\mathbf{\Sigma}}$ stands for a matrix with elements the square root ones of $\mathbf{\Sigma}$).

For the readers' convenience, next, we review the structure of $\mathbf{\Sigma}$ for the most popular correlation models met in practical wireless systems channels.

1) Exponential Model: The correlation matrix of this model is defined as $\mathbf{\Sigma}_{i,j} \equiv \rho^{|i-j|}$, $\forall\, i \neq j$, with $0 \leq \rho < 1$ being the correlation coefficient between adjacent channels [2]. This model corresponds to the scenario of multichannel reception by equispaced diversity antennas, in which the correlation between pairs of combined signals decays as the spacing between the antennas increases [1].

[1]Similarly to [13], positive integer or half-integer values for m are here assumed.

[2]A set of Gaussian RVs with zero mean and correlation matrix $\mathbf{M}$ is denoted as $\mathrm{N}(\mathbf{0}, \mathbf{M})$.

$$
\begin{aligned}
f_{\mathbf{R}}(\mathbf{r}) \leq & \frac{2^L \det(\mathbf{A}) \det(\mathbf{W})^m}{\Omega^{L+m-1}\Gamma(m)} r_1^{m-1} \left(\sum_{i=1}^{L} |q_{L,i}|\, r_i\right)^m \exp\left[-\frac{p_{L,L}}{\Omega}\left(\sum_{i=1}^{L}|q_{L,i}|\, r_i\right)^2\right] \prod_{k=1}^{L-1} |p_{k,k+1}|^{-(m-1)} \\
& \times \sum_{j=1}^{L} |q_{k,j}|\, r_j \exp\left[-\frac{p_{k,k}}{\Omega}\left(\sum_{i=1}^{L}|q_{k,i}|\, r_i\right)^2\right] I_{m-1}\left(\frac{2\,|p_{k,k+1}|}{\Omega}\sum_{l_1=1}^{L}\sum_{l_2=1}^{L}|q_{k,l_2}\, q_{k+1,l_1}|\, r_{l_1} r_{l_2}\right)
\end{aligned} \tag{5}
$$

2) Constant Model: The correlation matrix of the constant model, discussed in [1] and [2], is defined as $\mathbf{\Sigma}_{i,j} \equiv \rho, \forall\, i \neq j$, with $0 \leq \rho < 1$ being the correlation coefficient between any two channels. It corresponds to size-limited scenaria with diversity reception by an array of three antennas placed on an equilateral triangle or by closely placed antennas (other than linear arrays).

3) Circular Model: The circulant correlation matrix, presented in [14], is a Toeplitz matrix, i.e., $\mathbf{\Sigma}_{i,j} = \rho_{|i-j|}$, $\forall\, i \neq j$, with an Lth order symmetry, which implies that $\rho_{|i-j|} = \rho_{|L-i+j|}$. This model may apply to antennas lying on a circle or four antennas placed on a square edges. It is noted that the applicability of this model in practical situations is questionable, since the signal's incident angle has also an impact on the branch correlation and it is never symmetrical to all antennas.

4) Linearly Arbitrary Model: The correlation matrix of a linearly arbitrary model has a Toeplitz structure. This correlation model has a great practical value and corresponds to the practical situation of linear arrays, with the antennas to be placed evenly [26].

5) Arbitrary Model: The arbitrary correlation model is the most general one and considers arbitrary values for $\mathbf{\Sigma}_{i,j}$'s $\forall i \neq j$ [1]. This model corresponds to the scenario of multichannel reception with any diversity antennas configuration.

B. Joint PDF

In the proposed mathematical analysis, a class of orthogonal and symmetric matrices, known as Householder matrices [43], are used for the tridiagonal decomposition [44] of the inverse of the Gaussian correlation matrix, $\mathbf{W} = \mathbf{\Sigma_G}^{-1}$.

Definition 1 (Householder matrix): Let $\mathbf{a}, \mathbf{b} \in \Re^{L\times 1}$, $L \geq 3$, be two nonzero vectors having equal norms, i.e., $\|\mathbf{a}\| = \|\mathbf{b}\|$. There always exists an orthogonal and symmetric matrix $\mathbf{H} \in \Re^{L\times L}$ of the form

$$\mathbf{H} = \mathbf{I} - 2\,\mathbf{u}\,\mathbf{u}^{\mathrm{T}} \tag{2}$$

defined as Householder matrix, so that $\mathbf{b} = \mathbf{H}\,\mathbf{a}$, with $\mathbf{I} \in \Re^{L\times L}$ being the identity matrix and

$$\mathbf{u} = \frac{\mathbf{a}-\mathbf{b}}{\|\mathbf{a}-\mathbf{b}\|}. \tag{3}$$

By applying a similarity transformation

$$\mathbf{W}' = \mathbf{Q}^{\mathrm{T}}\,\mathbf{W}\,\mathbf{Q}, \tag{4}$$

$\mathbf{W}'$ becomes real, symmetric, and tridiagonal, where $\mathbf{Q} = [q_{i,j}] \in \Re^{L\times L}$ is an orthogonal matrix, given by $\mathbf{Q} = \prod_{k=1}^{L-2}\mathbf{H}_k$, with $\mathbf{H}_k$ being a Householder matrix which can be obtained using (2) and a computationally efficient method given in the Appendix A. Moreover, some properties concerning $\mathbf{Q}$ are described in the Appendix B. By using (4) and the analysis of [8], a closed-form union upper bound for the joint Nakagami-m PDF can be derived by means of the following theorem.

Theorem 1 (Upper Bound): A closed-form union upper bound for the joint PDF of $\mathbf{R} = [R_1\, R_2 \cdots R_L]$ with an arbitrary power correlation matrix is given by[3] (5) (top of this page) where $\mathbf{r} = [r_1\, r_2 \cdots r_L]$, $I_{m-1}(\cdot)$ is the $(m-1)$th-order modified Bessel function of the first kind [42, eq. (8.406/1)], $\det(\mathbf{W})$ denotes the determinant of $\mathbf{W}$, $p_{i,j} \in \Re$ are the elements of $\mathbf{W}'$, and $\mathbf{A} = [|q_{i,j}|] \in \Re^{L\times L}$, with $|\cdot|$ denoting absolute value.

Proof: The four steps shown in Fig. 1 are followed.

Step 1: Let us consider a set of Gaussian RVs such as $\mathrm{N}(\mathbf{0}, \mathbf{\Sigma_G})$.

Step 2: Let us also consider an orthogonal transformation of RVs

$$\mathbf{Y}'_k = \mathbf{Q}^{\mathrm{T}}\mathbf{Y}_k \tag{6}$$

with $\mathbf{Q}$ so as (4) to hold. Then, $\mathbf{Y}'_k$'s form another set of Gaussian RVs, $\mathrm{N}\left(\mathbf{0}, \mathbf{\Sigma_G}'\right)$, with correlation matrix [45]

$$\mathbf{\Sigma_G}' = \mathbf{Q}^{\mathrm{T}}\,\mathbf{\Sigma_G}\,\mathbf{Q}. \tag{7}$$

