The Trivariate and Quadrivariate Weibull Fading Distributions with Arbitrary Correlation and their Applications to Diversity Reception

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***Abstract*—The statistical characteristics of the trivariate and quadrivariate Weibull fading distribution with arbitrary correlation, non-identical fading parameters and average powers are analytically studied. Novel expressions for important joint statistics are derived using the Weibull power transformation. These expressions are used to evaluate the performance of selection combining (SC) and maximal ratio combining (MRC) diversity receivers in the presence of such fading channels.**

***Index Terms*—Multi-branch diversity, arbitrary correlation, Weibull fading.**

I. INTRODUCTION

IN the open technical literature there have been many publications concerning multivariate distributions in connection with performance analysis of digital communication systems in the presence of correlated fading channels (e.g. [1]–[3]). Most of these papers deal specifically with the constant and exponential correlation model. The arbitrary correlation model [1], used in this letter, is the most generic correlation model available as it allows for arbitrary correlation values between the signals received by different branches. Clearly it includes the constant and exponential correlation models as special cases. In [1], new infinite series representations for the joint probability density function (PDF) and the joint cumulative distribution function (CDF) of three and four arbitrarily correlated Rayleigh random variables (RVs) have been presented. Furthermore, in [4] a Green's matrix approximation for the multivariate Weibull distribution with arbitrary correlation has been presented and an analytical expression for the joint CDF has been derived. However, the performance analysis presented in [4] is restricted to selection combining (SC) receivers and outage probability (OP) evaluation.

This letter presents a thorough analytical study of the statistical characteristics of the arbitrary correlated trivariate and quadrivariate Weibull fading distributions and their applications to various diversity receivers. For both distributions we consider the arbitrary correlation model with non-identical fading parameters or average powers and without making any approximation for the covariance matrix. Novel expressions utilizing infinity series representations for the joint PDF, CDF and moment generating function (MGF) of both distributions will be presented. These analytical expressions will be conveniently used to evaluate the OP and the average bit error probability (ABEP) for SC and maximal ratio combining (MRC) diversity reception.

II. STATISTICAL CHARACTERISTICS

To investigate the trivariate and quadrivariate Weibull distributions, it is convenient to consider the multivariate Weibull distribution, $\mathbf{Z}_L = \{Z_1, Z_2, ...Z_L\}$. $\mathbf{Z}_L$ is assumed to be arbitrarily correlated according to a positive definite covariance matrix $\mathbf{\Psi}_L$, with elements $\psi_{i\kappa} = \mathbb{E}\langle G_i G_\kappa^*\rangle$, where $\mathbb{E}\langle\cdot\rangle$ denotes expectation, * complex conjugate, $i, \kappa \in \{1, 2, .., L\}$ and $\mathbf{G}_L = \{G_1, G_2, ..., G_L\}$ being joint complex zero mean Gaussian L RVs. Since $\psi_{i\kappa}$ can take arbitrary values, the analysis presented in this section refers to the most general correlation case.

A. Trivariate Weibull Distribution

For the case of the trivariate (i.e. $L = 3$) arbitrarily correlated[1] Weibull distribution and by applying the Weibull power transformation $Z = R^{2/\beta}$ [2, eq. (2)] in the infinite series representation of the Rayleigh distribution [1, eq. (5)], the joint PDF of $\mathbf{Z}_3 = \{Z_1, Z_2, Z_3\}$ can be conveniently expressed as

