

Switched Diversity Receivers over Generalized Gamma Fading Channels

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Abstract—A versatile envelope distribution which generalizes several commonly used fading models is the generalized Gamma (GG) distribution. This letter deals with the performance analysis of switch and stay combining (SSC) receivers operating over not necessarily identical GG fading channels. For these receivers, novel analytical expressions for the moments of the output signal-to-noise ratio (SNR) (including average SNR and amount of fading), outage probability, average bit error probability (ABEP), and Shannon average spectral efficiency (ASE) are derived. Moreover, closed-form expressions are obtained for the optimal average SNR, ABEP, and ASE switching thresholds. Special cases of the derived expressions agree with known results.

Index Terms—Generalized fading channel models, generalized Gamma, lognormal, Nakagami- m , Weibull, switched diversity.

I. INTRODUCTION

AMONG well-known diversity schemes, switch and stay combining (SSC) is one of the least complex to implement which can be used in conjunction with various coherent as well as noncoherent and differentially coherent modulations. According to this scheme, if the instantaneous signal-to-noise ratio (SNR) of a branch falls below a predetermined switching threshold, the combiner switches to and stays with another branch, regardless of whether the SNR of that branch is above or below the threshold. The performance of SSC receivers has been extensively studied in the past for the Rayleigh, Nakagami- m , and Weibull fading channel models [1]–[8]. A recently rediscovered versatile envelope distribution generalizing all these models is the so-called generalized Gamma (GG) distribution [9] which was introduced by Stacy [10] more than four decades ago. However, for this very generic fading model only average bit error probability (ABEP) expressions for single-branch receivers have been presented [11], [12]. To the best of the authors' knowledge, the performance of SSC receivers in GG fading has not been addressed yet. Motivated by the above, in this letter, the performance of SSC receivers operating over GG fading channels is analyzed and novel formulae for their performance are derived.

II. SYSTEM AND CHANNEL MODEL

Let us consider a dual-branch SSC receiver operating over independent, but not necessarily identically distributed (i.d.),

Manuscript received April 27, 2005. The associate editor coordinating the review of this letter and approving it for publication was Prof. George Karagiannidis.

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Digital Object Identifier 10.1109/LCOMM.2005.xxxxx.

GG fading channels. The probability density function (pdf) of the instantaneous SNR per symbol at the ℓ th input branch, γ_ℓ ($\ell = 1$ and 2), is [10, eq. (1)]

$$f_{\gamma_\ell}(\gamma) = \frac{\beta_\ell \gamma^{m_\ell \beta_\ell/2 - 1} / \Gamma(m_\ell)}{2 (\Xi_\ell \bar{\gamma}_\ell)^{m_\ell \beta_\ell/2}} \exp \left[- \left(\frac{\gamma}{\Xi_\ell \bar{\gamma}_\ell} \right)^{\beta_\ell/2} \right] \quad (1)$$

where $\beta_\ell > 0$ and $m_\ell \geq 1/2$ are two parameters related to the fading severity, $\bar{\gamma}_\ell$ is the average input SNR per symbol, and $\Xi_\ell = \Gamma(m_\ell) / \Gamma(m_\ell + 2/\beta_\ell)$, with $\Gamma(\cdot)$ being the Gamma function [13, eq. (8.310/1)]. The distribution in (1) is very flexible since it includes commonly used models such as Rayleigh ($\beta_\ell = 2$ and $m_\ell = 1$), Nakagami- m ($\beta_\ell = 2$), and Weibull ($m_\ell = 1$) as special cases. Moreover, for $\beta_\ell \rightarrow 0$ and $m_\ell \rightarrow \infty$, (1) becomes the well-known Lognormal pdf (as a limiting case). The cumulative distribution function (cdf) of γ_ℓ can be expressed as

$$F_{\gamma_\ell}(\gamma) = 1 - \frac{1}{\Gamma(m_\ell)} \Gamma \left[m_\ell, \left(\frac{\gamma}{\Xi_\ell \bar{\gamma}_\ell} \right)^{\beta_\ell/2} \right] \quad (2)$$

with $\Gamma(\cdot, \cdot)$ being the upper incomplete Gamma function [13, eq. (8.350/2)]. By following a similar method with that for deriving the moments-generating function (MGF) of the Weibull distribution [7], the MGF of γ_ℓ can be obtained as