Step 3: Let $R'_\ell = \|\mathbf{X}'_\ell\| = \sqrt{\sum_{k=1}^{2m} Y'^{\,2}_{k,\ell}}$ be the Euclidean norm of the $2m$-dimensional column vector $\mathbf{X}'_\ell = \left[Y'_{1,\ell}\, Y'_{2,\ell} \cdots Y'_{2m,\ell}\right]^{\mathrm{T}}$ composed by the ℓth components of $\mathbf{Y}'_k$'s. Clearly, R'_ℓ's are correlated Nakagami-m RVs with symmetric and positive-definite power correlation matrix, denoted by $\mathbf{\Sigma}' \in \Re^{L\times L}$. Since $\mathbf{Y}'_k$'s are independent, their joint PDF can be expressed as a product of marginal PDFs [46], i.e.,

$$
\begin{aligned}
f_{\mathbf{Y}'}(\mathbf{y}) &= \prod_{k=1}^{2m} f_{\mathbf{Y}'_k}(\mathbf{y}_k) \\
&= \frac{\det(\mathbf{W})^m}{(2\pi)^{m\,L}} \prod_{k=1}^{2m} \exp\left(-\frac{1}{2}\mathbf{y}_k{}^{T}\,\mathbf{W}'\,\mathbf{y}_k\right)
\end{aligned} \tag{8}
$$

with $\mathbf{Y}' = [\mathbf{Y}'_1\, \mathbf{Y}'_2 \cdots \mathbf{Y}'_{2m}]$, $\mathbf{y} = [\mathbf{y}_1\, \mathbf{y}_2 \cdots \mathbf{y}_{2m}]$, and $\mathbf{W}' = \left(\mathbf{\Sigma_G}'\right)^{-1}$ given by (4). Equation (8) has a respective form to [8, eq. (2.3)] with $\mathbf{W}'$ being tridiagonal, and hence, following a similar procedure such that in [8, Theorem I], the

[3] Equation (5) holds for any $m \geq 1/2$, except for the proof of the Theorem 1 where $m \geq 1/2$ is considered as a positive integer or half-integer.

$$f_{\mathbf{R}}(\mathbf{r}) \leq \frac{2^L \det(\mathbf{A}) \det(\mathbf{W})^m}{\Omega^{L\,m}\,\Gamma(m)} \exp\left\{-\frac{1}{\Omega}\left[p_{1,1}\,r_1^2 + \sum_{j=2}^{L} p_{j,j}\left(\sum_{i=1}^{L}|q_{j,i}|\;r_i\right)^2\right]\right\} \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} r_1^{2(k_1+m)-1} \times \left[\prod_{j=1}^{L-1}\frac{(p_{j,j+1}/\Omega)^{2k_j}}{k_j!\,\Gamma(k_j+m)}\right]\left(\sum_{i=1}^{L}|q_{L,i}|\;r_i\right)^{2(k_{L-1}+m)-1}\prod_{j=2}^{L-1}\left(\sum_{i=1}^{L}|q_{j,i}|\;r_i\right)^{2(k_{j-1}+k_j+m)-1} \tag{17}$$

joint PDF of $\mathbf{R}' = [R_1'\,R_2'\,\cdots\,R_L']$ can be easily obtained as

$$f_{\mathbf{R}'}(\mathbf{r}) = \det(\mathbf{W})^m\;\frac{2^L\,r_1^{m-1}\,r_L^m}{(\Omega')^{L+m-1}\,\Gamma(m)} \times \exp\left(-\frac{p_{L,L}}{\Omega'}\,r_L^2\right)\prod_{k=1}^{L-1}|p_{k,k+1}|^{-(m-1)}\;r_k \times \exp\left(-\frac{p_{k,k}}{\Omega'}\,r_k^2\right)\,I_{m-1}\left(\frac{2\,|p_{k,k+1}|}{\Omega'}\,r_k\,r_{k+1}\right) \tag{9}$$

with $\Omega' = \mathbb{E}\langle R_\ell'^2\rangle/m = \Omega$.

Step 4: Based on (B-1), (6) can be rewritten as

$$Y_{k,1}' = Y_{k,1} \tag{10a}$$

$$Y_{k,n}' = \sum_{i=1}^{L} q_{n,i}\,Y_{k,i}\;,\;\; n = 2,3,\ldots,L \tag{10b}$$

while starting from the definition of R_ℓ', R_ℓ, and after some algebraic manipulations, we obtain

$$R_1' = R_1 \tag{11a}$$

$$R_n' = \left\|\sum_{i=1}^{L} q_{n,i}\,\mathbf{X}_i\right\|. \tag{11b}$$

By applying the generalization of the triangle inequality in (11b), yields

$$R_n' \leq \sum_{i=1}^{L}|q_{n,i}|\;R_i \tag{12}$$

while following a standard method for RVs transformation, an upper bound for the joint PDF of $\mathbf{R}$ can be easily obtained as

$$f_{\mathbf{R}}(\mathbf{r})\;\leq\;\det(\mathbf{A})\;f_{\mathbf{R}'}\left(r_1, \sum_{i=1}^{L}|q_{2,i}|\;r_i, \ldots, \sum_{i=1}^{L}|q_{L,i}|\;r_i\right) \tag{13}$$

resulting to (5), with $\mathbf{A}$ being the Jacobian matrix of the RVs transformations of (11a) and (12). ■

The equality in (5) occurs when $\mathbf{W}$ is tridiagonal, i.e., for the exponential correlation model. At that special case, $\mathbf{Q} = \mathbf{I}$, and hence, $\mathbf{R}' = \mathbf{R}$, which agrees with [12, eq. (3)]. Also, for $L = 2$, since $\left(\sum_{i=1}^{2}|q_{j,i}|\right)^2 = 1$ ($j = 1, 2$), (5) reduces to the bivariate Nakagami-m PDF [6, eq. (1)].

The proposed closed-form union bound for the joint Nakagami-m PDF with an arbitrary correlation matrix, given by (5), is expressed as a product of elementary functions. As compared to the joint Nakagami-m PDF which can be derived using [9, eq. (3.7)], it is much simpler, since it involves Bessel functions as opposed to the use of the very complicated generalized Laguerre polynomials in [22, eq. (10)]. It must also be mentioned that $\det(\mathbf{W})$ can be recursively computed using (C-1) of the Appendix C. Note that the tridiagonalization of $\mathbf{W}$, needed in (5) for the computation of $q_{i,j}$'s, can be performed using the built-in function `TridiagonalForm` of the MAPLE mathematical software package.

In order to demonstrate the simplicity of (5), based on the similarity transformation in (4), we consider a typical example of an antennas configuration consisted of four arbitrarily spaced and placed antennas. Let us consider a 4×4 power correlation matrix being arbitrary, symmetric, and positive definite

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & 0.618 & 0.384 & 0.203 \\ 0.618 & 1 & 0.563 & 0.348 \\ 0.384 & 0.563 & 1 & 0.640 \\ 0.203 & 0.348 & 0.640 & 1 \end{bmatrix}. \tag{14}$$

The inverse matrix of $\mathbf{\Sigma_G}$ is $\mathbf{W}$=$\left(\sqrt{\mathbf{\Sigma}}\right)^{-1}$, yielding

$$\mathbf{W} = \begin{bmatrix} 4.796 & -3.909 & -0.996 & 0.676 \\ -3.909 & 7.189 & -2.772 & -0.421 \\ -0.996 & -2.772 & 8.414 & -4.729 \\ 0.676 & -0.421 & -4.729 & 5.099 \end{bmatrix}. \tag{15}$$

By applying an efficient algorithmic computation of (4) (see Appendix A), $\mathbf{W}$ is decomposed to tridiagonal form after two Householder transformations, resulting to

$$\mathbf{W}' = \begin{bmatrix} 4.796 & 4.090 & 0 & 0 \\ 4.090 & 6.428 & -1.926 & 0 \\ 0 & -1.926 & 2.403 & 2.596 \\ 0 & 0 & 2.596 & 11.871 \end{bmatrix}. \tag{16}$$

Note that for demonstration purposes the elements of matrices given by (15) and (16) have been rounded to the third decimal digit.