$$\begin{aligned} f_{\mathbf{Z}_3}(z_1, z_2, z_3) &= \frac{\beta_1\beta_2\beta_3 \det(\mathbf{\Phi}_3)}{z_1^{(2-\beta_1)/2} z_2^{(2-\beta_2)/2} z_3^{(2-\beta_3)/2}} \\ &\times \exp\left[-\left(z_1^{\beta_1}\phi_{11} + z_2^{\beta_2}\phi_{22} + z_3^{\beta_3}\phi_{33}\right)\right] \sum_{k=0}^{\infty} \epsilon_k(-1)^k \\ &\times \cos(k\chi) \sum_{\ell,m,n=0}^{\infty} \frac{|\phi_{12}|^{2\ell+k}}{\ell!(\ell+k)!} \frac{|\phi_{23}|^{2m+k}}{m!(m+k)!} \frac{|\phi_{31}|^{2n+k}}{n!(n+k)!} \\ &\times z_1^{\beta_1(\ell+n+k)+\beta_1/2} z_2^{\beta_2(\ell+m+k)+\beta_2/2} z_3^{\beta_3(m+n+k)+\beta_3/2} \end{aligned} \quad (1)$$

where ϵ_k is the Neumann factor ($\epsilon_0 = 1, \epsilon_k = 2$ for $k = 1, 2, \cdots$), $\chi = \chi_{12}+\chi_{23}+\chi_{31}$ and $\mathbf{\Phi_3}$ is the inverse covariance matrix given by

$$\mathbf{\Phi}_3 = {\mathbf{\Psi}_3}^{-1} = \begin{bmatrix} \phi_{11}, & \phi_{12}, & \phi_{13} \\ \phi_{12}^*, & \phi_{22}, & \phi_{23} \\ \phi_{13}^*, & \phi_{23}^*, & \phi_{33} \end{bmatrix} \quad (2)$$

[1]From now on and unless otherwise stated, it will be is assumed that the Weibull distributions under consideration are arbitrarily correlated.

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TABLE I
NUMBERS OF TERMS, N_T, NEEDED FOR THE OUTAGE PROBABILITY OF TRIPLE-BRANCH SC FOR 10^{-5} ACCURACY (*H*=*M*=*N*)

$\gamma_{th}/\overline{\gamma}$ (dB)	$\beta=2$	$\beta=2.7$	$\beta=4.1$
-15	*K*=1 *H*=2	*K*=1 *H*=2	*K*=*H*=1
-10	*K*=2 *H*=3	*K*=*H*=2	*K*=1 *H*=2
-5	*K*=*H*=4	*K*=*H*=3	*K*=2 *H*=3
0	*K*=7 *H*=8	*K*=7 *H*=8	*K*=7 *H*=8
5	*K*=*H*=15	*K*=19 *H*=20	*K*=33 *H*=34

where $\phi_{i\kappa} = |\phi_{i\kappa}|\exp(\jmath\chi_{i\kappa})$ with $i,\kappa \in \{1,2,3\}$ and $|\cdot|$ denoting absolute value.

By integrating (1), an infinite series representation for the CDF of $\mathbf{Z}_3$ is derived as

$$F_{\mathbf{Z}_3}(z_1,z_2,z_3) = \frac{\det(\mathbf{\Phi_3})}{\phi_{11}\phi_{22}\phi_{33}}\sum_{k=0}^{\infty}\epsilon_k(-1)^k \times\cos(k\chi)\sum_{\ell,m,n=0}^{\infty} C_3\nu_{12}^{\ell+k/2}\nu_{23}^{m+k/2}\nu_{31}^{n+k/2} \times\gamma\left(\delta_1,z_1^{\beta_1}\phi_{11}\right)\gamma\left(\delta_2,z_2^{\beta_2}\phi_{22}\right)\gamma\left(\delta_3,z_3^{\beta_3}\phi_{33}\right) \quad (3)$$

where $C_3 = [\ell!(\ell+k)!m!(m+k)!n!(n+k)!]^{-1}$, $\nu_{i\kappa} = |\phi_{i\kappa}|^2/\phi_{ii}\phi_{\kappa\kappa}$, $\delta_1 = \ell+n+k+1$, $\delta_2 = m+\ell+k+1$, and $\delta_3 = n+m+k+1$ with $\gamma(\cdot,\cdot)$ denoting the incomplete lower Gamma function [5, eq. (3.381/1)].