$$\mathcal{M}_{\gamma_\ell}(s) = \frac{\beta_\ell}{2\Gamma(m_\ell)} \frac{1}{(\Xi_\ell \bar{\gamma}_\ell s)^{m_\ell \beta_\ell/2}} \frac{l^{m_\ell \beta_\ell/2} \sqrt{k/l}}{(\sqrt{2\pi})^{k+l-2}} \times G_{l,k}^{k,l} \left[\frac{l^l/k^k}{(\Xi_\ell \bar{\gamma}_\ell s)^{k \beta_\ell/2}} \middle| \begin{matrix} \Delta(l; 1-m_\ell \beta_\ell/2) \\ \Delta(k; 0) \end{matrix} \right]. \quad (3)$$

In the above equation, $G[\cdot]$ is the Meijer's G-function [13, eq. (9.301)] and $\Delta(k; x)$ is defined as $\Delta(k; x) \triangleq x/k, (x+1)/k, \dots, (x+k-1)/k$, with x an arbitrary real value and k a positive integer. Moreover, k and l are positive integers so that $l/k = \beta_\ell/2$ holds. Depending upon the specific value of β_ℓ , a set of minimum k and l can be properly chosen. Note that for $m_\ell = 1$, (3) agrees with [7, eq. (3)].

By denoting with γ_τ the common switching threshold at both diversity branches and using (2), the cdf of the SSC output SNR, γ_{SSC} , can be expressed as [3, eq. (62)]

$$F_{\gamma_{SSC}}(\gamma) = \begin{cases} \frac{P_1 P_2}{P_1 + P_2} \sum_{i=1}^2 F_{\gamma_i}(\gamma), & \gamma \leq \gamma_\tau \\ \frac{P_1 P_2}{P_1 + P_2} \sum_{i=1}^2 \left[F_{\gamma_i}(\gamma) + \frac{F_{\gamma_i}(\gamma)}{P_i} - 1 \right], & \gamma > \gamma_\tau \end{cases} \quad (4)$$

where $P_\ell = F_{\gamma_\ell}(\gamma_\tau)$. By taking the first derivative of $F_{\gamma_{ssc}}(\gamma)$ with respect to γ , the pdf of γ_{ssc} can be obtained as

$$f_{\gamma_{ssc}}(\gamma) = \begin{cases} \frac{P_1 P_2}{P_1 + P_2} \sum_{i=1}^2 f_{\gamma_i}(\gamma), & \gamma \leq \gamma_\tau \\ \frac{P_1 P_2}{P_1 + P_2} \sum_{i=1}^2 f_{\gamma_i}(\gamma) \left(1 + \frac{1}{P_i}\right), & \gamma > \gamma_\tau \end{cases}. \quad (5)$$

III. PERFORMANCE ANALYSIS

A. Moments of the Output SNR

Starting from the definition of the n th order moment of γ_{ssc} , $\mu_n \triangleq \mathcal{E}\langle \gamma_{ssc}^n \rangle$ ($\mathcal{E}\langle \cdot \rangle$ denotes expectation), and by using [13, eq. (8.350/1)], μ_n can be easily derived in closed-form as

$$\mu_n = \frac{P_1 P_2}{P_1 + P_2} \sum_{i=1}^2 \frac{(\Xi_i \bar{\gamma}_i)^n}{\Gamma(m_i)} \left[\Gamma(d_{i,n}) + \frac{\Gamma(d_{i,n}, \xi_i)}{P_i} \right] \quad (6)$$

with $\xi_\ell = [\gamma_\tau / (\Xi_\ell \bar{\gamma}_\ell)]^{\beta_\ell/2}$ and $d_{\ell,n} = m_\ell + 2n/\beta_\ell$. For $m_\ell = 1$ and $\beta_\ell = \beta \forall \ell$, (6) reduces to a known expression [6, eq. (6)] for the Weibull channel model.

