Equal-Gain and Maximal-Ratio Combining Over Nonidentical Weibull Fading Channels

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***Abstract*—We study the performance of L-branch equal-gain combining (EGC) and maximal-ratio combining (MRC) receivers operating over nonidentical Weibull-fading channels. Closed-form expressions are derived for the moments of the signal-to-noise ratio (SNR) at the output of the combiner and significant performance criteria, for both independent and correlative fading, such as average output SNR, amount of fading and spectral efficiency at the low power regime, are studied. We also evaluate the outage and the average symbol error probability (ASEP) for several coherent and noncoherent modulation schemes, using a closed-form expression for the moment-generating function (mgf) of the output SNR for MRC receivers and the Padé approximation to the mgf for EGC receivers. The ASEP of dual-branch EGC and MRC receivers is also obtained in correlative fading. The proposed mathematical analysis is complimented by various numerical results, which point out the effects of fading severity and correlation on the overall system performance. Computer simulations are also performed to verify the validity and the accuracy of the proposed theoretical approach.**

***Index Terms*—Amount of fading (AoF), correlated fading, equal-gain combining (EGC), maximal-ratio combining (MRC), outage probability, spectral efficiency (SE), Weibull fading channels.**

I. Introduction

DIVERSITY combining is one of the most practical, effective and widely employed technique in digital communications receivers for mitigating the effects of multipath fading and improving the overall wireless systems performance. The most popular diversity techniques are equal-gain combining (EGC), maximal-ratio combining (MRC), selection combining (SC) and a combination of MRC and SC, called generalized-selection combining (GSC). The performance of EGC and MRC diversity receivers has been extensively studied in the open technical literature for several well-known fading statistical models, such as Rayleigh, Rice and Nakagami-m assuming independent or correlative fading (see [1]–[6] and references therein). However, another well known fading channel model, namely the Weibull model, has not yet received as much attention as the others, despite the fact that it exhibits an excellent fit to experimental fading channel measurements, for both indoor, as well as for outdoor environments [7]–[10]. Considering related works on diversity combining in Weibull fading, Alouini and Simon [11] have been presented an analysis for the evaluation of the GSC performance over independent Weibull fading channels. Recently, some other contributions dealing with switched and selection diversity, as well as second-order statistics over Weibull fading channels have been presented by Sagias *et al.* in [12]–[15]. In these works, useful performance criteria including the average output SNR, outage probability, and the bit-error rate performance have been studied.

In this paper, we present a moments-based approach to the performance analysis of L-branch EGC and MRC receivers, operating in independent or correlated, not necessarily identically distributed (i.d.), Weibull fading. For both EGC and MRC receivers the moments of the output signal-to-noise ratio (SNR) are obtained in closed-form. An accurate approximate expression is derived for the moment-generating function (mgf) of the output SNR of the EGC receiver utilizing the Padé approximants theory [16], while a closed-form expression for the corresponding mgf of the MRC, is obtained. Significant performance criteria, such as average output SNR, amount of fading (AoF) and spectral efficiency (SE) at the low power regime, are extracted in closed-forms, using the moments of the output SNR for both independent and correlative fading. Moreover, using the well-known mgf approach [1], the outage and the average symbol error probability (ASEP) for several coherent, noncoherent, binary, and multilevel modulation schemes, are studied. The ASEP of dual-branch EGC and MRC receivers is also obtained when correlative fading is considered in the diversity input branches. The proposed mathematical analysis is illustrated by various numerical results and validated by computer simulations.

II. System and Channel Model

We consider an L-branch diversity receiver operating in a flat fading environment. The baseband received signal at the ℓth, $\ell = 1, 2, \ldots, L$, diversity branch is

$$z_\ell = s a_\ell \exp(\jmath\theta_\ell) + n_\ell \tag{1}$$

where s is the transmitted symbol, a_ℓ is the fading envelope, $\jmath = \sqrt{-1}$, n_ℓ is the additive white Gaussian noise (AWGN) with a single-sided power spectral density N_0 and θ_ℓ is the random phase due to Doppler shift and oscillators frequency mismatch. The phase θ_ℓ is uniformly distributed over the range $[0, 2\pi]$ and

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the noise components are assumed to be statistically independent of the signal and uncorrelated with each other. The channel is considered slowly time varying and, thus, the phase can be easily estimated.