Although the exact numerical evaluation of (3) requires the summation of an infinite number of terms, in practice only a few terms are required to be evaluated. Such truncation in (3) results in an error, E_T, given by

$$E_T = \sum_{k=K}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}G(k,l,m,n) + \sum_{k=0}^{K-1}\sum_{l=H}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}G(k,l,m,n) + \sum_{k=0}^{K-1}\sum_{l=0}^{H-1}\sum_{m=M}^{\infty}\sum_{n=0}^{\infty}G(k,l,m,n) + \sum_{k=0}^{K-1}\sum_{l=0}^{H-1}\sum_{m=0}^{M-1}\sum_{n=N}^{\infty}G(k,l,m,n) \quad (4)$$

where

$$G(k,l,m,n) = \frac{\det(\mathbf{\Phi}_3)}{\phi_{11}\phi_{22}\phi_{33}}\epsilon_k(-1)^k\cos(k\chi) \times C_3\,\nu_{12}^{\ell+k/2}\nu_{23}^{m+k/2}\nu_{31}^{n+k/2} \times\gamma\left(\delta_1,z_1^{\beta_1}\phi_{11}\right)\gamma\left(\delta_2,z_2^{\beta_2}\phi_{22}\right)\gamma\left(\delta_3,z_3^{\beta_3}\phi_{33}\right) \quad (5)$$

and K,H,M and N are the number of terms required to achieve certain accuracy. Noting that $\gamma(a,x)\leq\Gamma(a)$, where $\Gamma(\cdot)$ is the gamma function [5, eq. (8.310/1)] and following a similar approach as the one presented in [1], an upper bound for E_T has been obtained. Although, due to space limitation, the expression will not be given here, typical performance evaluation results are presented in Table I.

To verify the correctness of the above general expressions, the two previously used spatial correlation models, i.e. constant and exponential, will be studied as special cases.

1) Constant Correlation Model: Its normalized correlation matrix consists of the elements $\psi_{i\kappa} = \rho\ (i\neq\kappa)$ and $\psi_{ii}=1$, where $-1/2\leq\rho<1$ [6]. Moreover, it can been shown that in this case $\chi = \chi_{12}+\chi_{23}+\chi_{31} = 3\pi$ [1]. As a consequence, for the constant correlation model, (3) simplifies to

$$F_{\mathbf{Z}_3}(z_1,z_2,z_3) = \frac{(1-\rho)(1+2\rho)^2}{(1+\rho)^3}\sum_{k=0}^{\infty}\epsilon_k \times\sum_{\ell,m,n=0}^{\infty} C_3\left(\frac{\rho}{1+\rho}\right)^{\delta_1+\delta_2+\delta_3-3}\lambda_1\lambda_2\lambda_3 \quad (6)$$

where $\lambda_\ell = \gamma\left[\delta_\ell,\frac{(1+\rho)z_\ell^{\beta_\ell}}{(1+\rho-2\rho^2)\Omega_\ell}\right]$ and $\Omega_i = \mathbb{E}\left\langle Z_i^{\beta_i}\right\rangle$.

For $\Omega_1=\Omega_2=\Omega_3=\Omega$, (6) is consistent with the analysis presented in [2] for the multivariate Weibull distribution with constant correlation and for identical average powers.

2) Exponential Correlation Model: In this case, the normalized correlation matrix is $\psi_{i\kappa} = \rho^{|i-\kappa|}$, where $0\leq\rho<1$ and $\phi_{31}=\phi_{13}=0$[2]. Thus, (3) simplifies to

$$F_{\mathbf{Z}_3}(z_1,z_2,z_3) = \frac{(1-\rho^2)}{(1+\rho^2)}\sum_{\ell,m=0}^{\infty}\frac{1}{(\ell!)^2(m!)^2} \times\left(\frac{\rho^2}{1+\rho^2}\right)^{\ell+m}\gamma\left[\ell+1,\frac{z_1^{\beta_1}}{(1-\rho^2)\Omega_1}\right] \times\gamma\left[\ell+m+1,\frac{(1+\rho^2)z_2^{\beta_2}}{(1-\rho^2)\Omega_2}\right] \times\gamma\left[m+1,\frac{z_3^{\beta_3}}{(1-\rho^2)\Omega_3}\right]. \quad (7)$$

Similar to the constant correlation model, for $\Omega_1=\Omega_2=\Omega_3=\Omega$, (7) simplifies to a previously known expression for $L=3$ [2, eq. (28)]. It is underlined though that, since the CDF expressions (6) and (7) include the case of non-identical average powers, they are more general than those presented in [2].