1) *Average SNR*: The average output SNR can be obtained in closed-form as $\bar{\gamma}_{ssc} = \mu_1$. The optimum γ_τ for maximizing $\bar{\gamma}_{ssc}$ can be derived by solving $\partial \bar{\gamma}_{ssc} / \partial \gamma_\tau |_{\gamma_\tau^*} = 0$. It can be easily shown that for i.d. input channels ($\bar{\gamma} = \bar{\gamma}_\ell$, $m = m_\ell$, $\beta = \beta_\ell$, and $P = P_\ell$), $\gamma_\tau^* = \bar{\gamma}$. For non i.d. input branches, γ_τ^* can be obtained using root-finding techniques available in most of the popular mathematical software packages.

2) *Amount of fading*: The amount of fading, defined as the ratio of the variance to the square average SNR per symbol, i.e., $A_F \triangleq \text{var}(\gamma_{ssc}) / \bar{\gamma}_{ssc}^2$, can be expressed using (6) in a simple closed-form expression as $A_F = \mu_2 / \mu_1^2 - 1$.

B. Outage Probability

If γ_{th} is a specified threshold, then the outage probability is defined as the probability that γ_{ssc} falls below γ_{th} . This probability can be obtained using (4) as $P_{out}(\gamma_{th}) = F_{\gamma_{ssc}}(\gamma_{th})$.

C. Average Bit Error Probability (ABEP)

A convenient approach to evaluate the ABEP for a great variety of modulation schemes is to use the MGF-based approach [1]. By averaging (5) over $\exp(-s\gamma_{ssc})$, the MGF of γ_{ssc} can be obtained as

$$\mathcal{M}_{\gamma_{ssc}}(s) = \frac{P_1 P_2}{P_1 + P_2} \sum_{i=1}^2 \left[\mathcal{M}_{\gamma_i}(s) + \frac{\mathcal{M}_{\gamma_i}(s)}{P_i} - \frac{1}{P_i} \int_0^{\gamma_\tau} \exp(-s\gamma) f_{\gamma_i}(\gamma) d\gamma \right]. \quad (7)$$

For i.d. input branches, (7) simplifies to

$$\mathcal{M}_{\gamma_{ssc}}(s) = (1 + P) \mathcal{M}_\gamma(s) - \int_0^{\gamma_\tau} \exp(-s\gamma) f_\gamma(\gamma) d\gamma \quad (8)$$

with $\gamma = \gamma_\ell \forall \ell$. The above two equations include finite integrals, which can be easily evaluated via numerical integration. The optimum γ_τ for minimizing the ABEP can be obtained as $\partial \bar{P}_{be} / \partial \gamma_\tau |_{\gamma_\tau^*} = 0$. Although analytical expressions for γ_τ^*

with specific signalling constellations and i.d. input branches can be easily obtained, a unified ABEP expression for the many modulation schemes which can be evaluated by the MGF-based approach cannot be derived. For example, for binary phase-shift keying (PSK) with i.d. input branches, γ_τ^* can be obtained by numerically solving $\int_0^\infty \mathcal{M}_\gamma(1/\sin^2 \varphi) d\varphi = \int_0^\infty \exp(-\gamma_\tau^*/\sin^2 \varphi) d\varphi$. Additionally, for binary differential phase-shift keying (DPSK) and noncoherent frequency-shift keying (NFSK) signalings, γ_τ^* can be obtained in closed-form as $\gamma_\tau^* = -\ln[\mathcal{M}_\gamma(B)]/B$ where for DPSK $B = 1$ and for NFSK $B = 1/2$. For non i.d. input channels, γ_τ^* can be derived by employing root-finding analytical techniques.

IV. AVERAGE SPECTRAL EFFICIENCY (ASE)

By averaging $\log_2(1 + \gamma_{ssc})$ over (5) and following a similar method as that in [14], the average spectral efficiency (ASE) [15] at the ℓ th input branch can be obtained as

$$\bar{S}_{\gamma_\ell} = \frac{1/\ln(2)}{(\sqrt{\Xi_\ell \bar{\gamma}_\ell})^{m_\ell \beta_\ell}} \frac{1/\sqrt{k}}{(\sqrt{2\pi})^{k+2l-3}} \frac{1}{\Gamma(m_\ell)} \times G_{2l,k+2l}^{k+2l,l} \left[\frac{k^{-k}}{(\sqrt{\Xi_\ell \bar{\gamma}_\ell})^{\beta_\ell k}} \left| \begin{array}{c} \Delta(l, -\frac{m_\ell \beta_\ell}{2}), \Delta(l, 1 - \frac{m_\ell \beta_\ell}{2}) \\ \Delta(k, 0), \Delta(l, -\frac{m_\ell \beta_\ell}{2}), \Delta(l, -\frac{m_\ell \beta_\ell}{2}) \end{array} \right. \right]. \quad (9)$$