Let us assume that a_ℓ is a two-parameter Weibull random variable (rv), with probability density function (pdf) given by [17]

$$f_{a_\ell}(a_\ell) = \frac{\beta}{\omega_\ell}\left(\frac{a_\ell}{\omega_\ell}\right)^{\beta-1}\exp\left[-\left(\frac{a_\ell}{\omega_\ell}\right)^{\beta}\right] \quad (2)$$

where β and ω_ℓ are the fading and scaling parameters, respectively, and $\overline{a_\ell^2}$ is the average power of fading. The scaling parameter is related to the average power of fading as $\omega_\ell = \sqrt{\overline{a_\ell^2}/\Gamma(d_2)}$, where $d_\tau = 1 + \tau/\beta, \tau$ is a real constant and $\Gamma(\cdot)$ is the Gamma function [18, eq. (6.1.1)]. Moreover, β expresses the fading severity. As β increases the severity of fading decreases, while for $\beta = 2$, (2) reduces to the well-known Rayleigh pdf. The cumulative distribution function (cdf) and the moments of a_ℓ are given by

$$F_{a_\ell}(a_\ell) = 1 - \exp\left[-\left(\frac{a_\ell}{\omega_\ell}\right)^{\beta}\right] \quad (3)$$

and

$$E\langle a_\ell^n\rangle = \omega_\ell^n \Gamma(d_n) \quad (4)$$

respectively, where n is a positive integer and $E\langle\cdot\rangle$ denotes expectation. The instantaneous output SNR of EGC or MRC receivers can be written as

$$\gamma_{out} = \lambda_{\xi,1}\frac{E_s}{N_0}\left(\sum_{i=1}^{L} a_i^{-\xi+2}\right)^{\xi+1} \quad (5)$$

where for MRC $\xi = 0$ and for EGC $\xi = 1, \lambda_{\xi,n} = (L^{-n} - 1)\xi + 1$ and E_s is the transmitted symbol-energy.

Next, we briefly present the necessary theoretical framework for the bivariate Weibull distribution, which will be used to study the performance of diversity receivers in correlative fading. The complementary cdf (or survival function) of the bivariate Weibull distribution has the form [19]

$$\tilde{F}_{a_1,a_2}(a_1,a_2) = \exp\left\{-\left[\left(\frac{a_1}{\omega_1}\right)^{\frac{\beta}{\delta}} + \left(\frac{a_2}{\omega_2}\right)^{\frac{\beta}{\delta}}\right]^{\delta}\right\} \quad (6)$$

where $\delta, 0 \le \delta \le 1$, is a parameter which is related to the correlation coefficient, defined $\rho \triangleq \mathrm{cov}(a_1,a_2)/\sqrt{\mathrm{var}(a_1)\mathrm{var}(a_2)}$, as

$$\rho = \frac{\Gamma^2(d_\delta)\Gamma(d_2) - \Gamma^2(d_1)\Gamma(d_2\delta)}{\Gamma(d_{2\delta})[\Gamma(d_2) - \Gamma^2(d_1)]}. \quad (7)$$

By substituting (3) and (6) in [20, eq. (6.22)], the cdf of a_1 and a_2 can be derived as

$$F_{a_1,a_2}(a_1,a_2) = 1 + \exp\left\{-\left[\left(\frac{a_1}{\omega_1}\right)^{\frac{\beta}{\delta}} + \left(\frac{a_2}{\omega_2}\right)^{\frac{\beta}{\delta}}\right]^{\delta}\right\} - \exp\left[-\left(\frac{a_1}{\omega_1}\right)^{\beta}\right] - \exp\left[-\left(\frac{a_2}{\omega_2}\right)^{\beta}\right]. \quad (8)$$

For independent input paths, $\rho = 0$ (i.e., $\delta = 1$), (8) can be written as a product of two Weibull cdf's. Differentiating (8) the joint pdf of a_1 and a_2 can be extracted in a rather complicated form, while the covariance of a_1 and a_2 (joint moments of order $n+m$) is [19]

$$E\langle a_1^n, a_2^m\rangle = \omega_1^n\omega_2^m\frac{\Gamma(d_{n\delta})\Gamma(d_{m\delta})\Gamma(d_{n+m})}{\Gamma[d_{(n+m)\delta}]}. \quad (9)$$

III. Moments of the Output SNR

By definition and using (5), the nth order moment of the combiner's output SNR is

$$E\langle\gamma_{out}^n\rangle = \lambda_{\xi,n}\left(\frac{E_s}{N_0}\right)^n E\left\langle\left(\sum_{i=1}^{L} a_i^{-\xi+2}+\right)^{n(\xi+1)}\right\rangle. \quad (10)$$