The joint MGF of $\mathbf{Z}_3$ can expressed as $M_{\mathbf{Z}_3}(s_1,s_2,s_3) = \mathbb{E}\langle\exp(-s_1Z_1-s_2Z_2-s_3Z_3)\rangle$. From (1) and following the integral solutions using the Meijer G-function presented in [2,

[2]It can be easily proved (e.g. see [7]) that for the exponential correlation model the inverse covariance matrix is tridiagonal, i.e. $\phi_{i\kappa}=0$ for $|i-\kappa|>1\ \forall i\neq\kappa$.

pp. 3610], the following novel expression has been obtained

$$\begin{aligned} M_{\mathbf{Z}_3}(s_1,s_2,s_3) &= \beta_1\beta_2\beta_3 \det(\mathbf{\Phi}_3) \sum_{k=0}^{\infty} \epsilon_k (-1)^k \cos(k\chi) \\ &\times \sum_{\ell,m,n=0}^{\infty} C_3 \frac{|\phi_{12}|^{2\ell+k} |\phi_{23}|^{2m+k} |\phi_{31}|^{2n+k}}{s_1^{\beta_1(\ell+n+k+1)} s_2^{\beta_2(\ell+m+k+1)} s_3^{\beta_3(m+n+k+1)}} \\ &\times \Upsilon\left[\frac{\phi_{11}}{s_1^{\beta_1}}, \beta_1(\ell+n+k+1)\right] \\ &\times \Upsilon\left[\frac{\phi_{22}}{s_2^{\beta_2}}, \beta_2(\ell+m+k+1)\right] \\ &\times \Upsilon\left[\frac{\phi_{33}}{s_3^{\beta_3}}, \beta_3(m+n+k+1)\right] \end{aligned} \quad (8)$$

where $\Upsilon(\cdot)$ is given in [2, eq. 8].

B. Quadrivariate Weibull Distribution

For the case of the quadrivariate (i.e. $L = 4$) Weibull distribution, we consider the inverse covariance matrix, $\mathbf{\Phi}_4$, expressed as

$$\mathbf{\Phi}_4 = \mathbf{\Psi}_4^{-1} = \begin{bmatrix} \phi_{11}, & \phi_{12}, & \phi_{13}, & 0 \\ \phi_{12}^*, & \phi_{22}, & \phi_{23}, & \phi_{24} \\ \phi_{13}^*, & \phi_{23}^*, & \phi_{33}, & \phi_{34} \\ 0, & \phi_{24}^*, & \phi_{34}^*, & \phi_{44} \end{bmatrix} \quad (9)$$

where the $\phi_{i\kappa}$ $i,\kappa \in \{1,2,3,4\}$ can take arbitrary values with the restriction of $\phi_{14} = \phi_{14}^* = 0$. Although this restriction is mainly a mathematical assumption, necessary for the derivation of the equivalent statistics and does not necessarily correspond to a physical explanation, it is underlined that our approach is more general than of [8] for the multivariate Rayleigh distribution. More specifically, the statistical properties derived in [8] hold only under the assumption that $\mathbf{\Phi}_L$ is tridiagonal, i.e. when $\phi_{i\kappa} = 0$ for $|i-\kappa| > 1$ $\forall i \neq \kappa$. The same assumption was used in [4], where the correlation matrix was approached by the tridiagonal Green matrix.

