Serial Free-Space Optical Relaying Communications Over Gamma-Gamma Atmospheric Turbulence Channels

Christos K. Datsikas, Kostas P. Peppas, Nikos C. Sagias, and George S. Tombras

***Abstract*—In this paper, a study on the end-to-end performance of multihop free-space optical wireless systems over turbulence-induced fading channels, modeled by the gamma-gamma distribution, is presented. Our analysis is carried out for systems employing amplify-and-forward channel-state-information-assisted or fixed-gain relays. To assess the statistical properties of the end-to-end signal-to-noise ratio for both considered systems, we derive novel closed-form expressions for the moment-generating function, the probability density function, and the cumulative distribution function of the product of rational powers of statistically independent squared gamma-gamma random variables. These statistical results are then applied to studying the outage probability and the average bit error probability of binary modulation schemes. Also, for the case of channel-state-information-assisted relays, an accurate asymptotic performance analysis at high SNR values is presented. Numerical examples compare analytical and simulation results, verifying the correctness of the proposed mathematical analysis.**

***Index Terms*—Free-space optics; Multihop relaying; Gamma-gamma fading; Outage probability; Average bit error probability**.

I. Introduction

Free-space optical (FSO) communication has gained significant research and commercial attention in recent years, due to the necessity of a cost-effective, license-free, and high-bandwidth access communication technique [1,2]. The performance of FSO systems depends strongly on the atmospheric conditions between the transmitter and the receiver. Thus, for an outdoor line-of-sight, point-to-point optical channel link, the existence of atmospheric turbulence, which takes place because of the variations in the refractive index due to inhomogeneities in temperature and pressure fluctuations, can degrade the performance of the communication system mainly over distances of 1 km or longer [3].

Various statistical models have been proposed in order to describe the optical channel characteristics with respect to the atmospheric turbulence strength [2]. It has been observed that for weak fluctuations, the distribution of received intensities is close to a lognormal. For strong turbulence conditions Al-Habash *et al.* proposed the gamma-gamma distribution, which is a multiplicative random process and is used to model both small-scale and large-scale fluctuations [4]. The lognormal and the gamma-gamma distributions have been found to be suitable for modeling the irradiance of the optical channels for weak-to-moderate and moderate-to-strong turbulence channels, respectively, since they provide good agreement between theoretical and experimental data [4,5].

Recently, multihop routing between the transmitter and the receiver has been introduced in the literature in order to increase the reliability of a FSO link [6,7]. This technique is common in various wireless RF communication systems where relayed transmission is providing broader and more efficient coverage. Moreover, the so-called subcarrier intensity modulation (SIM) has been proposed as an alternative modulation scheme for FSO systems [8–12]. The idea of subcarrier modulation is borrowed from the widely used multiple-carrier radio communication (e.g., orthogonal frequency-division multiplexing systems). An advantage of SIM over conventional on–off keying (OOK) systems is that SIM avoids the need for the adaptive threshold required by OOK-modulated FSO systems [8]. Moreover, SIM leverages on the already developed and evolved radio frequency (RF) communi-

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C. K. Datsikas (e-mail: cdatsikas@gmail.com) and G. S. Tombras are with the Department of Electronics, Computers, Telecommunications and Control, Faculty of Physics, University of Athens 15784, Greece.

K. P. Peppas is with the Laboratory of Wireless Communications, Institute of Informatics and Telecommunications, National Centre for Scientific Research "Demokritos," Agia Paraskevi, 15310 Athens, Greece.

N. C. Sagias is with the Department of Telecommunications Science and Technology, School of Applied Sciences and Technology, University of Peloponnese, End of Karaiskaki Street, 22100 Tripolis, Greece.

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cation components such as stable oscillators and narrow filters [11], and hence, any of the evolved digital modulation techniques such as phase-shift keying (PSK) can in principle be used. Finally, with SIM, it is possible to achieve higher throughput or increase the system capacity by modulating several digital streams from the same source onto different RF subcarrier frequencies. The composite signal is then used to vary the laser irradiance by allotting each RF subcarrier frequency to a different user/data source.

In this paper, an extensive study on the end-to-end performance of multihop FSO wireless systems with amplify-and-forward (AF) relays operating over nonidentical gamma-gamma fading channels and employing subcarrier PSK or differential phase-shift keying (DPSK) modulation is presented. Our analysis is carried out for systems with channel-state-information-assisted (CSI-assisted) or fixed-gain relays. For the case of CSI-assisted relays, closed-form performance bounds for the end-to-end signal-to-noise ratio (SNR) are presented by making use of the well-known inequality between harmonic and geometric means of positive random variables [13]. In order to study the statistical properties of the end-to-end SNR of a multihop system with fixed-gain relays, we make use of the fact that the resulting relay channel is cascaded and can be modeled as the product of N squared gamma-gamma fading distributions. Motivated by the fact that, in both considered systems, the statistics of the end-to-end SNR, in their general form, are products of rational powers of statistically independent squared gamma-gamma random variables (RVs), we derive novel closed-form expressions for their moment-generating function (MGF), the probability density function (PDF), and the cumulative distribution function (CDF). These statistical results are then applied to the study of important system performance metrics such as the outage probability (OP) and the average bit error probability (ABEP) of binary modulation schemes. Moreover, for the case of CSI-assisted relays, an accurate error rate performance analysis at high SNR values is presented. Various numerical and computer simulation results are also presented that verify the accuracy of the proposed mathematical analysis.

The paper is organized as follows: In Section II, the statistical properties of the products of rational powers of statistically independent squared gamma-gamma distributions are presented. Section III gives a detailed description of the PSK subcarrier intensity modulation in FSO systems. In Section IV, the system and channel models are described in detail for multihop FSO systems with CSI-assisted and fixed-gain relays. In Section V, the end-to-end performance of the considered systems is addressed in terms of OP and the ABEP of binary modulation schemes. In Section VI numerical and computer simulation results are presented, and Section VII concludes the paper.

In this paper the following notation is used: $K_\alpha(\cdot)$ for the modified Bessel function of the second kind and order α, $\Gamma(\cdot)$ for the Gamma function, $G_{m,n}^{p,q}[\cdot]$ for the Meijer-G function, $\mathrm{erfc}(\cdot)$ for the complementary error function, $\mathbb{E}\langle\cdot\rangle$ for the expectation operator, $\mathcal{L}^{-1}\{\cdot,s;t\}$ for the inverse Laplace transform, and $\Delta(k,\alpha)\triangleq\{\alpha/k,(\alpha+1)/k,\ldots,(\alpha+k-1)/k\}$, where k is a positive integer.