Expanding the term $(\sum_{i=1}^{L} a_i^{-\xi+2}+)^{n(\xi+1)}$, using the multinomial identity [18, eq. (24.1.2)], (10) can be rewritten as

$$E\langle\gamma_{out}^n\rangle = \lambda_{\xi,n}\left(\frac{E_s}{N_0}\right)^n[(\xi+1)n]! \times \sum_{\substack{k_1,\ldots,k_L=0\\ k_1+\cdots+k_L=n(\xi+1)}}^{n(\xi+1)}\frac{E\left\langle a_1^{k_1(2-\xi)}\cdots a_L^{k_L(2-\xi)}\right\rangle}{k_1!\cdots k_L!} \quad (11)$$

or, in terms of the instantaneous SNR of each diversity path $\gamma_\ell = a_\ell^2 E_s/N_0$, as

$$E\rangle\gamma_{out}^n\rangle = \lambda_{\xi,n}[n(\xi+1)]! \times \sum_{\substack{k_1,\cdots,k_L=0\\ k_1+\cdots+k_L=n(\xi+1)}}^{n(\xi+1)}\frac{E\left\langle\gamma_1^{\frac{k_1}{\xi+1}}\cdots\gamma_L^{\frac{k_L}{\xi+1}}\right\rangle}{k_1!\cdots k_L!}. \quad (12)$$

For uncorrelated input paths, the mean product term $E\langle\gamma_1^{k_1/(\xi+1)}\cdots\gamma_L^{k_L/(\xi+1)}\rangle$ can be expressed as

$$E\left\langle\gamma_1^{\frac{k_1}{\xi+1}}\cdots\gamma_L^{\frac{k_L}{\xi+1}}\right\rangle = \prod_{i=1}^{L}E\left\langle\gamma_i^{\frac{k_i}{\xi+1}}\right\rangle \quad (13)$$

where $E\langle\gamma_\ell^n\rangle$ is obtained from (4) as

$$E\langle\gamma_\ell^n\rangle = \frac{\Gamma(d_{2n})}{\Gamma^n(d_2)}\bar{\gamma}_\ell^n \quad (14)$$

and $\overline{\gamma}_\ell$ is the ℓth average input SNR. By substituting (13) and (14) into (12), the moments of the EGC or MRC output SNR for independent but not necessarily i.d. input branches can be written in a simple and closed-form expression given by

$$E\left\langle \gamma_{out}^n \right\rangle = \frac{\lambda_{\xi,n}}{\Gamma^n(d_2)}[n(\xi+1)]! \times \sum_{\substack{k_1,\ldots,k_L=0 \\ k_1+\cdots+k_L=n(\xi+1)}}^{n(\xi+1)} \prod_{j=1}^{L} \frac{1}{k_j!}\Gamma\left(d_{\frac{2k_j}{\xi+1}}\right)\bar{\gamma}_j^{\frac{k_j}{\xi+1}}. \quad (15)$$

For dual diversity ($L = 2$) with correlated input paths, the term $E\langle\gamma_1^{k_1/(\xi+1)}\gamma_2^{k_2/(\xi+1)}\rangle$, appearing in (12), can be evaluated, using (9), as

$$E\left\langle \gamma_1^n \gamma_2^m \right\rangle = \bar{\gamma}_1^n \bar{\gamma}_2^m \frac{\Gamma(d_{2n\delta})\Gamma(d_{2m\delta})\Gamma\left[d_{2(n+m)}\right]}{\Gamma^{n+m}(d_2)\Gamma\left[d_{2(n+m)\delta}\right]} \quad (16)$$

and, thus, the nth moment of the output SNR for dual EGC or MRC can be expressed in closed-form by substituting (16) in (12) for $L = 2$, as

$$E\left\langle \gamma_{out}^n \right\rangle = \frac{2^{-\xi n}\Gamma(d_{2n})}{\Gamma^n(d_2)\Gamma^n(d_{2\delta n})} \times \sum_{k=0}^{n(\xi+1)} \binom{n(\xi+1)}{k} \bar{\gamma}_1^{\frac{k}{\xi+1}} \bar{\gamma}_2^{n-\frac{k}{\xi+1}} \times \Gamma\left(d_{\frac{2\delta k}{\xi+1}}\right)\Gamma\left[d_{2\delta\left(n-\frac{k}{\xi+1}\right)}\right] \quad (17)$$

where $\binom{n(\xi+1)}{k} = [n(\xi+1)]!/\{k![n(\xi+1)-k]!\}$. To the best of the authors' knowledge, (15) and (17) are novel.

A. Average Output SNR

When the receiver employs MRC, the average output SNR $\bar{\gamma}_{\mathrm{MRC}}$, can be easily obtained both for independent and correlative fading, setting $\xi = 0$ and $n = 1$ in (12), which leads to the well-known formula $\bar{\gamma}_{\mathrm{MRC}} = \sum_{i=1}^{L} \bar{\gamma}_i$. In this case, the fading correlation does not affect the average output SNR performance.