In principle, an expression for the joint PDF of $\mathbf{Z}_4 = \{Z_1, Z_2, Z_3, Z_4\}$ can be derived using [1, eq. (16)] and by applying the power transformation described in [2, eq. (2)] as a product of the modified Bessel function of the first kind $I_n(u)$. However, this approach will not be adopted since expressions containing modified Bessel functions are difficult to be mathematically manipulated, e.g. performing integrations. Instead, a more convenient approach is to use its infinite series expansion [5, eq. (8.447/1)]. Thus, the following PDF is obtained

$$\begin{aligned} f_{\mathbf{Z}_4}(z_1,z_2,z_3,z_4) &= \beta_1\beta_2\beta_3\beta_4 \det(\mathbf{\Phi}_4) \\ &\times \exp\left[-\left(z_1^{\beta_1}\phi_{11} + z_2^{\beta_2}\phi_{22}\right)\right] \\ &\times \exp\left[-\left(z_3^{\beta_3}\phi_{33} + z_4^{\beta_4}\phi_{44}\right)\right] \\ &\times \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} \epsilon_j (-1)^{j+k} \cos(A) \sum_{\ell,m,n,p,q=0}^{\infty} C_4 \\ &\times |\phi_{12}|^{2\ell+j} |\phi_{13}|^{2m+j} |\phi_{24}|^{2n+|k|} |\phi_{34}|^{2p+|k|} \\ &\times |\phi_{23}|^{2q+|j+k|} z_1^{\beta_1(\ell+n+j+1)-1} z_2^{\beta_2[J_1]-1} \\ &\times z_3^{\beta_3[J_2]-1} z_4^{\beta_4(n+p+|k|/2+1)-1} \end{aligned} \quad (10)$$

where $C_4 = [\ell!(\ell+j)!m!(m+j)!n!(n+|k|)!p!(p+|k|)!q!(q+|k+j|)!]^{-1}$, $A = j(\chi_{12}+\chi_{23}+\chi_{31}) + k(\chi_{23}+\chi_{34}+\chi_{42})$, $J_1 = \ell+n+q+(j+|k|+|j+k|)/2+1$ $J_2 = m+p+q+(j+|k|+|j+k|)/2+1$.

By integrating (10), the corresponding CDF becomes

$$\begin{aligned} F_{\mathbf{Z}_4}(z_1,z_2,z_3,z_4) &= \det(\mathbf{\Phi}_4) \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} \epsilon_j \\ &\times (-1)^{j+k} \cos(A) \sum_{\ell,m,n,p=0}^{\infty} C_4 |\phi_{12}|^{2\ell+j} |\phi_{13}|^{2m+j} \\ &\times |\phi_{24}|^{2n+|k|} |\phi_{34}|^{2p+|k|} \frac{\gamma\left(\ell+m+j+1, z_1^{\beta_1}\phi_{11}\right)}{\phi_{11}^{\ell+m+j+1}} \\ &\times \frac{\gamma\left(n+p+|k|+1, z_4^{\beta_4}\phi_{44}\right)}{\phi_{44}^{n+p+|k|+1}} \sum_{q=0}^{\infty} |\phi_{23}|^{2q+|j+k|} \\ &\times \frac{\gamma\left[J_1, z_2^{\beta_2}\phi_{22}\right]}{\phi_{22}^{J_1}} \frac{\gamma\left[J_2, z_3^{\beta_3}\phi_{33}\right]}{\phi_{33}^{J_2}}. \end{aligned} \quad (11)$$

Considering the exponential correlation model as special case[3] and by substituting $\psi_{i\kappa} = \rho^{|i-\kappa|}$ in (11), the equivalent CDF for the exponential correlation has been found to be identical with a previously known expression [2, eq. (28)] for $L = 4$.