C. Joint CDF

By using an infinite series representation for Bessel functions [42, eq. (8.445)], a union upper bound for the joint Nakagami-m PDF of $\mathbf{R}$ can be reexpressed as (17) (top of this page). After L integrations of (17), a union upper bound for the joint Nakagami-m CDF of $\mathbf{R}$ can be derived as (18) (top of the next page) where $\gamma(\cdot,\cdot)$ is the lower incomplete Gamma function [42, eq. (8.350)] and

$$b_1 = k_1 + m \tag{19a}$$

$$b_j = k_{j-1} + k_j + m\,,\,\forall j = 2,3,\ldots,L-1 \tag{19b}$$

$$b_L = k_{L-1} + m. \tag{19c}$$

It is useful to note that for integer m, $\gamma(\cdot,\cdot)$ can be simplified to standard functions using [42, eq. (8.352/1)]. For the special

$$F_{\mathbf{R}}(\mathbf{r}) \leq \frac{\det(\mathbf{W})^m}{\Gamma(m)} \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \frac{\prod_{i=1}^{L-1} p_{i,i+1}^{2k_i} / [k_i!\,\Gamma(k_i+m)]}{p_{1,1}^{k_1+m} \left(\prod_{i=2}^{L-1} p_{i,i}^{k_{i-1}+k_i+m}\right) p_{L,L}^{k_{L-1}+m}} \gamma\left(b_1, \frac{p_{1,1}}{\Omega} r_1^2\right) \times \left\{\prod_{j=2}^{L-1} \gamma\left[b_j, \frac{p_{j,j}}{\Omega}\left(\sum_{i=1}^{L}|q_{j,i}|\, r_i\right)^2\right]\right\} \gamma\left[b_L, \frac{p_{L,L}}{\Omega}\left(\sum_{i=1}^{L}|q_{L,i}|\, r_i\right)^2\right] \tag{18}$$

$$P_{\text{out}}(\gamma_{\text{th}}) \leq \frac{\det(\mathbf{W})^m}{\Gamma(m)} \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \frac{\prod_{i=1}^{L-1} p_{i,i+1}^{2k_i} / [k_i!\,\Gamma(k_i+m)]}{p_{1,1}^{k_1+m} \left(\prod_{i=2}^{L-1} p_{i,i}^{k_{i-1}+k_i+m}\right) p_{L,L}^{k_{L-1}+m}} \gamma\left(b_1, \frac{p_{1,1}\, m}{\overline{\gamma}_s} \gamma_{\text{th}}\right) \times \left\{\prod_{j=2}^{L-1} \gamma\left[b_j, \frac{p_{j,j}\, m}{\overline{\gamma}_s}\left(\sum_{i=1}^{L}|q_{j,i}|\right)^2 \gamma_{\text{th}}\right]\right\} \gamma\left[b_L, \frac{p_{L,L}\, m}{\overline{\gamma}_s}\left(\sum_{i=1}^{L}|q_{L,i}|\right)^2 \gamma_{\text{th}}\right] \tag{23}$$

case of exponential correlations, the equality in (18) occurs, which agrees with [12, eq. (6)]. Moreover, for $L = 2$, since $\left(\sum_{i=1}^{2}|q_{j,i}|\right)^2 = 1$ $(j = 1, 2)$, (18) reduces to the bivariate Nakagami-m CDF [6, eq. (3)].

III. Performance Analysis of Multibranch Receivers

We consider an L-branch diversity receiver operating over i.d. and arbitrarily correlated Nakagami-m fading channels. Let a signal transmission over the ℓth flat Nakagami-m fading channel corrupted by additive white Gaussian noise (AWGN), with E_s being the transmitted symbols' energy and N_0 the single-sided noise power spectral density of the AWGN. The instantaneous signal-to-noise ratio (SNR) per symbol of the ℓth diversity channel can be expressed by $\gamma_\ell = R_\ell^2\, E_s/N_0$, with its corresponding average value being $\overline{\gamma}_\ell = \mathbb{E}\langle R_\ell^2\rangle\, E_s/N_0 = m\,\Omega\, E_s/N_0 = \overline{\gamma}_s\ \forall\ \ell$. Union bound expressions for the joint PDF and CDF of $\boldsymbol{\gamma} = [\gamma_1\, \gamma_2\, \cdots\, \gamma_L]$ can be easily obtained, through (5) and (18), as

$$f_{\boldsymbol{\gamma}}(\boldsymbol{\gamma}) = \frac{f_{\boldsymbol{R}}\left(\sqrt{\frac{m\,\Omega}{\overline{\gamma}_s}\gamma_1}, \sqrt{\frac{m\,\Omega}{\overline{\gamma}_s}\gamma_2}, \ldots, \sqrt{\frac{m\,\Omega}{\overline{\gamma}_s}\gamma_L}\right)}{2^L\left[\overline{\gamma}_s/(m\,\Omega)\right]^{L/2}\prod_{\ell=1}^{L}\sqrt{\gamma_\ell}} \tag{20}$$

and

$$F_{\boldsymbol{\gamma}}(\boldsymbol{\gamma}) = F_{\boldsymbol{R}}\left(\sqrt{\frac{m\,\Omega}{\overline{\gamma}_s}\gamma_1}, \sqrt{\frac{m\,\Omega}{\overline{\gamma}_s}\gamma_2}, \ldots, \sqrt{\frac{m\,\Omega}{\overline{\gamma}_s}\gamma_L}\right) \tag{21}$$

respectively. The above expressions can be used in the study of several performance criteria of diversity receivers such as the OP and average symbol error probability (ASEP).

A. Multibranch SC Receivers

The instantaneous SNR per symbol at the output of an L-branch SC receiver will be the one with the highest instantaneous value among the L branches, i.e., $\gamma_{\text{sc}} = \max\{\gamma_1, \gamma_2, \ldots, \gamma_L\}$. The OP, P_{out}, is defined as the probability that the SC output SNR falls below a given outage threshold, γ_{th}. This probability can be easily obtained as

$$P_{\text{out}}(\gamma_{\text{th}}) = F_{\boldsymbol{\gamma}}(\gamma_{\text{th}}, \gamma_{\text{th}}, \ldots, \gamma_{\text{th}}) \tag{22}$$

where using (21) yields (23) (top of this page).

B. Multibranch MRC Receivers

An exact performance analysis of multibranch MRC receivers operating over i.d. and arbitrarily correlated Nakagami-m fading channels can be carried out using the generic expression derived for $\mathbf{R}'$, due to the following theorem.

Theorem 2 (Equal norms): Both groups of Nakagami-m RVs, $\mathbf{R}'$ and $\mathbf{R}$, have the same norm, i.e.,

$$\|\mathbf{R}'\| = \|\mathbf{R}\|. \tag{24}$$

Proof: Since $\mathbf{Y}_k'$ is an orthogonal transformation of $\mathbf{Y}_k$, as shown in (6), they both have the same norm [44], i.e.,

$$\|\mathbf{Y}_k'\| = \|\mathbf{Y}_k\|. \tag{25}$$

By square raising both sides of (25), adding by parts the associated $2m$ equations, and using the definition of R_ℓ' and R_ℓ, (24) is extracted. ■

Consequently, it directly follows from (24) that $\sum_{\ell=1}^{L} R_\ell'^2 = \sum_{\ell=1}^{L} R_\ell^2$, which implies that the instantaneous MRC output SNR per symbol can be expressed as

$$\gamma_{\text{mrc}} = \frac{E_s}{N_0}\sum_{\ell=1}^{L} R_\ell'^2. \tag{26}$$

The advantage of (26) is the following: In a standard treatment, in order to evaluate the performance of MRC, the joint PDF of $\mathbf{R}$ is needed, which is given in the form of a multiple series of generalized Laguerre polynomials [9, eq. (3.7)], and hence, is very complicated to be used for the performance analysis of MRC. Thus, based on (26), we avoid employing the distribution of $\mathbf{R}$, managing to use the distribution of $\mathbf{R}'$ given by (9), which is significantly simpler. Also, for i.d. fading parameters, the proposed approach seems to be less complicated than the PDF-based one presented in [14].

1) Error Probability: By using (9), Theorem 2, and the definition of the MGF of the output SNR per symbol of an L-branch MRC, i.e., $\mathcal{M}_{\gamma_{\text{mrc}}}(s) = \mathbb{E}\langle\exp(-s\,\gamma_{\text{mrc}})\rangle$, $\mathcal{M}_{\gamma_{\text{mrc}}}(s)$ can be obtained as (27) (top of the next page).