II. Statistical Background

Let us consider $N>1$ squared gamma-gamma random variables μ_i, $i=1,\ldots,N$, with the PDF given by

$$f_{\mu_i}(\mu)=\frac{(a_ib_i)^{(a_i+b_i)/2}}{\Gamma(a_i)\Gamma(b_i)\sqrt{\bar{\mu}_i\mu}}\left(\sqrt{\frac{\mu}{\bar{\mu}_i}}\right)^{(a_i+b_i)/2-1}\times K_{a_i-b_i}\left(2\sqrt{a_ib_i\sqrt{\frac{\mu}{\bar{\mu}_i}}}\right), \tag{1}$$

where a_i, b_i are the distribution-shaping parameters. In optical communication theory, μ_i represents the instantaneous electrical SNR at the receiver of an optical communication system, whereas $\bar{\mu}_i$ is the corresponding average electrical SNR defined in [14]. We define a new random variable Y as the scaled product of rational powers of μ_i, namely,

$$Y\triangleq\frac{1}{L}\prod_{i=1}^{N}\mu_i^{p/q}, \tag{2}$$

where p and q are positive integers and L is a real number.

Theorem 1 (Moment-generating function of Y): The MGF of Y is given by

$$\mathcal{M}_Y(s|N,L,p,q) = \frac{(2p)^{\sum_{i=1}^{N}a_i+\sum_{i=1}^{N}b_i-N(a_N+b_N+1)}}{(2\pi)^{q/2-1/2+N(2p-1)}} \times \prod_{i=1}^{N}\frac{\Xi_i^{a_N+b_N}}{\Gamma(a_i)\Gamma(b_i)}\left(\frac{L}{s}\right)^{q(a_N+b_N)/4p} q^{1/2+q(a_N+b_N)/4p} \times G_{q,4Np}^{4Np,q}\left[\frac{\left(\prod_{i=1}^{N}\Xi_i^{4p}\right)L^q q^q}{(2p)^{4pN}s^q}\middle|_{b_{(4Np)}}^{\Delta(q,1-q(a_N+b_N)/4p)}\right], \tag{3}$$

where

$$b_{(4Np)} \triangleq \left\{\Delta\left(2p,a_N-\frac{a_N+b_N}{2}\right),\Delta\left(2p,b_N-\frac{a_N+b_N}{2}\right),\ldots,\Delta\left(2p,a_1-\frac{a_N+b_N}{2}\right),\Delta\left(2p,b_1-\frac{a_N+b_N}{2}\right)\right\} \tag{4}$$

and $\Xi_i \triangleq \sqrt{a_i b_i/\sqrt{\bar{\mu}_i}}$.

Proof: The MGF of Y is defined as the Laplace transform of Y, i.e.,

$$\mathcal{M}_Y(s|N,L,p,q) = \int_0^{\infty}\exp(-sy)f_Y(y)\mathrm{d}y. \tag{5}$$

Using the definition of Y, the MGF can be expressed by means of the following N-fold integral:

$$\mathcal{M}_Y(s|N,L,p,q) = \int_0^{\infty}\cdots\int_0^{\infty}\exp\left(-\frac{s}{L}\prod_{\ell=1}^{N}\mu_\ell^{p/q}\right)\times\left[\prod_{\ell=1}^{N}f_{\mu_\ell}(\mu_\ell)\right]\mathrm{d}\mu_1\cdots\mathrm{d}\mu_N. \tag{6}$$

To evaluate this integral, we first express the exponential and the modified Bessel function in terms of Meijer-G functions, i.e., $\exp((-s/L)\Pi_{\ell=1}^{N}\mu_\ell^{p/q}) = G_{0,1}^{1,0}[(s/L)\Pi_{\ell=1}^{N}\mu_\ell^{p/q}|_0^-]$ [[15], Eq. (8.4.3.1)], and $K_{a_i-b_i}(2\Xi_i\mu_i^{1/4}) = (1/2)G_{0,2}^{2,0}[\Xi_i^2\,\mu_i^{1/2}|_{(a_i-b_i)/2,(b_i-a_i)/2}^-]$ [[15], Eq. (8.4.23.1)]. Using the fact that the integral of the product of a power and two Meijer G-functions is also a Meijer G-function [[15], Eq. (2.24.1.1)], Eq. (6) is expressed in closed form yielding Eq. (3).

Lemma 1 (Probability density function of Y): The PDF of Y is given by

$$f_Y(y|N,L,p,q) = \frac{q}{2p}L^{q(a_N+b_N)/4p}y^{q(a_N+b_N)/4p-1}\prod_{i=1}^{N}\frac{\Xi_i^{a_N+b_N}}{\Gamma(a_i)\Gamma(b_i)}\times G_{0,2N}^{2N,0}\left[\left(\prod_{i=1}^{N}\Xi_i^2\right)(Ly)^{q/2p}\middle|_{c_{(2N)}}^{-}\right], \tag{7}$$

where

$$c_{(2N)} \triangleq \left\{a_N-\frac{a_N+b_N}{2},b_N-\frac{a_N+b_N}{2},\ldots,a_1-\frac{a_N+b_N}{2},b_1-\frac{a_N+b_N}{2}\right\}. \tag{8}$$

Proof:

The PDF of Y can be obtained by applying the inverse Laplace transform to Eq. (3):

$$f_Y(y|N,L,p,q) = \mathcal{L}^{-1}\{\mathcal{M}_Y(s|N,L,p,q),s;y\}. \tag{9}$$

Using Eq. (3.40.1.1) of [16], the PDF of Y can be expressed in closed form yielding Eq. (7).

Lemma 2 (Cumulative distribution function of Y): The CDF of Y is given by

$$F_Y(y|N,L,p,q) = \frac{(2p)^{\sum_{i=1}^{N}a_i+\sum_{i=1}^{N}b_i-N(a_N+b_N+1)}}{(2\pi)^{N(2p-1)}} \times \prod_{i=1}^{N}\frac{\Xi_i^{a_N+b_N}}{\Gamma(a_i)\Gamma(b_i)}(Ly)^{q(a_N+b_N)/4p} \times G_{q,4Np+q}^{4Np,q}\left[\frac{\left(\prod_{i=1}^{N}\Xi_i^{4p}\right)L^q y^q}{(2p)^{4pN}}\middle|_{b_{(4Np)},\Delta(q,-q(a_N+b_N)/4p)}^{\Delta(q,1-q(a_N+b_N)/4p)}\right], \tag{10}$$

where $b_{(4Np)}$ is defined in Eq. (4).