Finally, using (10) and [2, eq. 8] the equivalent MGF has been obtained as

$$\begin{aligned} M_{\mathbf{Z}_4}(s_1,s_2,s_3,s_4) &= \beta_1\beta_2\beta_3\beta_4 \det(\mathbf{\Phi}_4) \\ &\times \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} \epsilon_j (-1)^{j+k} \cos(A) \sum_{\ell,m,n,p,q=0}^{\infty} C_4 \\ &\times |\phi_{12}|^{2\ell+j} |\phi_{23}|^{2m+j} |\phi_{24}|^{2n+|k|} |\phi_{34}|^{2p+|k|} \\ &\times |\phi_{23}|^{2q+|j+k|} \frac{\Upsilon\left[\phi_{11}/s_1^{\beta_1}, \beta_1(\ell+m+j+1)\right]}{s_1^{\beta_1(\ell+m+j+1)}} \\ &\times \frac{\Upsilon\left[\phi_{22}/s_2^{\beta_2}, \beta_2(J_1)\right]}{s_2^{\beta_2(J_1)}} \frac{\Upsilon\left[\phi_{33}/s_3^{\beta_3}, \beta_3(J_2)\right]}{s_3^{\beta_3(J_2)}} \\ &\times \frac{\Upsilon\left[\phi_{44}/s_4^{\beta_4}, \beta_4(n+p+|k|+1)\right]}{s_4^{\beta_4(n+p+|k|+1)}}. \end{aligned} \quad (12)$$

III. Performance Analysis

In this section important performance criteria for diversity receivers with three or four arbitrarily correlated diversity branches operating over Weibull fading and additive white Gaussian noise (AWGN) channels will be studied. In particular, by using the previously derived expressions for the statistical characteristics of the trivariate and quadrivariate Weibull distribution, the performance of MRC and SC diversity receivers will be studied and their OP and ABEP will be derived.

For the system model considered, the equivalent baseband signal received at the ℓth branch can be mathematically

[3]The constant correlation model is not considered as a special case for the quadrivariate case since for the matrix with elements $\psi_{i\kappa} = \rho$ $(i \neq \kappa)$ and $\psi_{ii} = 1$, the restriction $\phi_{14} = \phi_{14}^* = 0$ for its inverse does not hold.

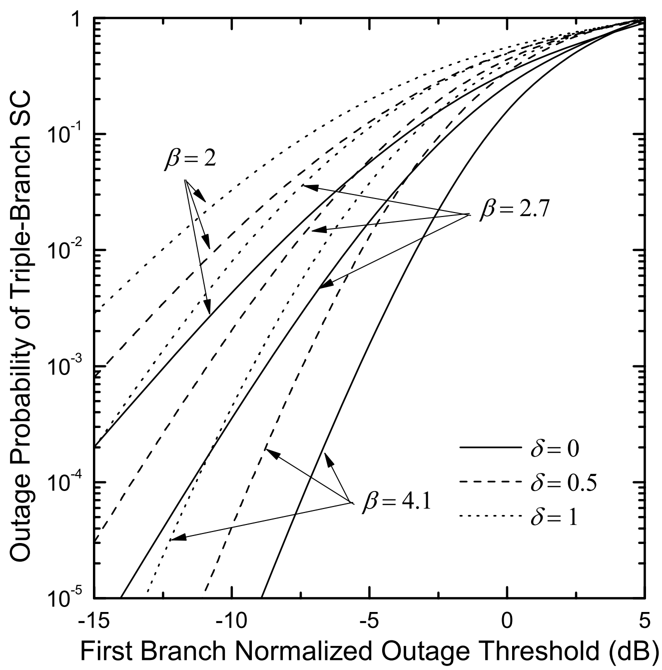

Fig. 1. Outage probability of triple-branch SC receiver as a function of the first branch normalized outage threshold for different values of β and δ.