Based on the above MGF expression, the ASEP, $\overline{P}_{se}$, at the output of an L-branch MRC receiver, for non-coherent binary frequency shift keying (NBFSK) and differential binary phase shift keying (DBPSK) modulation schemes can be directly calculated (e.g. the average bit error rate probability (ABEP) for

$$\mathcal{M}_{\gamma_{\text{mrc}}}(s) = \frac{\det(\mathbf{W})^m}{\Gamma(m)} \left(\frac{m}{\overline{\gamma}_s}\right)^{Lm} \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \left[\prod_{i=1}^{L-1} \frac{(m\, p_{i,i+1}/\overline{\gamma}_s)^{2k_i}}{k_i!\,\Gamma(k_i+m)}\right] \prod_{\ell=1}^{L} \left(s + \frac{m p_{\ell,\ell}}{\overline{\gamma}_s}\right)^{-b_\ell} \Gamma(b_\ell) \tag{27}$$

$$\begin{aligned} P_{\text{out}}(\gamma_{\text{th}}) =& \frac{\det(\mathbf{W})^m}{(m-1)!} \left(\frac{m}{\overline{\gamma}_s}\right)^{Lm} \sum_{k_1,k_2,\ldots,k_{L-1}=0}^{\infty} \left[\prod_{i=1}^{L-1} \frac{(m\, p_{i,i+1}/\overline{\gamma}_s)^{2k_i}}{k_i!\,(k_i+m-1)!}\right] \left[\prod_{\ell=1}^{L} (b_\ell - 1)!\right] \\ & \times \sum_{\ell=1}^{L} \sum_{q=1}^{b_\ell} \frac{B_{\ell,q}}{(m\, p_{\ell,\ell}/\overline{\gamma}_s)^q} \left[1 - \exp\left(-m\, p_{\ell,\ell} \frac{\gamma_{\text{th}}}{\overline{\gamma}_s}\right) \sum_{n=0}^{q-1} \frac{1}{n!} \left(m\, p_{n,n} \frac{\gamma_{\text{th}}}{\overline{\gamma}_s}\right)^n\right] \end{aligned} \tag{32}$$

TABLE I
NUMBER OF REQUIRED TERMS FOR CONVERGENCE TO THE SIXTH SIGNIFICANT DIGIT OF THE UNION UPPER BOUND OF THE OP OF SC RECEIVERS WITH A LINEARLY ARBITRARY MODEL ($L = 3$).

$\gamma_{\text{th}}/\overline{\gamma}_s$ (dB)	$m = 1$	$m = 2$	$m = 4$
5	14	21	37
0	11	19	30
-5	7	14	21
-10	5	7	12
-15	2	2	5
-20	1	1	1

TABLE II
NUMBER OF REQUIRED TERMS FOR CONVERGENCE TO THE SIXTH SIGNIFICANT DIGIT OF THE ABEP OF MRC RECEIVERS WITH DBPSK SIGNALLING AND A LINEARLY ARBITRARY MODEL ($L = 5$).

$\overline{\gamma}_b$ (dB)	$m = 1$	$m = 2$	$m = 4$
-5	18	24	35
0	13	20	29
5	9	11	17
10	6	7	9
15	3	4	6

DBPSK is given by $\overline{P}_{be} = 0.5\, \mathcal{M}_{\gamma_{\text{mrc}}}(1)$). For other schemes, including binary phase shift keying (BPSK), M-ary phase shift keying (M-PSK), quadrature amplitude modulation (M-QAM), amplitude modulation (M-AM), and differential phase shift keying (M-DPSK), single integrals with finite limits and integrands composed of elementary functions (exponential and trigonometric) have to be readily evaluated via numerical integration [1]. For example, the ASEP for M-PSK is given by

$$\overline{P}_{se} = \frac{1}{\pi} \int_0^{\pi-\pi/M} \mathcal{M}_{\gamma_{\text{mrc}}} \left(\frac{g_{\text{PSK}}}{\sin^2\varphi}\right) \mathrm{d}\varphi \tag{28}$$

where $g_{\text{PSK}} = \sin^2(\pi/M)$, while for square M-QAM, the ASEP can be derived as

$$\begin{aligned} \overline{P}_{se} =& \frac{4}{\pi} \left(1 - \frac{1}{\sqrt{M}}\right) \left[\int_0^{\pi/2} \mathcal{M}_{\gamma_{\text{mrc}}} \left(\frac{g_{\text{QAM}}}{\sin^2\varphi}\right) \mathrm{d}\varphi \right. \\ & \left. - \left(1 - \frac{1}{\sqrt{M}}\right) \int_0^{\pi/4} \mathcal{M}_{\gamma_{\text{mrc}}} \left(\frac{g_{\text{QAM}}}{\sin^2\varphi}\right) \mathrm{d}\varphi \right] \end{aligned} \tag{29}$$

with $g_{\text{QAM}} = 3/[2(M-1)]$.

2) Outage Probability: The OP at the output of MRC can be obtained as [1]

$$P_{\text{out}}(\gamma_{\text{th}}) = F_{\gamma_{\text{mrc}}}(\gamma_{\text{th}}) = \mathbb{L}^{-1}\left[\frac{\mathcal{M}_{\gamma_{\text{mrc}}}(s)}{s}; \gamma_{\text{mrc}}\right]\Big|_{\gamma_{\text{mrc}}=\gamma_{\text{th}}} \tag{30}$$

where $F_{\gamma_{\text{mrc}}}(\cdot)$ is the CDF of γ_{mrc} and $\mathbb{L}^{-1}[\cdot;\cdot]$ denotes the inverse Laplace transform. Due to the complicated form of $\mathcal{M}_{\gamma_{\text{mrc}}}(s)/s$ in (30), the so-called Euler summation-based algorithm for the inversion of the CDFs may be applied [42, Appendix 9B.1], [47]. For m integer, the integration theory of rational functions [42, Section 2.102] can be applied. Moreover, inverse Laplace transformations of the form $\mathbb{L}^{-1}[(s + m\, p_{\ell,\ell}/\overline{\gamma}_s)^{-q}/s; t]$ (q integer) need to be performed, as [42, Section 17.1]

$$\mathbb{L}^{-1}\left[\frac{(s + m\, p_{\ell,\ell}/\overline{\gamma}_s)^{-q}}{s}; t\right] = \frac{\gamma(q, t\, m\, p_{\ell,\ell}/\overline{\gamma}_s)}{(q-1)!\,(m\, p_{\ell,\ell}/\overline{\gamma}_s)^q}. \tag{31}$$

Hence, for distinct values of $p_{\ell,\ell}$'s, occurring in most practical cases in which $\rho_{i,j} \neq 0$, an analytical expression for the OP at the output of MRC can be derived as in (32) (top of this page) with

$$B_{\ell,q} = \frac{1}{(b_\ell - q)!} \Psi_\ell(s)^{(b_\ell - q)} \Big|_{s=-m\, p_{\ell,\ell}/\overline{\gamma}_s} \tag{33a}$$

and

$$\Psi_\ell(s) = \left(s + \frac{m\, p_{\ell,\ell}}{\overline{\gamma}_s}\right)^{b_\ell} \prod_{i=1}^{L} \left(s + \frac{m\, p_{i,i}}{\overline{\gamma}_s}\right)^{-b_i}. \tag{33b}$$

IV. NUMERICAL AND COMPUTER SIMULATION RESULTS

In this section, by using the previous mathematical analysis, numerically evaluated results are presented for the OP of SC as well as the OP and ABEP of MRC receivers operating over i.d. and arbitrarily correlated Nakagami-m fading channels. In order to verify the tightness of the proposed bound for the OP of SC and the validity of the exact outage and error performance of MRC receivers, curves obtained by extensive computer simulations are also presented for comparison purposes.