Proof: The CDF of Y may be obtained as $F_Y(y|N,L,p,q)=\int_0^y f_Y(t|N,L,p,q)\mathrm{d}t$. Using Eqs. (8.4.2.1) and (2.24.1.1) of [15] and Eq. (7), the CDF of Y can be expressed in closed form yielding Eq. (10).

III. PSK Subcarrier Intensity Modulation

In an optical SIM-PSK system, the data sequence is initially modulated by an electrical modulator using PSK, which can be easily implemented with existing microchips. The PSK signal is then upconverted to an intermediate frequency (IF), f_c, and the upconverted signal modulates the intensity of a laser [8,17]. Due to the fact that the upconverted electrical signal is a sum of modulated sinusoids, it takes on both negative and positive values. Optical intensity (instantaneous power) must be nonnegative. Therefore, a DC bias must be added to the electrical signal in order to modulate it onto the intensity of an optical carrier [18].

At the receiver side the received optical field is focused onto a photodetector. The photodetector converts the received optical field to an electrical signal, which is then demodulated. At the demodulator, an estimate of the absolute phase of the RF subcarrier signal is extracted for coherent demodulation in order to recover the source data estimate. In some cases, the phase extraction might be very demanding and therefore differential PSK should be adopted.

In this work we assume that the electrical signal is modulated using binary PSK (BPSK) or binary differential PSK (BDPSK). The transmitted optical intensity can be written as

$$s(t) = 1 + \alpha m(t)\cos(2\pi f_c t), \qquad (11)$$

where $0 < \alpha \leqslant 1$ is the modulation index, and $m(t) = d(t)g(t)$ with $d(t)$ being the data signal and $g(t)$ being the shaping pulse. Directly modulated laser diodes such as the JDS Uniphase CTR915 Series can be employed for high efficiency $\alpha=1$ and low cost.

The received optical intensity can be expressed as [8]

$$P(t) = A_0 I(u,t)[1 + \alpha m(t)\cos(2\pi f_c t)], \qquad (12)$$

where A_0 is a constant and $I(u,t)$ is a stationary random process for the signal scintillation caused by the atmospheric turbulence, with u being an event in the sample space. The sample $I=I(u,t=t_0)$ at any time instant $t=t_0$ is a random variable, which is assumed to follow a gamma-gamma distribution.

On the receiver end, the optical power that is incident on the photodetector is converted into an electrical signal through direct detection. We assume operation in the high-SNR region where shot noise caused by ambient light is much stronger than thermal noise in the electronics following the photodetector. In this case, the additive white Gaussian noise (AWGN) model can be used as an accurate approximation of the Poisson photon-counting detection model [14].

The electrical signal at the photodetector output can be expressed as

$$r(t) = \eta I(u,t)[1 + \alpha m(t)\cos(2\pi f_c t)] + n(t), \qquad (13)$$

where η corresponds to the receiver's optical-to-electrical efficiency and $n(t)$ is an additive white Gaussian process with power spectral density N_0. The DC component in Eq. (13) can be filtered out by a bandpass filter. Downconverting the resulting signal to the bandpass gives the signal at the demodulator input as

$$r(t) = \eta\alpha I(u,t)m(t) + n(t). \qquad (14)$$

Using Eq. (14), the instantaneous electrical SNR, μ, can be derived as

$$\mu = \eta^2\alpha^2 I^2/N_0. \qquad (15)$$

IV. Statistics of the End-to-End SNR

In this section the statistical properties of the end-to-end SNR of FSO multihop communication systems operating over gamma-gamma fading channels with CSI-assisted and fixed-gain relays are studied in detail.

A. System and Channel Model

We consider a FSO multihop system, as shown in Fig. 1, with a source node **S** communicating with a destination node **D** via $N-1$ relay nodes R_i in series and employing subcarrier BPSK or DPSK modulation. Also let L_i be the distance between the ith and $(i+1)$th hop.

Assume that terminal **S** is transmitting a signal x using SIM-PSK modulation. Note that x is real for SIM-BPSK and SIM-BDPSK, whereas it is complex

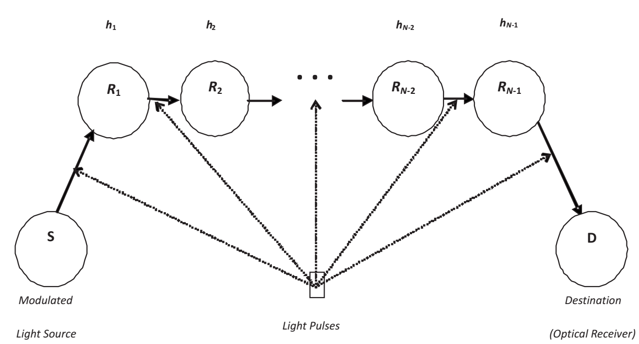

Fig. 1. The considered multihop FSO communication system.

for SIM M-PSK, $M>2$. Then, using Eq. (14), the received signal at the first intermediate node, $\mathbf{R}_1$, can be written as

$$y_1 = s_1 x + n_1 \ , \tag{16}$$

where $s_1 = \alpha\eta I_1$, I_1 is the turbulence-induced light intensity at the first hop, and n_1 is the AWGN at the input of the first relay having power spectral density N_{01}. The signal y_1 is then multiplied by the gain h_1 of the node $\mathbf{R}_1$ and retransmitted to node $\mathbf{R}_2$. The received signal in $\mathbf{R}_2$ is expressed as

$$y_2 = h_1 s_2 (s_1 x + n_1) + n_2, \tag{17}$$

where $s_2 = \alpha\eta I_2$, I_2 is the turbulence-induced light intensity at the second hop, and n_2 is the AWGN at the input of $\mathbf{R}_2$. The received signal at the ith relay $\mathbf{R}_i$ ($i = 1, 2, \ldots, N-1$) is given by

$$y_i = h_{i-1} s_i y_{i-1} + n_i, \tag{18}$$

where $s_i = \alpha\eta I_i$ is the instantaneous intensity gain of the ith hop, h_i is the gain of the ith relay, and n_i is the AWGN at the input of $\mathbf{R}_i$. Using Eq. (18), the received signal at the final destination can be written as

$$y_N = \prod_{i=1}^{N} h_{i-1} s_i x + \sum_{i=1}^{N} n_i \prod_{t=i+1}^{N} h_{t-1} s_t, \quad h_0 = 1. \tag{19}$$