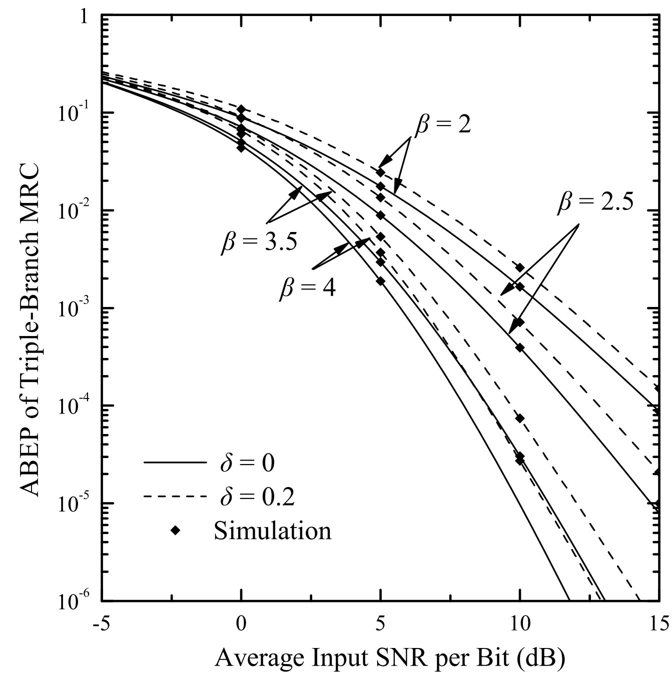

Fig. 2. ABEP of triple-branch MRC receiver as a function of the average input SNR for different values of β and δ.

expressed as $\zeta_\ell = wh_\ell + n_\ell$ where w is the complex transmitted symbol having average energy $E_s = \mathbb{E}\langle|w|^2\rangle$, h_ℓ is the complex channel fading envelope with its magnitude $Z_\ell = |h_\ell|$ being a Weibull distributed RV and n_ℓ is the AWGN with single-sided power spectral density N_0. The instantaneous, per symbol, SNR of the ℓth diversity channel is $\gamma_\ell = Z_\ell^2 E_s/N_0$, while its average is $\overline{\gamma}_\ell = \mathbb{E}\langle Z_\ell^2\rangle E_s/N_0 = \Gamma(d_{2,\ell})\Omega_\ell^{2/\beta_\ell} E_s/N_0$ where in general $d_{\tau,\ell} = 1 + \tau/\beta_\ell$ with $\tau > 0$. Note that it is straightforward to obtain expressions for the statistics of γ_ℓ by replacing at the previously mentioned expressions for the fading envelope Z_ℓ (e.g. in (3), (8), (11) and (12)), β_ℓ with $\beta_\ell/2$ and Ω_ℓ with $(\alpha_\ell\overline{\gamma}_\ell)^{\beta_\ell/2}$ [2]. Thus, denoting $\boldsymbol{\gamma_L}$={$\gamma_1, \gamma_2, ...\gamma_L$}, the CDF $F_{\boldsymbol{\gamma_L}}(\gamma_1, \gamma_2, ..., \gamma_L)$ and the MGF $M_{\boldsymbol{\gamma_L}}(s_1, s_2, ..., s_L)$ of the SNR for the trivariate and quadrivariate Weibull distribution can be easily obtained, but will not be presented here due to space limitation.

1) Performance of MRC Receivers: For MRC receivers the output, per symbol, SNR (SNR_o), is $\gamma_{mrc} = \sum_{\ell=1}^{L}\gamma_\ell$ [9]. To obtain the ABEP performance it is convenient to use the MGF-based approach. Hence, the MGF of the L-branch MRC output can be derived as $M_{\gamma_{mrc}}(s) = M_{\boldsymbol{\gamma_L}}(s, s, .., s)$. By using the MGF-based approach, the ASEP of noncoherent binary frequency-shift keying (NBFSK) and binary differential phase-shift keying (BDPSK) modulation signaling can be directly calculated. For other types of modulation formats, numerical integration is needed in order to evaluate single integrals with finite limits.

2) Outage Probability of SC Receivers: The instantaneous SNR at the output of a L-branch SC receiver will be the SNR with the highest instantaneous value between all branches, i.e. $\gamma_{sc} = \max\{\gamma_1, \gamma_2, ..., \gamma_L\}$ [10]. Since the CDF of γ_{sc}, $F_{\boldsymbol{\gamma_{sc}}}(\gamma_{sc}) = F_{\boldsymbol{\gamma}}(\gamma_{sc}, \gamma_{sc}, ..., \gamma_{sc})$, P_{out} can be easily obtained as $P_{out}(\gamma_{th}) = F_{\gamma_{sc}}(\gamma_{th})$ for both trivariate and quadrivariate cases.