The numerical evaluation of several expressions in Section III requires the summation of an infinite number of terms. As indicative examples, Tables I and II summarize the number of terms needed for SC and MRC receivers, so as the expressions for the OP using (23) and the ABEP using (27) to converge after the truncation of the infinite series, respectively. Note that in Table II as well as in the examples for the error performance that follow, when the modulation order $M > 2$, Gray encoding is assumed, resulting to $\overline{P}_{be} = \overline{P}_{se}/\log_2(M)$.

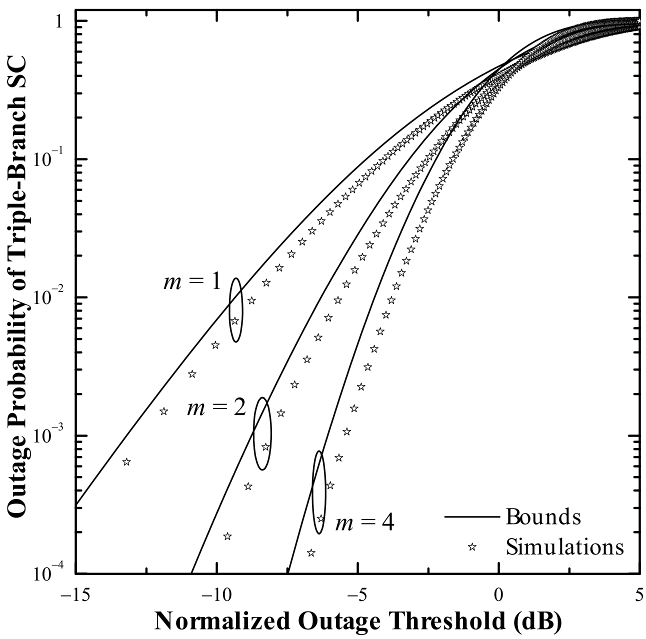

Fig. 2. Upper bound for the OP of a triple-branch SC with a linearly arbitrary model as a function of the normalized outage threshold. The derived bounds are tight compared with simulation results for different values of m.

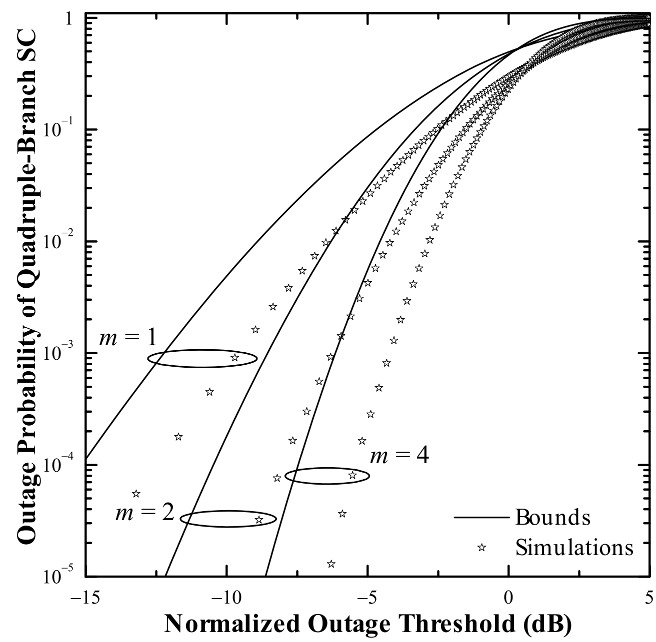

Fig. 3. Upper bound for the OP of a quadruple-branch SC with a circular model as a function of the normalized outage threshold. The tightness of the proposed bounds decreases with an increase on L and/or m.

A linearly arbitrary correlation model with $L = 3$ [13, p. 886] has been considered in Table I, while in Table II, a linearly arbitrary model with $L = 5$, in which the correlation matrix is given by [26, eq. (40)], has been assumed. As Table I indicates, the number of required terms depends strongly on the normalized outage threshold, $\gamma_{\rm th}/\overline{\gamma}_s$. As $\gamma_{\rm th}/\overline{\gamma}_s$ decreases, less terms are required to be summed. Moreover, for a fixed $\gamma_{\rm th}/\overline{\gamma}_s$, an increase on m results to an increase on the required number of terms that are essential to be summed in order the upper bound for the OP to converge. Similar conclusions for the convergence of the ABEP for DBPSK can be also extracted from Table II. An increase on the average SNR per bit, $\overline{\gamma}_b$, results to a decrease of the required number of terms, and for a fixed $\overline{\gamma}_b$, the required number of terms for convergence increases with increasing m. It is interesting to be mentioned that additional convergence experiments have been conducted for the OP and the ABEP, and the following findings have been obtained. *i*) The convergence rate depends slightly on the diversity order and *ii*) an increase on any of the correlation coefficients results to an increase of the required number of terms needed for convergence.

Having numerically evaluated (23), in Fig. 2, upper bounds for $P_{\rm out}$ are plotted as a function of $\gamma_{\rm th}/\overline{\gamma}_s$, for a triple-branch SC receiver, different values of m, and a linearly arbitrary correlation matrix given in [13, p. 886]. It can be easily verified that $P_{\rm out}$ degrades with a decrease of m and/or an increase of $\gamma_{\rm th}/\overline{\gamma}_s$. More importantly, the obtained results clearly show that the proposed bounds for $P_{\rm out}$ are tight, compared with extensive computer simulations for the exact OP. For example, for $m = 2$ and $P_{\rm out} = 10^{-4}$, the distance between the two curves is less than 0.1 dB. In Fig. 3, upper bounds for $P_{\rm out}$ are presented as a function of the $\gamma_{\rm th}/\overline{\gamma}_s$ for a quadruple-branch SC receiver with a circulant correlation matrix given by [13, p. 888]. It is clearly shown that the

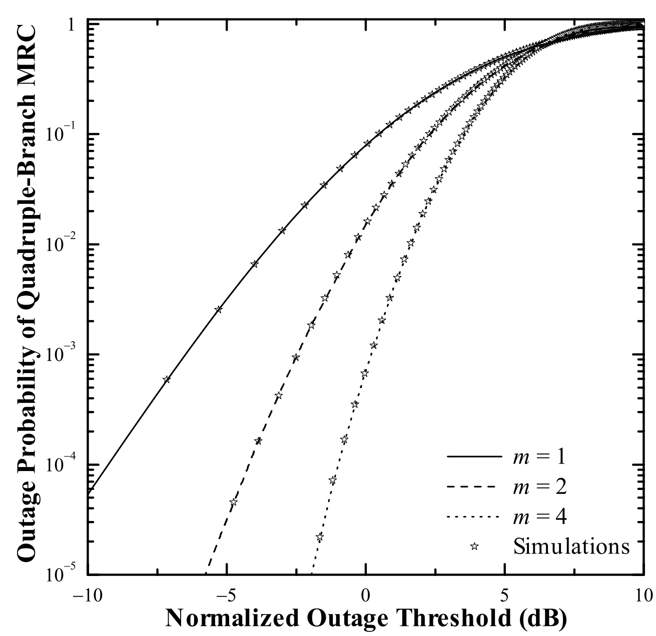

Fig. 4. OP of a quadruple-branch MRC with an arbitrary model as a function of the normalized outage threshold. Numerically evaluated results for the OP of MRC perfectly match with equivalent simulation ones for $L = 4$.

difference between the numerical results for the bounds and the equivalent computer simulation ones for the exact $P_{\rm out}$ increases with an increase on L and/or m. From Figs. 2 and 3, it is evident that the smaller the L and/or m are, the tighter the bounds are.