The instantaneous electrical SNR at the ith hop, μ_i, is defined as $\mu_i \triangleq \eta^2 \alpha^2 I_i^2 / N_{0i}$. The PDF of μ_i is given by Eq. (1), where the parameters a_i, b_i, assuming spherical wave propagation, are directly related to atmospheric conditions using Eqs. (60) and (61), respectively, of [2]:

$$a_i = \left[\exp\left(\frac{0.49\beta_{0i}^2}{(1 + 0.18 d_i^2 + 0.56 \beta_{0i}^{12/5})^{7/6}} \right) - 1 \right]^{-1}, \tag{20}$$

$$b_i = \left[\exp\left(\frac{0.51\beta_{0i}^2 (1 + 0.69 \beta_{0i}^{12/5})^{-5/6}}{1 + 0.90 d_i^2 + 0.62 d_i^2 \beta_{0i}^{12/5}} \right) - 1 \right]^{-1}. \tag{21}$$

In Eqs. (20) and (21), $d_i = (kD^2/4L_i)^{1/2}$, where $k = 2\pi/\lambda$ is the optical wave number, with λ being the wavelength and D being the diameter of the receiver collecting lens aperture. Also, $\beta_{0i}^2 = 0.5 C_n^2 k^{7/6} L_i^{11/6}$ is the Rytov variance for a plane wave in weak scintillation theory, where C_n denotes the index of refraction structure parameter. For FSO links near the ground, C_n^2 is approximately $1.7 \times 10^{-14}\ \mathrm{m}^{-2/3}$ during daytime and $8.4 \times 10^{-15}\ \mathrm{m}^{-2/3}$ at night. Generally, C_n^2 varies from $10^{-13}\ \mathrm{m}^{-2/3}$ for strong turbulence to $10^{-17}\ \mathrm{m}^{-2/3}$ for weak turbulence, with $10^{-15}\ \mathrm{m}^{-2/3}$ often defined as a typical average value [5].

B. CSI-Assisted Relays

Using Eq. (19), the equivalent end-to-end SNR can be written as [[13], Eq. (8)]

$$\mu_{equ} = \frac{\prod_{i=1}^{N} s_i^2 h_{i-1}^2}{\sum_{i=1}^{N} n_i \left(\prod_{j=i+1}^{N} h_{j-1}^2 s_j^2 \right)}. \tag{22}$$

By adjusting the gain as the inverse of the fading state $h_i^2 = 1/s_i^2$, the relay just amplifies the incoming signal with the inverse of the channel of the previous hop, regardless of the noise of that hop. In [19], it was proved that such a relay serves as a benchmark for all practical multihop RF systems employing AF relays. In this case, the end-to-end SNR of the considered multihop FSO system can be expressed as

$$\mu_{equ} = \left(\sum_{i=1}^{N} \frac{1}{\mu_i} \right)^{-1}. \tag{23}$$

The end-to-end SNR expression given by Eq. (23) is not mathematically tractable in its current form, due to the difficulty in finding the statistics associated with it. In order to study important performance metrics based on the end-to-end SNR statistics, we use an upper bound for Eq. (23) proposed in [20]. This bound is based on the well-known inequality between geometric and harmonic means for RVs, $\mu_1, \ldots, \mu_N$, given by

$$\mathcal{H}_N \leqslant \mathcal{G}_N, \tag{24}$$

where $\mathcal{H}_N \triangleq N(\sum_{i=1}^{N} 1/\mu_i)^{-1}$ and $\mathcal{G}_N \triangleq \prod_{i=1}^{N} \mu_i^{1/N}$. In Eq. (24) the equality holds when $\mu_1 = \mu_2 = \cdots = \mu_N$. Therefore, the end-to-end SNR can be upper-bounded as

$$\mu_{equ} \leqslant \mu_b = \frac{1}{N} \prod_{i=1}^{N} \mu_i^{1/N}. \tag{25}$$

We may observe that the MGF, the PDF, and the CDF of μ_b may be readily obtained from Eqs. (3), (7), and (10), respectively, by setting $p=1$, $q=N$, and $L=N$ in the corresponding equations.

C. Fixed-Gain Relays

The fixed-gain relays provide reduced implementation complexity in the CSI part, at the expense of the requirements for high-transmission-power amplifiers, which may be very costly in practice. In a fixed-gain relay multihop system, each relay terminal just multiplies the received signal from the previous terminal by a constant gain, h_i, determined by fading loss.

Using Eq. (18), the total fading gain at the destination station $\mathbf{D}$, is obtained as

$$\mathcal{G} = \prod_{i=1}^{N} s_i h_{i-1}. \tag{26}$$

Similarly to [21], we assume that the receiver at **D** can perfectly estimate the amplitude of the N-product fading $\Pi_{i=1}^{N} s_i$. However, it has no capability to estimate the amplitude of each fading one by one. In such a case, the instantaneous end-to-end SNR at the destination terminal is described by a random process defined as the product of N independent squared gamma-gamma random variables, and hence the overall communication channel may be considered as a cascaded channel. The instantaneous end-to-end SNR at **D** is given by Eq. (20) of [21]:

$$\mu_{equ} = \frac{E_s}{N_T} \prod_{i=1}^{N} s_i^2 h_{i-1}^2, \tag{27}$$

where E_s is the average energy of the transmitted symbols and N_T is the total noise power at **D**.

In [22], a specific class of fixed-gain relays that consume the same average power with the corresponding CSI-based relays was proposed. This "semi-blind" relay benefits from the knowledge of the ith hop average fading power. In such a scenario, the fixed gain is considered equal to the average of CSI-assisted gain, namely,

$$h_i^2 = \mathbb{E}\left\langle \frac{1}{s_i^2 + n_i^2} \right\rangle. \tag{28}$$

Performing the required statistical average in Eq. (28), h_i^2 is expressed as

$$h_i^2 = \int_0^\infty \frac{\Xi_i^{(a_i+b_i)}}{(\mu+1)\Gamma(a_i)\Gamma(b_i)} \mu^{(a_i+b_i)/4-1} K_{a_i-b_i}(2\Xi_i \mu^{1/4}) \mathrm{d}\mu. \tag{29}$$