Note that for $\beta_\ell = 2$ and $m_\ell = 1$, (9) reduces to [8, eq. (3)] for Nakagami- m and [8, eq. (6)] for Weibull, respectively. From (5), the ASE at the output of the SSC can be expressed as

$$\bar{S}_e = \frac{P_1 P_2}{P_1 + P_2} \sum_{i=1}^2 \left[\bar{S}_{\gamma_i} + \frac{\bar{S}_{\gamma_i}}{P_i} - \frac{1}{P_i} \int_0^{\gamma_\tau} \log_2(1 + \gamma) f_{\gamma_i}(\gamma) d\gamma \right]. \quad (10)$$

For i.d. input branches ($\bar{S}_\gamma = \bar{S}_{\gamma_\ell} \forall \ell$), (10) can be reduced to

$$\bar{S}_e = (1 + P) \bar{S}_\gamma - \int_0^{\gamma_\tau} \log_2(1 + \gamma) f_\gamma(\gamma) d\gamma. \quad (11)$$

The optimum γ_τ for maximizing \bar{S}_e can be obtained as $\partial \bar{S}_e / \partial \gamma_\tau |_{\gamma_\tau^*} = 0$, where after some mathematical manipulations, yields $\gamma_\tau^* = 2^{\bar{S}_e} - 1$. For the non i.d. case, γ_τ^* can be derived as described above for $\max\{\bar{\gamma}_{ssc}\}$ or $\min\{\bar{P}_{be}\}$.

V. NUMERICAL RESULTS AND DISCUSSION

In Fig. 1, $\bar{\gamma}_{ssc}/\bar{\gamma}_1$ is plotted as a function of β for non i.d. input branches (i.e., $\bar{\gamma}_1 = 0.5 \bar{\gamma}_2$), optimum γ_τ for $\max\{\bar{\gamma}_{ssc}\}$, and several values of m . From these curves it can be observed that the larger m and/or β is, the lower is the gain of the combiner (e.g. for the non i.d. case and $\beta \geq 4$, $\bar{\gamma}_{ssc}/\bar{\gamma}_1 \approx 1$). Additionally in the same figure, curves for i.d. input branches (i.e., $\bar{\gamma}_1 = \bar{\gamma}_2$) are also included for best performance. In Fig. 2, P_{out} is plotted as a function of $\gamma_{th}/\bar{\gamma}$ for i.d. input branches, $\beta = 2.5$, and the three different γ_τ^* derived in Sections III and IV. As it can be seen, for fixed $\gamma_{th}/\bar{\gamma}$ and m , the best P_{out} is obtained for that γ_τ^* leading to $\min\{\bar{P}_{be}\}$. Moreover, the worst P_{out} is provided for that γ_τ^* which maximizes $\bar{\gamma}_{ssc}$, while an intermediate solution between these two γ_τ^* is the optimum γ_τ for maximizing the ASE. From these results, it turns out that the proper choice of γ_τ^* among

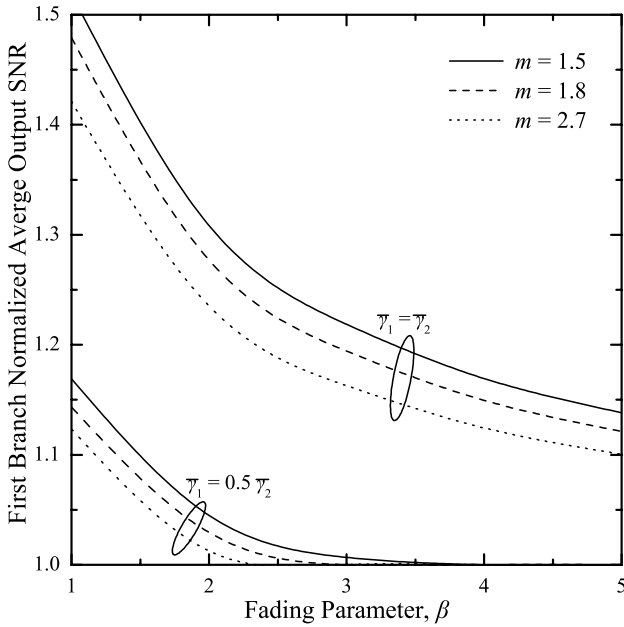


Fig. 1. First branch normalized average output SNR as a function of β .