IV. Performance Evaluation Results and Discussion

Using the previous mathematical analysis, in this section performance evaluation results for the SC and MRC receivers will be presented. Non-identical distributed Weibull channels, i.e., $\overline{\gamma}_\ell = \overline{\gamma}_1 \exp[-(\ell-1)\delta]$ where δ is the power decay factor are considered and for the convenience of the presentation, but without any loss of generality, $\beta_\ell = \beta \ \forall \ \ell$ will be assumed.

Considering a triple-branch diversity receiver with the linearly arbitrary normalized covariance matrix[4] given in [3, pp. 886] and SC diversity, the OP has been obtained as a function of the first branch normalized outage threshold $\gamma_{th}/\overline{\gamma}_1$ for different values of β and δ. The performance evaluation results, illustrated in Fig. 1, indicate that P_{out} degrades with increasing $\gamma_{th}/\overline{\gamma}_1$ and δ and/or decreasing β. Note that for $\beta = 2$ and $\delta = 0$ the obtained results are in agreement with previously known performance evaluation results presented in [4]. The number of terms, N_T, needed for the OP to achieve an accuracy better than 10^{-5}, after the truncation of the infinite series and for the case $\delta = 0$ are shown in Table I, revealing the series convergence behavior. Clearly, N_T strongly depends on the normalized outage threshold $\gamma_{th}/\overline{\gamma}$, as an increase in $\gamma_{th}/\overline{\gamma}$ leads to an increase of N_T.

For MRC receivers and BDPSK signaling, the ABEP has been obtained and is illustrated in Figs. 2 and 3 for three or four receiving branches, assuming the covariance matrices presented in [3, pp. 886] and [1, eq. (34)], respectively. As expected, the ABEP improves as the first branch average input SNR $\overline{\gamma}_1$ increases, while for a fixed value of $\overline{\gamma}_1$, similar to the SC diversity, a decrease of β and/or an increase of δ degrades the ABEP. Furthermore, performance evaluation

[4]Note that the covariance matrix specifies the fading correlation between two complex Gaussian RVs.

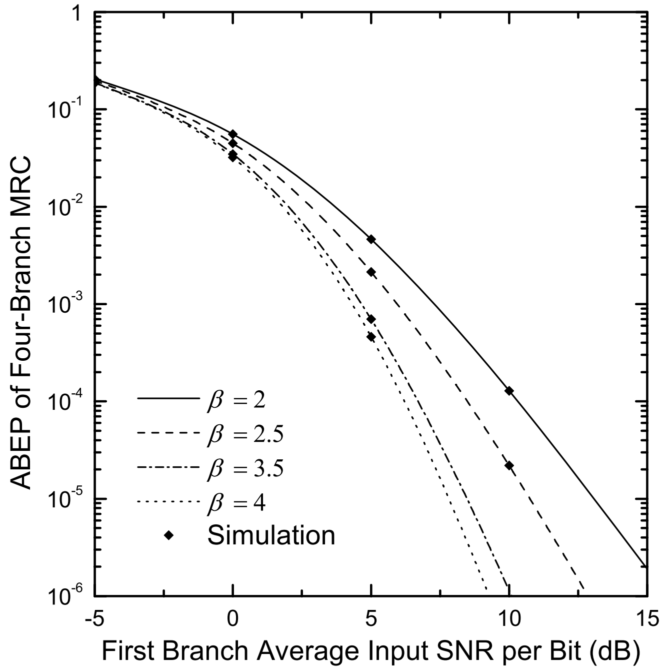

Fig. 3. ABEP of four-branch MRC receiver as a function of the first branch average input SNR per bit for different values of β.

results obtained by means of computer simulation are also shown in Figs. 2 and 3 and have verified the accuracy of the analysis. It is finally noted that for the four-branch diversity reception and $\overline{\gamma}_1 > 5$ dB, only one term is required to achieve ABEP accuracy better than 10^{-5}.

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