Using the identities $K_{a_i-b_i}(2\Xi_i\mu_i^{1/4}) = (1/2) G_{0,2}^{2,0}[\Xi_i^2\ \mu_i^{1/2}|_{(a_i-b_i)/2,(b_i-a_i)/2}^{-}]$ [[15], Eq. (8.4.23.1)], $(1+\mu)^{-1} = G_{1,1}^{1,1}[\ \mu|_0^0]$ [[15], Eqs. (2.24.2.5) and (2.24.1.1)], h_i^2 is obtained in closed form as

$$h_i^2 = \frac{\Xi_i^{a_i+b_i}}{4\pi\Gamma(a_i)\Gamma(b_i)} \times G_{1,5}^{5,1}\left[\frac{\Xi_i^4}{16} \middle|_{(a_i-b_i)/4,(a_i-b_i)/4+1/2,(b_i-a_i)/4,(b_i-a_i)/4+1/2,1-(a_i+b_i)/4}^{1-(a_i+b_i)/4} \right]. \tag{30}$$

V. End-to-End Performance Analysis

In this section, using the previously derived formulas, useful performance metrics of multihop communication systems operating over gamma-gamma fading channels are presented. The performance metrics of interest are the end-to-end OP and the ABEP of binary modulation schemes. For the case of a multihop system with CSI-assisted relays, lower bounds for the considered performance metrics are derived. Moreover, for high SNR values an accurate asymptotic error rate analysis of the considered system is also presented. For the case of fixed-gain relays, the expressions of the performance metrics of interest are exact.

A. Outage Probability

The OP is defined as the probability that the end-to-end output SNR falls below a specified threshold μ_{th}. This threshold is a minimum value of the SNR above which the quality of service is satisfactory. For CSI-assisted relays, a lower bound of the OP may be obtained in closed form as follows:

$$P_{out} \geqslant F_Y(\mu_{th}|N,N,1,N). \tag{31}$$

For fixed-gain relays, the OP may be obtained as

$$P_{out} = F_Y(\mu_{th}|N,1/\lambda_s,1,1), \tag{32}$$

where $\lambda_s = (E_s/N_T)\Pi_{i=1}^{N} h_{i-1}^2$.

B. Average Bit Error Probability

1) CSI-Assisted Relays: For subcarrier BDPSK a lower bound for the ABEP may be readily obtained from Eq. (3) as $\bar{P}_{be} \geqslant 0.5\mathcal{M}_Y(1|N,N,1,N)$. For coherent binary signal constellations, a lower bound for the ABEP may be obtained by averaging the corresponding bit error probability for AWGN channels, $P_e(\mu)$, over the PDF of μ_b, namely,

$$\bar{P}_{be} \geqslant \int_0^\infty P_e(\mu) f_Y(\mu|N,N,1,N) \mathrm{d}\mu. \tag{33}$$

The error probability $P_e(\mu)$ may be formulated as

$$P_e(\mu) = \frac{1}{2} \operatorname{erfc}(\sqrt{\psi\mu}), \tag{34}$$

where $\psi = 1$ for subcarrier BPSK. The integral (33) can be solved in closed form (see Appendix A), and therefore a lower bound for the ABEP of binary modulation schemes may be obtained as follows:

$$\bar{P}_{be} \geqslant \mathcal{I}(1/2,\psi;N,N,1,N), \tag{35}$$

where $\mathcal{I}$ is defined in Appendix A in Eq. (44).

2) Asymptotic Error Rate Analysis of Multihop-Systems With CSI-Assisted Relays at High SNR: In [23] the authors presented a generic analytical framework for the asymptotic performance analysis of multihop systems at high SNR. In this work the authors proved the following theorem: Let X_i, $i=1,2,\ldots,N$, be N independent and nonnegative random variables. If the series expansion of the PDF of X_i in the neighborhood zero can be expressed as[1]

$$P_{X_i}(x) = A_i x^{t_i} + o(x^{t_i+\epsilon}), \tag{36}$$

where $A_i > 0$, $t_i \geqslant 0$, $\epsilon > 0$, then the series expansion of the PDF of $Z=(\Sigma_{i=1}^{N} 1/X_i)^{-1}$ is given by

$$f_Z(z) = \sum_{i=1,t_i=t_{min}}^{K} A_i z^{t_{min}} + o(z^{t_{min}+\zeta}), \tag{37}$$

where $t_{min}=\min_i\{t_i\}$ and ζ is a constant that depends on t_i and ϵ. Following a similar approach and by making use of the fact that $K_t(z) \simeq (\Gamma(t)/2)(2/z)^t$ when $z \to 0$, the PDF of μ_i may be expressed as

$$f_{\mu_i}(\mu) \simeq \frac{\Xi_i^{2b_i} \mu^{b_i/2-1} \Gamma(a_i - b_i)}{2\Gamma(a_i)\Gamma(b_i)}, \tag{38}$$

and hence the PDF of μ_b is given by

$$f_{\mu_b}(\mu) \simeq \frac{\mu^{b_m/2-1}}{2} \sum_{i=1}^{N} \frac{\Xi_i^{2b_i}\Gamma(a_i - b_i)}{\Gamma(a_i)\Gamma(b_i)}, \tag{39}$$

where $b_m=\min\{b_i\}$, $i=1, \ldots ,N$. The MGF of μ_b can be evaluated using Eq. (2.1.1.1) of [16] as

$$\mathcal{M}_{\mu_b}(s) \simeq \frac{\Gamma\left(\frac{b_m}{2}\right)}{2s^{bm/2}} \sum_{i=1}^{N} \Xi_i^{2b_i} \frac{\Gamma(a_i - b_i)}{\Gamma(a_i)\Gamma(b_i)}. \tag{40}$$

The asymptotic ABEP of subcarrier BDPSK modulation at high SNR values can readily be obtained from Eq. (40) as $\bar{P}_{be}=0.5\mathcal{M}_{\mu_b}(1)$. For subcarrier BPSK modulation, the asymptotic ABEP is obtained using the well-known MGF approach [24] and after some algebraic manipulations as

[1]It is noted that for two functions f and g of a real argument x we write $f(x)=o(g(x))$ when $x \to x_0$ if $\lim_{x\to x_0} f(x)/g(x)=0$

$$\begin{aligned}\bar{P}_{be} &= \frac{1}{\pi}\int_0^{\pi/2} \mathcal{M}_{\mu_b}\left(\frac{1}{\sin^2\theta}\right)\mathrm{d}\theta \\ &\simeq \frac{\Gamma\left(\frac{b_m}{2}\right)}{2\pi}\sum_{i=1}^{N} \Xi_i^{2b_i}\frac{\Gamma(a_i - b_i)}{\Gamma(a_i)\Gamma(b_i)}\int_0^{\pi/2} \sin^{b_m}\theta \mathrm{d}\theta \\ &= \frac{\Gamma\left(\frac{1}{2}+\frac{b_m}{2}\right)}{2\sqrt{\pi}b_m}\sum_{i=1}^{N}\frac{\Xi_i^{2b_i}\Gamma(a_i - b_i)}{\Gamma(a_i)\Gamma(b_i)}.\end{aligned} \tag{41}$$