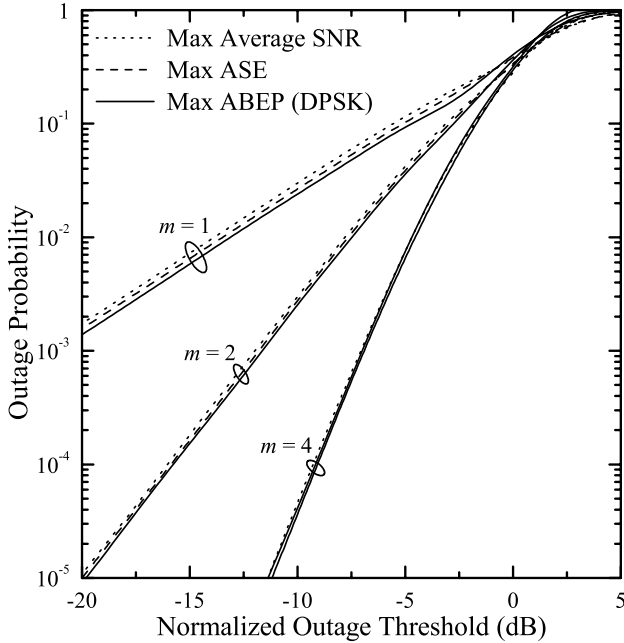


Fig. 2. Outage probability as a function of $\gamma_{th}/\bar{\gamma}$ for $\beta = 2.5$.

the three of them is very important especially in bad fading conditions. In Fig. 3, the ABEP of SSC is plotted as a function of $\bar{\gamma}$ (bottom axis) using γ_{τ}^* in the minimum ABEP sense. As for m and β , it can be observed that the ABEP improves as m and/or β increases. In the same figure, the ABEP is also plotted as a function of $10 \log_{10}(\gamma_{\tau}/\gamma_{\tau}^*)$ (top axis) for $\bar{\gamma} = 5$ dB and $\beta = 2.5$. It can be seen that by properly adjusting γ_{τ} the minimum ABEP can be obtained.

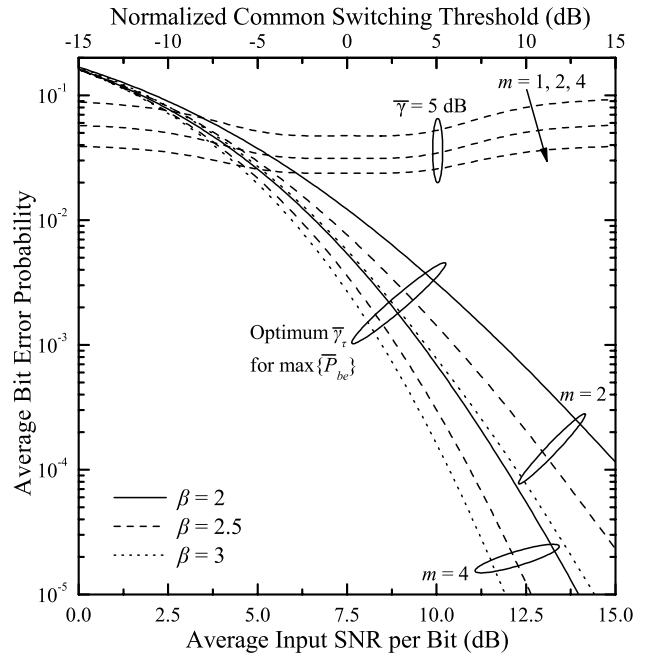


Fig. 3. ABEP of SSC as a function of $\bar{\gamma}$ (bottom axis) and $\gamma_{\tau}/\gamma_{\tau}^*$ (top axis).

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