Based on (32), Fig. 4 demonstrates the numerically evaluated results for $P_{\rm out}$ as a function of the $\gamma_{\rm th}/\overline{\gamma}_s$, for a quadruple-branch MRC receiver, with different values of m and an arbitrary correlation matrix given by (14). As expected, $P_{\rm out}$ degrades with a decrease of m and/or an increase of

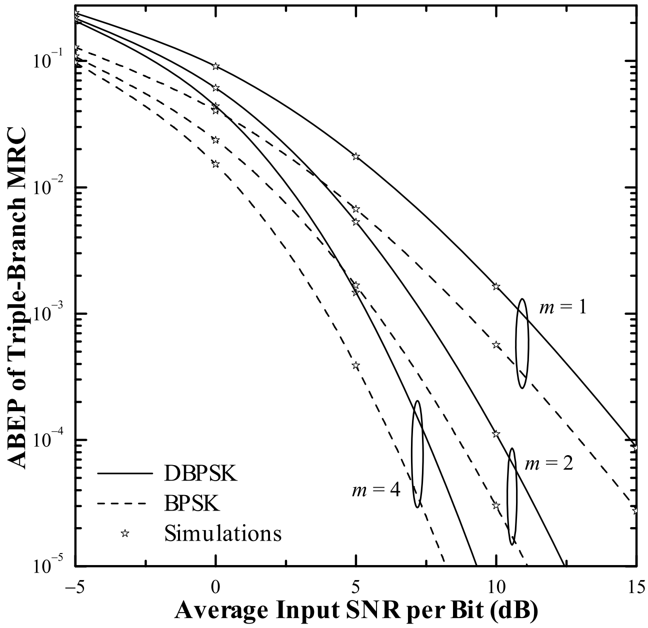

Fig. 5. ABEP of DBPSK and BPSK for a triple-branch MRC with a linearly arbitrary model as a function of the average input SNR per bit. Numerically evaluated results for the ABEP of DBPSK and BPSK perfectly match with equivalent simulation ones for $L = 3$.

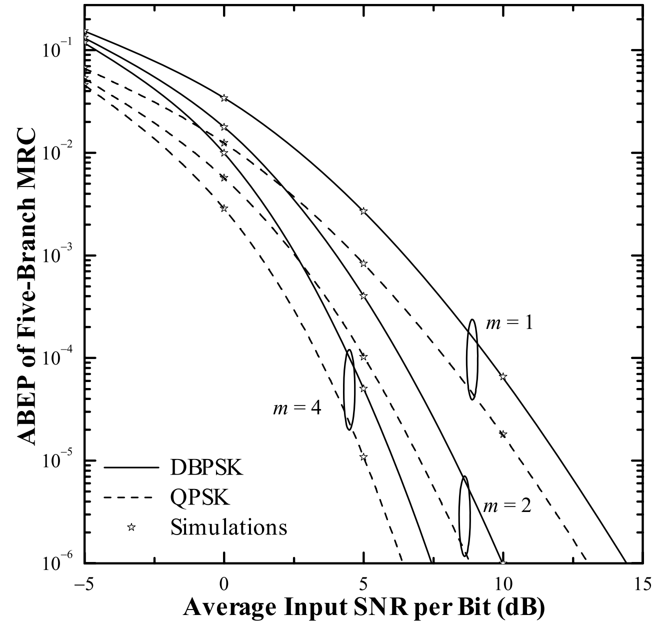

Fig. 7. ABEP of DBPSK and Gray encoded QPSK for a five-branch MRC with a linearly arbitrary model as a function of the average input SNR per bit. Numerically evaluated results for the ABEP of DBPSK and Gray encoded QPSK perfectly match with equivalent simulation ones for $L = 5$.

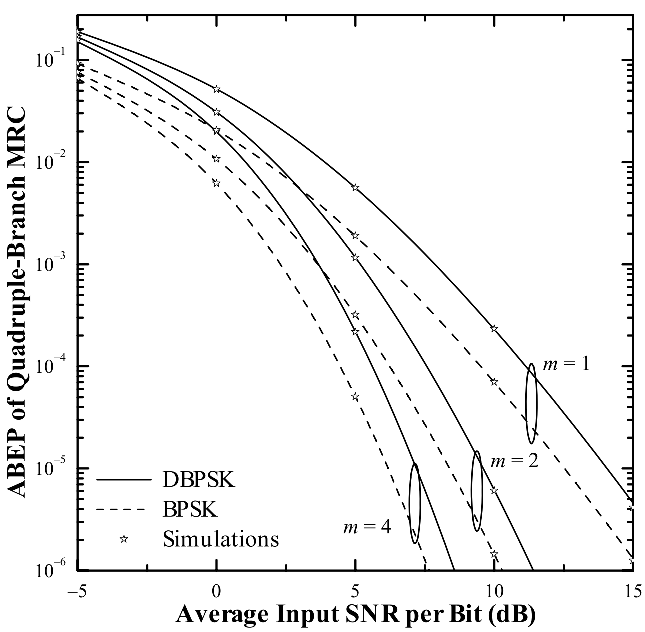

Fig. 6. ABEP of DBPSK and BPSK for a quadruple-branch MRC with a circular model as a function of the average input SNR per bit. Numerically evaluated results for the ABEP of DBPSK and BPSK perfectly match with equivalent simulation ones for $L = 4$.

$\gamma_{\rm th}/\overline{\gamma}_s$. It is obvious that the numerically evaluated curves for $P_{\rm out}$ perfectly match with the equivalent computer simulation results.

In Figs. 5–8, by using (27), a few curves for the ABEP performance are plotted as a function of $\overline{\gamma}_b = \overline{\gamma}_s/\log_2(M)$, for multibranch MRC receivers, several modulation schemes, different values of m, and several correlation matrices. In Fig. 5, the ABEP of DBPSK and BPSK signalling is plotted as a function of $\overline{\gamma}_b$, for a triple-branch MRC receiver with a linearly arbitrary correlation matrix given in [13, p. 886]. As expected, the ABEP improves with an increase of $\overline{\gamma}_b$. Furthermore, in Fig. 6, ABEP is also presented for a quadruple-branch MRC receiver with a circulant correlation matrix given in [13, p. 888], while in Fig. 7, the ABEP performance of DBPSK and Gray-encoded quadrature phase shift keying (QPSK) signalling is plotted as a function of $\overline{\gamma}_b$ for a five-branch MRC receiver with a linearly arbitrary correlation matrix given by [26, eq. (40)]. Finally, in Fig. 8, the ABEP is plotted versus $\overline{\gamma}_b$, for Gray-encoded square M-QAM modulation format with $M = 4, 16$, and 64, for a six-branch MRC receiver with an arbitrary correlation matrix given by

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & 0.632 & 0.366 & 0.203 & 0.141 & 0.080 \\ 0.632 & 1 & 0.562 & 0.403 & 0.160 & 0.123 \\ 0.366 & 0.562 & 1 & 0.540 & 0.476 & 0.250 \\ 0.203 & 0.403 & 0.540 & 1 & 0.672 & 0.342 \\ 0.141 & 0.160 & 0.476 & 0.672 & 1 & 0.533 \\ 0.080 & 0.123 & 0.250 & 0.342 & 0.533 & 1 \end{bmatrix}. \tag{34}$$

In Figs. 5–8, it is clearly shown that numerically evaluated curves for the ABEP coincide to the equivalent computer simulation results.