Note that the asymptotic expressions for the ABEP can be used to obtain insight as to what parameters determine system performance in the presence of turbulence channels. The ABEP is quantified in terms of two parameters: diversity gain and coding gain. The diversity gain, G_d, determines the slope of the ABEP versus the $\bar{\mu}$ curve, at high SNR, in a log-log scale. The coding gain determines the shift of the curve in SNR relative to a reference ABEP curve of $(\bar{\mu}^{-G_d})$. Using Eq. (41) one can observe that the diversity gain is determined only by the parameters b_i. The coding gain is determined by both a_i and b_i.

3) Fixed-Gain Relays: For subcarrier BDPSK, a lower bound for the ABEP may be readily obtained from Eq. (3) as $\bar{P}_{be}=0.5\mathcal{M}_Y(1|N,1/\lambda_s,1,1)$. For coherent binary signal constellations, the ABEP may be obtained as follows:

$$\bar{P}_{be} = \mathcal{I}(1/2,\psi;N,1/\lambda_s,1,1). \tag{42}$$

VI. Numerical and Computer Simulation Results

In this section, numerical and computer simulation results that demonstrate the proposed mathematical analysis are presented. In our analysis we assume a wavelength of $\lambda=1550$ nm, in accordance with [5]. Moreover, the aperture diameter for the receiver D has been taken to be 0.01 m, which is an acceptable value for these systems [25].

In Fig. 2, lower bounds for the end-to-end OP of a multihop system with CSI-assisted relays is presented as a function of the first hop inverse normalized outage threshold $\bar{\mu}_1/\mu_{th}$ with unbalanced ($\bar{\mu}_i=i\bar{\mu}_1$, $i=1,\ldots,N$) hops. The refractive index is considered equal to $C_n^2=5\times10^{-14}$ m$^{-2/3}$. The distances between hops are equal and selected as $L=3000$ m, resulting in channel parameters of $(a,b)=(2.07882,1.63848)$. Dashed curves for the exact outage performance, obtained via Monte Carlo simulations based on Eq. (23), are also included for comparison purposes. As can be observed, the difference between the exact value of

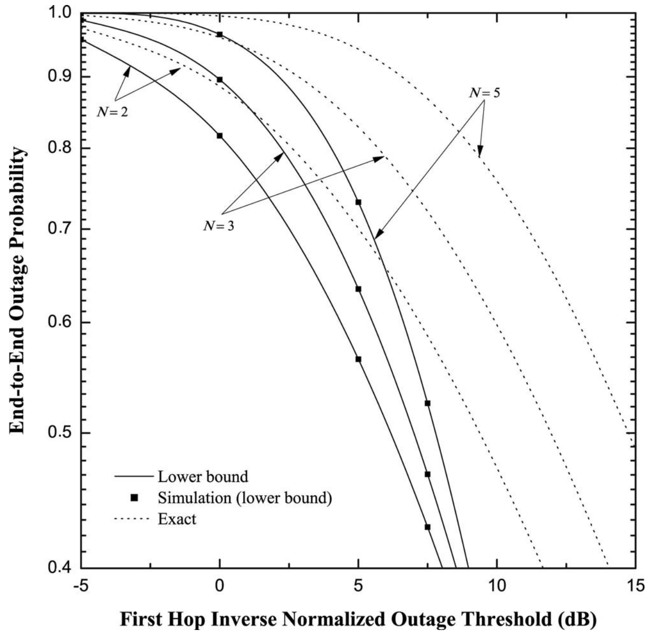

Fig. 2. End-to-end outage probability of multihop FSO systems with CSI-assisted relays operating over gamma-gamma fading channels for different numbers of hops N as a function of the first hop inverse normalized outage threshold $\bar{\mu}_1/\mu_{th}$ ($L=3000$ m, $C_n^2=5\times10^{-14}$ $\text{m}^{-2/3}$, $a=2.07882$, $b=1.63848$, $\bar{\mu}_i=i\bar{\mu}_1$, $i=1,\ldots,N$).

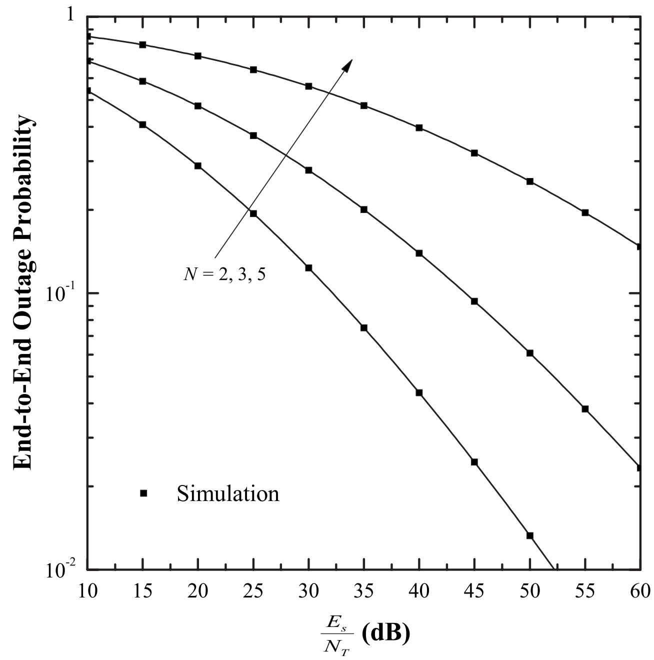

Fig. 3. End-to-end outage probability of multihop FSO systems with fixed-gain relays operating over gamma-gamma fading channels for different numbers of hops N as a function of E_s/N_T [$L_i=3000+500(i-1)$ (m), $C_n^2=5\times10^{-14}$ $\text{m}^{-2/3}$, $\mu_{th}=0$ dB, $\bar{\mu}_i=1$, $i=1,\ldots,N$].

the OP and the obtained bound is tighter at low values of $\bar{\mu}_1/\mu_{th}$. However, the bounds lose tightness as $\bar{\mu}_1/\mu_{th}$ increases. It is noted that this result is in agreement with [13,20]. Also, as is evident, the outage performance of the considered system deteriorates as the number of hops N increases.