V. Conclusions

In this paper, new results for the multivariate Nakagami-m fading channel model with arbitrary correlation structures were presented. By using an efficient tridiagonalization method based on Householder matrices, the inverse of the Gaussian correlation matrix was transformed to tridiagonal, managing to derive a closed-form union upper bound for the joint Nakagami-m PDF and an exact analytical expression for the MGF of the sum of i.d. and correlated gamma RVs. Arbitrary correlation structures were considered, including the

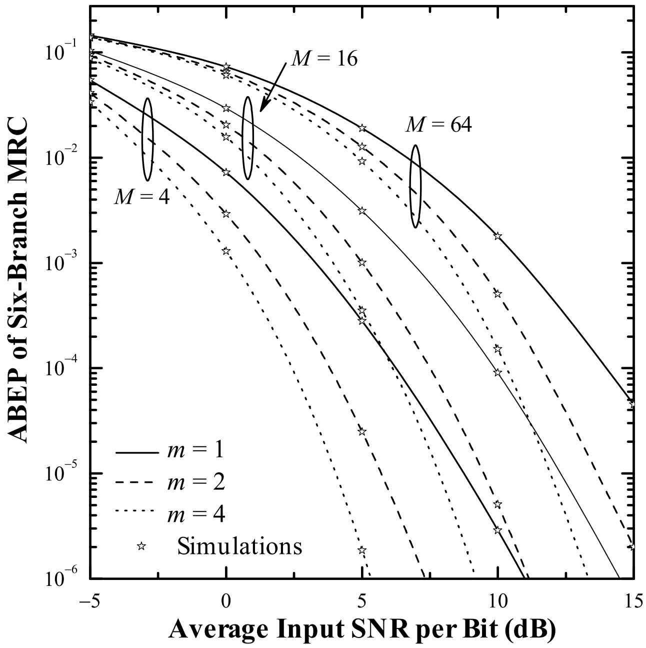

Fig. 8. ABEP of Gray encoded star M-QAM for a six-branch MRC with an arbitrary model as a function of the average input SNR per bit. Numerically evaluated results for the ABEP of Gray encoded star M-QAM perfectly match with equivalent simulation ones for $L = 6$.

exponential, constant, circular, and linear correlation ones as special cases. Based on the proposed mathematical analysis, a tight union upper bound for the OP of multibranch SC as well as exact analytical expressions for the OP and the ASEP of multibranch MRC receivers were obtained. Comparisons between numerically evaluated and extensive computer simulation performance evaluation results verified the validity of our approach.

APPENDIX A
EFFICIENT ALGORITHMIC COMPUTATION OF (4)

Let $\mathbf{W} \in \Re^{L\times L}$ be a real symmetric matrix. Starting with $\mathbf{W}_0 = \mathbf{W}$, the kth $(k = 1, 2, \ldots, L-2)$ Householder transformation can be obtained using the following recursive formula

$$\mathbf{W}_k = \mathbf{H}_k\,\mathbf{W}_{k-1}\,\mathbf{H}_k \tag{A-1}$$

which, alternatively, can be efficiently computed performing only vector multiplications as

$$\mathbf{W}_k = \mathbf{W}_{k-1} - 2\mathbf{u}_k\,\mathbf{z}_k^{\mathrm{T}} - 2\mathbf{z}_k\,\mathbf{u}_k^{\mathrm{T}} \tag{A-2}$$

where $\mathbf{z}_k = \mathbf{v}_k - c_k\,\mathbf{u}_k$ with $\mathbf{v}_k = \mathbf{W}_{k-1}\,\mathbf{u}_k$ and $c_k = \mathbf{u}_k^{\mathrm{T}}\,\mathbf{v}_k$. In order to derive the vector $\mathbf{u}_k$, a useful property of the Householder matrix is used. A similar vector, $\mathbf{b}_k$, to $\mathbf{a}_k = \mathbf{W}_{k-1}(:,k)$ ($\mathbf{W}_{k-1}(:,k)$ denotes the kth column of $\mathbf{W}_{k-1}$) may be constructed as

$$\begin{aligned}\mathbf{b}_k = \mathbf{H}_k\,\mathbf{a}_k &= \mathbf{H}_k\left[a_1\;\; a_2\;\;\cdots\;\; a_k\;\; a_{k+1}\;\; a_{k+2}\;\;\cdots\;\; a_L\right]^{\mathrm{T}}\\ &= \left[a_1\;\; a_2\;\;\cdots\;\; a_k\;\; -s_k\;\; \underbrace{0\;\;\cdots\;\;0}_{L-(k+1)}\right]^{\mathrm{T}}\end{aligned} \tag{A-3}$$

where $s_k = \operatorname{sign}(a_{k+1})\sqrt{\sum_{i=k+1}^{L} a_i^2}$, so as vectors $\mathbf{b}_k$ and $\mathbf{a}_k$ to have identical norms. Note that the sign of s_k is chosen to be equal to that of a_{k+1} for less round-off error propagation [44]. By using (A-3), $\mathbf{u}_k$ can be efficiently obtained based on (3), avoiding the computation of $\mathbf{H}_k$, as

$$\mathbf{u}_k = \frac{1}{d_k}\left[\underbrace{0\;\;0\;\;\cdots\;\;0}_{k}\;\; s_k + a_{k+1}\;\; a_{k+2}\;\;\cdots\;\; a_L\right]^{\mathrm{T}} \tag{A-4}$$

where $d_k = \|\mathbf{a}_k - \mathbf{b}_k\| = \sqrt{2a_{k+1}s_k + 2s_k^2}$. After using (A-2) $L-2$ times, $\mathbf{W}' = \mathbf{W}_{L-2}$ is finally formed.

APPENDIX B
SOME PROPERTIES FOR THE ORTHOGONAL MATRIX

The orthogonal matrix used for the transformation in (4) (and in (6)) is of the form

$$\mathbf{Q}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & q_{2,2} & \cdots & q_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & q_{L,2} & \cdots & q_{L,L}\end{bmatrix} \tag{B-1}$$

having the following properties:

i) for any row, the sum of the squared elements is unity, i.e. $\sum_{j=1}^{L} q_{i,j}^2 = 1$,

ii) for any two rows i and k with $i \neq k$, the sum of products of corresponding elements is zero, i.e. $\sum_{j=1}^{L} q_{i,j}\, q_{k,j} = 0$, and

iii) $\mathbf{Q}^{-1} = \mathbf{Q}^{\mathrm{T}}$, $\det\left(\mathbf{Q}^{\mathrm{T}}\right) = \pm 1$.

APPENDIX C
DETERMINANT RECURSIVE CALCULATION OF TRIDIAGONAL MATRICES

Taking advantage of the tridiagonal form of $\mathbf{W}'$, the determinant of $\mathbf{W}$ can be efficiently computed using the recursive formula

$$\begin{aligned}\det(\mathbf{W}) =& p_{L,L}\,\det\left(\left[\mathbf{W}'\right]_{\{1,2,\ldots,L-1\}}\right) - \\ & - p_{L,L-1}^2\,\det\left(\left[\mathbf{W}'\right]_{\{1,2,\ldots,L-2\}}\right)\end{aligned} \tag{C-1}$$

where $[\mathbf{W}']_{\{1,2,\ldots,n\}}$ is the submatrix formed by the first $n \leq L-1$ rows and columns of $\mathbf{W}'$.

REFERENCES

[1] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[2] V. A. Aalo, “Performance of maximal-ratio diversity systems in a correlated Nakagami-fading environment,” *IEEE Trans. Commun.*, vol. 43, no. 8, pp. 2360–2369, Aug. 1995.

[3] V. V. Veeravalli, “On performance analysis for signaling on correlated fading channels,” *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1879–1883, Nov. 2001.

[4] M. Nakagami, “The m-distribution–a general formula of intensity distribution of rapid fading,” in *Statistical Methods in Radio Wave Propagation*. Oxford, U.K.: Pergamon Press, 1960, pp. 3–36.

[5] U. Charash, “Reception through Nakagami fading multipath channels with random delays,” *IEEE Trans. Commun.*, vol. 27, no. 4, pp. 657–670, Apr. 1979.

[6] C. C. Tan and N. C. Beaulieu, “Infinite series representations of the bivariate Rayleigh and Nakagami-m distributions,” *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1159–1161, Oct. 1997.

[7] M. K. Simon and M.-S. Alouini, “A simple integral representation of the bivariate Rayleigh distribution,” *IEEE Commun. Lett.*, vol. 2, no. 5, pp. 128–130, May 1998.

[8] L. E. Blumenson and K. S. Miller, "Properties of generalized Rayleigh distributions," *Ann. Math. Statist.*, vol. 34, no. 3, pp. 903–910, Sep. 1963.

[9] A. S. Krishnamoorthy and M. Parthasarathy, "A multivariate gamma-type distribution," *Ann. Math. Statist.*, vol. 22, pp. 549–557, 1951.

[10] O. C. Ugweje and V. A. Aalo, "Performance of selection diversity system in correlated Nakagami fading," in *Proc. IEEE Veh. Technol. Conf. (VTC '97)*, vol. 3, New York, USA, May 1997, pp. 1488–1492.

[11] R. K. Mallik, "On multivariate Rayleigh and exponentials distributions," *IEEE Trans. Inform. Theory*, vol. 49, no. 6, pp. 1499–1515, June 2003.

[12] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoulos, "On the multivariate Nakagami-m distribution with exponential correlation," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1240–1244, Aug. 2003.

[13] ——, "An efficient approach to multivariate Nakagami-m distribution using Green's matrix approximation," *IEEE Trans. Wireless Commun.*, vol. 2, no. 5, pp. 883–889, Sept. 2003.

[14] M.-S. Alouini, A. Abdi, and M. Kaveh, "Sum of gamma variates and performance of wireless communication systems over Nakagami-fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1471–1480, Nov. 2001.

[15] G. K. Karagiannidis, "Performance analysis of SIR-based dual selection diversity over correlated Nakagami-m fading channels," *IEEE Trans. Veh. Technol.*, vol. 52, no. 5, pp. 1207–1216, Sept. 2003.

[16] ——, "Moments-based approach to the performance analysis of equal gain diversity in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 52, no. 5, pp. 685–690, May 2004.

[17] N. C. Sagias and G. K. Karagiannidis, "Gaussian class multivariate Weibull distributions: theory and applications in fading channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 10, pp. 3608–3619, Oct. 2005.

[18] G. K. Karagiannidis, N. C. Sagias, and T. A. Tsiftsis, "Closed-form statistics for the sum of squared Nakagami-m variates and its applications," *IEEE Trans. Commun.*, vol. 54, no. 8, pp. 1353–1359, Aug. 2006.

[19] G. C. Alexandropoulos, N. C. Sagias, and K. Berberidis, "On the multivariate Weibull fading model with arbitrary correlation matrix," *IEEE Antennas Wireless Propag. Lett.*, vol. 6, pp. 93–95, 2007.

[20] X. Zhang and N. C. Beaulieu, "SER of threshold-based hybrid selection/maximal-ratio combining in correlated Nakagami fading," *IEEE Trans. Commun.*, vol. 53, no. 9, pp. 1423–1426, Sep. 2005.

[21] ——, "Performance analysis of generalized selection combining in generalized correlated Nakagami-m fading," *IEEE Trans. Commun.*, vol. 54, no. 11, pp. 2103–2112, Nov. 2006.

[22] R. A. A. de Souza, C. Fraidenraich, and M. D. Yacoub, "On the multivariate $\alpha - \mu$ distribution with arbitrary correlation," in *Proc. VI Inter. Telecommun. Symp. (ITS'06)*, Fortaleza-CE, Brazil, Sep. 2006.

[23] M. D. Yacoub, "The $\alpha - \mu$ distribution: A physical fading model for the Stacy distribution," *IEEE Trans. Veh. Technol.*, vol. 56, no. 1, pp. 27–34, Jan. 2007.

[24] T. Piboongungon, V. A. Aalo, C.-D. Iskander, and G. P. Efthymoglou, "Bivariate generalised Gamma distribution with arbitrary fading parameters," *Electron. Lett.*, vol. 41, no. 12, pp. 709–710, Nov. 2005.

[25] V. A. Aalo and T. Piboongungon, "On the multivariate generalized Gamma distribution with exponential correlation," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM'05)*, vol. 3, St. Louis, Missouri, USA, Nov. 2005.

[26] Q. T. Zhang, "Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1141–1150, July 1999.

[27] P. Sahu and A. K. Chaturvedi, "Performance analysis of a predetection EGC receiver in exponentially correlated Nakagami-m fading channels for noncoherent binary modulations," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1634–1638, July 2006.

[28] I. Ghareeb, "Noncoherent MT-MFSK signals with diversity reception in arbitrary correlated and unbalanced Nakagami-m fading channels," *IEEE J. Select. Areas Commun.*, vol. 23, no. 9, pp. 1839–1850, Sept. 2005.

[29] M. K. Simon and M.-S. Alouini, "A unified performance analysis of digital communication with dual selective combining diversity over correlated Rayleigh and Nakagami-m fading channels," *IEEE Trans. Commun.*, vol. 47, no. 1, pp. 33–43, Jan. 1999.

[30] M. Z. Win, G. Chrisikos, and J. H. Winters, "MRC performance for M-ary modulation in arbitrarily correlated Nakagami fading channels," *IEEE Commun. Lett.*, vol. 4, no. 10, pp. 301–303, Oct. 2000.

[31] Q. T. Zhang, "A decomposition technique of efficient generation of correlated Nakagami fading channels," *IEEE J. Select. Areas Commun.*, vol. 18, no. 11, pp. 2385–2392, Nov. 2000.

[32] ——, "A generic correlated Nakagami fading model for wireless communications," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1745–1748, Nov. 2003.

[33] J. Luo, J. R. Zeidler, and S. McLaughlin, "Performance analysis of compact antenna arrays with MRC in correlated Nakagami fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, no. 1, pp. 267–277, Jan. 2001.

[34] J. Luo, J. R. Zeidler, and J. G. Proakis, "Error probability performance for W-CDMA systems with multiple transmit and receive antennas in correlated Nakagami fading channels," *IEEE Trans. Veh. Technol.*, vol. 51, no. 6, pp. 1502–1516, Nov. 2002.

[35] R. K. Mallik and M. Z. Win, "Analysis of hybrid selection/maximal-ratio combining in correlated Nakagami fading," *IEEE Trans. Commun.*, vol. 50, no. 8, pp. 1372–1383, Aug. 2002.

[36] Y. Chen and C. Tellambura, "Distribution functions of selection combiner output in equally correlated Rayleigh, Rician, and Nakagami-m fading channels," *IEEE Trans. Commun.*, vol. 52, no. 11, pp. 1948–1956, Nov. 2004.

[37] ——, "Moment analysis of the equal gain combiner output in equally correlated fading channels," *IEEE Trans. Veh. Technol.*, vol. 54, no. 6, pp. 1971–1979, Nov. 2005.

[38] ——, "Infinite series representation of the trivariate and quadrivariate Rayleigh distribution and their applications," *IEEE Trans. Commun.*, vol. 53, no. 12, pp. 2092–2101, Dec. 2005.

[39] ——, "Performance analysis of three-branch selection combining over arbitrarily correlated Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 861–865, May 2005.

[40] ——, "Performance analysis of L-branch equal gain combiners in equally correlated Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 8, no. 3, pp. 150–152, Mar. 2005.

[41] K. D. P. Dharmawansa, R. M. A. P. Rajatheva, and C. Tellambura, "Infinite series representations of the trivariate and quadrivariate Nakagami-m distributions," in *Proc. IEEE Inter. Conf. Commun. (ICC'07)*, Glasgow, Scotland, June 2007, pp. 1114–1118.

[42] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.

[43] A. S. Householder, "Unitary triangularization of a nonsymmetric matrix," *Journal of the ACM*, vol. 5, no. 4, pp. 339–342, Oct. 1958.

[44] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD: The Johns Hopkins University Press, 1996.

[45] C. W. Helstrom, *Probability and Stochastic Processes for Engineers*, 2nd ed. New York: Macmillian, 1991.

[46] J. G. Proakis, *Digital Communications*, 3rd ed. NY, USA: McGraw-Hill, 1995.

[47] W. Grassman, *Computational Probability*. Boston: Kluwer, 1999.

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