In Fig. 3 the end-to-end OP for the case of fixed-gain relays is illustrated as a function of E_s/N_T and for different numbers of relays N, assuming $\mu_{th}=0$ dB. The distances between successive hops are assumed to be $L_i=3000+500(i-1)$ (m), $i=1,\ldots,N$, resulting in different values of parameters a_i and b_i. Also, $C_n^2=5\times10^{-14}$ $\text{m}^{-2/3}$. We also assumed that the average electrical SNR of each hop is $\bar{\mu}_i=1$, $\forall i$. One can observe that the outage performance deteriorates with an increase in the number of relays. Equivalent results, obtained via Monte Carlo simulations are also included, and a perfect match with the analytical results is observed.

In Fig. 4 lower-bound curves for the ABEP of subcarrier BDPSK and BPSK modulations of a multihop system with CSI-assisted relays are plotted as a function of the first hop average SNR $\bar{\mu}_1$ for various values of N and unbalanced hops. The distances between successive hops are assumed to be $L_i=3000+500(i-1)$ (m), $i=1,\ldots,N$. The refractive index is considered equal to $C_n^2=1.7\times10^{-14}$ $\text{m}^{-2/3}$. It is obvious that ABEP improves as N decreases or $\bar{\mu}_1$ increases. The analytically derived lower bounds are compared with the exact error performance results, obtained through Monte Carlo simulations. Similarly to the case of the outage performance, the obtained bounds are tighter at low values of $\bar{\mu}_1$.

In Figs. 5 and 6 the ABEP of subcarrier BDPSK and

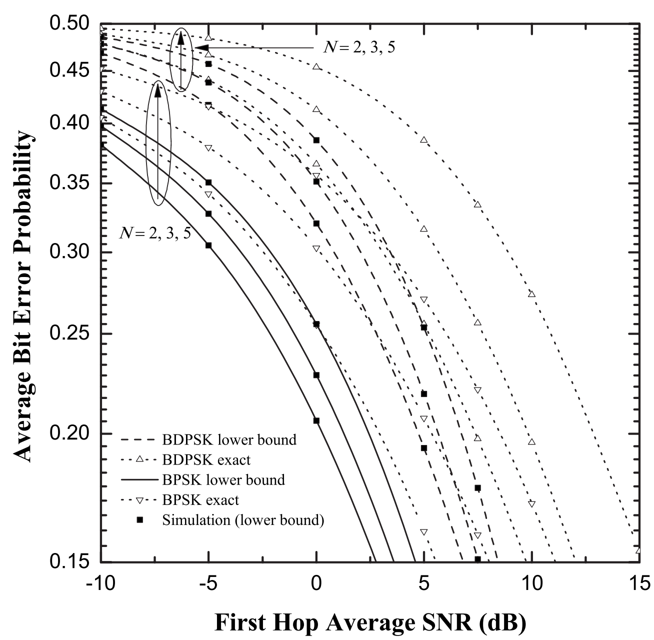

Fig. 4. End-to-end ABEP of subcarrier BDPSK and BPSK for multihop FSO systems with CSI-assisted relays operating over gamma-gamma fading channels for different numbers of hops N as a function of the first hop average SNR $\bar{\mu}_1$ [$C_n^2=1.7\times10^{-14}$ $\text{m}^{-2/3}$ $L_i=3000+500(i-1)$ m, $\bar{\mu}_i=i\bar{\mu}_1$, $i=1,\ldots,N$].

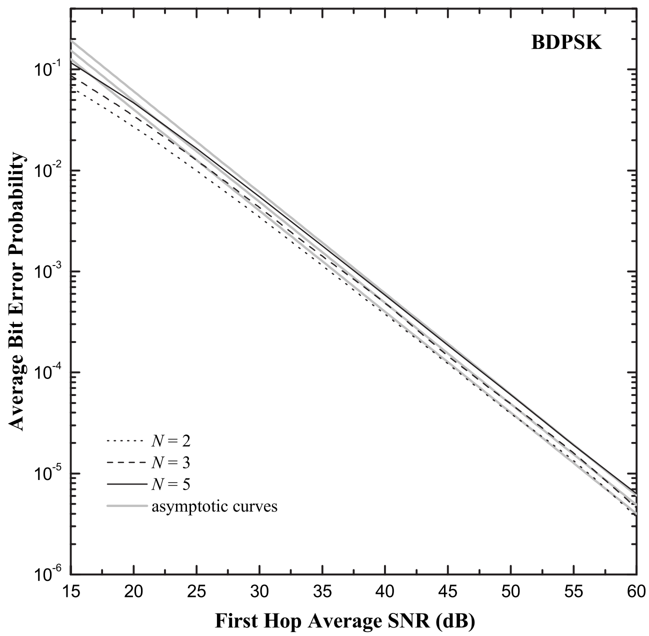

Fig. 5. End-to-end asymptotic ABEP of subcarrier BDPSK for multihop FSO systems with CSI-assisted relays operating over gamma-gamma fading channels for different numbers of hops N as a function of the first hop average SNR $\bar{\mu}_1$ ($a=4$, $b=2$, $\bar{\mu}_i=i\bar{\mu}_1$, $i=1,\ldots,N$).

BPSK modulations of a multihop system with CSI-assisted relays and their corresponding asymptotic values obtained with the help of Eqs. (40) and (41), respectively, are plotted as a function of $\bar{\mu}_1$, for various values of N, $a=2$ and $b=4$, and unbalanced hops. As can be observed, our asymptotic analysis yields accurate results for high values of $\bar{\mu}_1$, especially when $\bar{\mu}_1 > 30$ dB.

Finally, in Fig. 7 the ABEP of subcarrier BDPSK and BPSK modulations for the case of fixed-gain relays is illustrated as a function of E_s/N_T for $C_n^2=1.7\times10^{-14}$ m$^{-2/3}$ and for different numbers of relays N. Again, we assume $\bar{\mu}_i=1$, $i=1,\ldots,N$. The distances between successive hops are assumed to be $L_i=3000+500(i-1)$ (m). As can be observed, the error performance deteriorates with an increase in the number of relays. Equivalent ABEP results, obtained via Monte Carlo simulations are also included, and, as is obvious, a perfect match with the analytical results is observed.

VII. Conclusions

In this paper, a thorough performance analysis of multihop FSO systems operating over gamma-gamma channels has been presented. Both CSI-assisted and fixed-gain relays were considered. A general analytical framework for the performance evaluation of the above-mentioned systems was provided by studying the statistical properties of the distribution of the product of rational powers of squared gamma-gamma distributions. Novel closed-form expressions for the MGF, PDF, and CDF of this distribution were derived. The previously mentioned formulas provide lower bounds for important performance metrics of multi-

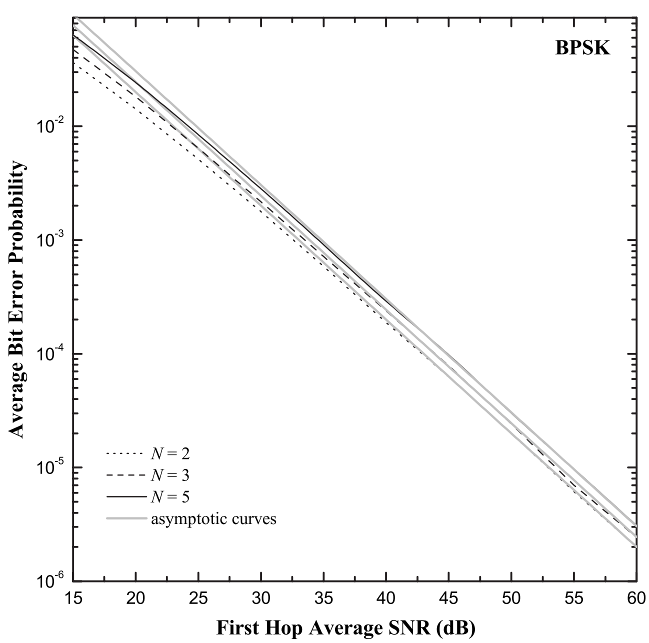

Fig. 6. End-to-end asymptotic ABEP of subcarrier BPSK for multihop FSO systems with CSI-assisted relays operating over gamma-gamma fading channels for different numbers of hops N as a function of the first hop average SNR $\bar{\mu}_1$ ($a=4$, $b=2$, $\bar{\mu}_i=i\bar{\mu}_1$, $i=1,\ldots,N$).

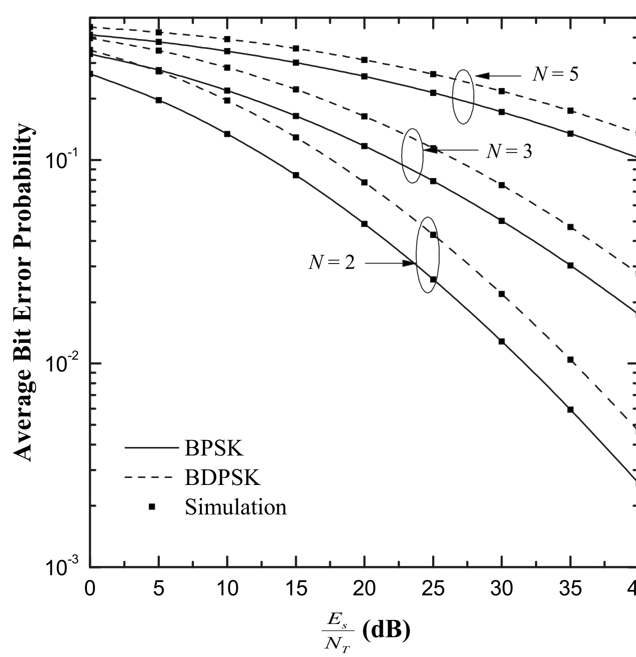

Fig. 7. End-to-end ABEP of subcarrier BDPSK and BPSK for multihop FSO systems with fixed-gain relays operating over gamma-gamma fading channels for different numbers of hops N as a function of E_s/N_T [$C_n^2=1.7\times10^{-14}$ m$^{-2/3}$, $L_i=3000+500(i-1)$ m, $\bar{\mu}_i=1$, $i=1,\ldots,N$].

hop systems with CSI-assisted relays and exact expressions for the corresponding metrics of multihop systems with fixed-gain relays. In the latter case, it was assumed that the resulting channel model is a cascaded one. The performance metrics of interest are the outage probability and the average bit error probability, assuming that subcarrier PSK or subcarrier BDPSK is employed. For the specific case of CSI-assisted relays, it was proved that the proposed bounds are tight at low SNR values; however, these bounds are loose at high SNR. Also, an accurate error-rate asymptotic analysis of multihop systems with CSI-assisted relays at high SNR has been presented. Extensive numerical and computer simulation results were presented that verified the correctness of the proposed mathematical analysis.

Appendix A: Integrals Consisting of Products of $f_Y(y|N,L,p,q)$ and the Complementary Error Function

We consider the following integral:

$$\mathcal{I}(A,B,N,L,p,q) = A\int_0^\infty \operatorname{erfc}(\sqrt{By}) f_Y(y|N,L,p,q)\mathrm{d}y, \tag{43}$$

where A and B are constants and $f_Y(y|N,L,p,q)$ is given by Eq. (7). Then by expressing the complementary error function in terms of Meijer-G functions, i.e., $\operatorname{erfc}(\sqrt{B\mu}) = 1/\sqrt{\pi} G_{1,2}^{2,0}[B\mu|_{0,1/2}^{1}]$ [[15], Eq. (8.4.14.2)], and with the help of Eq. (2.24.1.1) of [15], $\mathcal{I}(A,B;N,L,p,q)$ can be solved in closed form, yielding

$$\mathcal{I}(A,B;N,L,p,q) = \frac{A}{\sqrt{\pi}} \frac{(2p)^{\sum_{i=1}^N a_i + \sum_{i=1}^N b_i - N(a_N+b_N+1)}}{(2\pi)^{q/2-1/2+N(2p-1)}} \times \prod_{i=1}^{N} \frac{\Xi_i^{a_N+b_N}}{\Gamma(a_i)\Gamma(b_i)} \left(\frac{Lq}{B}\right)^{q(a_N+b_N)/4p} \times G_{2q,4Np+q}^{4Np,2q}\left[\frac{\left(\prod_{i=1}^{N}\Xi_i^{4p}\right)L^q q^q}{(2p)^{4Np}B^q}\middle|_{(d_{4Np+q})}^{(c_{2q})}\right], \tag{44}$$

where $c_{2q} = \Delta(q, 1-q(a_N+b_N)/4p)$, $\Delta(q, \frac{1}{2} - q(a_N + b_N)/4p)$, and $d_{4Np+q} = b_{(4Np)}$, $\Delta(q, -q(a_N+b_N)/4p)$.

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