For independent input paths with EGC diversity, after straightforward mathematical manipulations, the average output SNR can be derived in closed-form setting $n = 1$ and $\xi = 1$ in (15) as

$$\bar{\gamma}_{\mathrm{EGC}} = \frac{1}{L}\left[\sum_{i=1}^{L}\bar{\gamma}_i + 2\frac{\Gamma^2(d_1)}{\Gamma(d_2)}\sum_{i=2}^{L}\sum_{j=1}^{i-1}\sqrt{\bar{\gamma}_i\bar{\gamma}_j}\right] \quad (18)$$

while for independent and identically distributed (i.i.d.) input paths ($\bar{\gamma}_\ell = \bar{\gamma}_0, \forall\ell$), (18) reduces to

$$\bar{\gamma}_{\mathrm{EGC}} = \left[1 + (L-1)\frac{\Gamma^2(d_1)}{\Gamma(d_2)}\right]\bar{\gamma}_0. \quad (19)$$

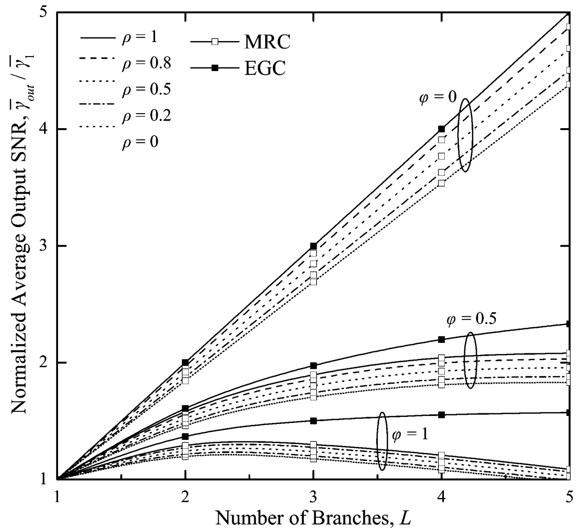

Fig. 1. First branch normalized average output SNR of EGC and MRC, versus L, with constant correlation, exponentially decaying pdp and $\beta = 2.5$.

For correlated input paths, the average output SNR of EGC $\bar{\gamma}_{\mathrm{EGC}}$ can be obtained by setting $n = 1$ and $\xi = 1$ in (12), yielding

$$\bar{\gamma}_{\mathrm{EGC}} = \frac{2}{L}\sum_{\substack{k_1,\ldots,k_L=0 \\ k_1+\cdots+k_L=2}}^{2} \frac{E\left\langle \gamma_1^{\frac{k_1}{2}}\cdots\gamma_L^{\frac{k_L}{2}}\right\rangle}{k_1!\cdots k_L!}. \quad (20)$$

It can be easily recognized that in the above equation only the terms of the form $E\langle\gamma_i^{1/2}\gamma_j^{1/2}\rangle$ have to be evaluated. Therefore, using (20), the average output SNR of the L-branch EGC receiver over correlated Weibull fading channels can be expressed in a simple and closed-form expression as

$$\bar{\gamma}_{\mathrm{EGC}} = \frac{1}{L}\left[\sum_{i=1}^{L}\bar{\gamma}_i + 2\frac{\Gamma^2(d_\delta)}{\Gamma(d_{2\delta})}\sum_{i=2}^{L}\sum_{j=1}^{i-1}\sqrt{\bar{\gamma}_i\bar{\gamma}_j}\right]. \quad (21)$$

Note, that for $\delta = 1$, (21) reduces to (18), while for i.d. input branches simplifies to

$$\bar{\gamma}_{\mathrm{EGC}} = \left[1 + (L-1)\frac{\Gamma^2(d_\delta)}{\Gamma(d_{2\delta})}\right]\bar{\gamma}_0. \quad (22)$$

Assuming constant correlation among the EGC and MRC branches and an exponentially decaying power delay profile (pdp) ($\bar{\gamma}_\ell = \bar{\gamma}_1\exp[-\varphi(\ell-1)]$), Fig. 1 plots the first branch normalized average output SNR of EGC and MRC, as a function of L, for $\beta = 2.5$, various values of ρ and power decay factor φ. In contrary to the behavior of the average SNR at the MRC output—which is unaffected by the correlation—the average output EGC increases as the correlation increases. Also, the combining loss of the receiver gets more accentuated as φ increases. Note, that with an increase of ρ it can be easily verified that, not only the normalized average output SNR increases, but also the variance of the output SNR increases.

B. Amount of Fading (AoF)

The AoF is a unified measure of the severity of fading, which is typically independent of the average fading power and is defined as [1]

$$A_F \triangleq \frac{\mathrm{var}(\gamma_{out})}{\bar{\gamma}_{out}^2} = \frac{E\left\langle \gamma_{out}^2 \right\rangle}{\bar{\gamma}_{out}^2} - 1. \tag{23}$$

When the receiver employs MRC, the first and second moments of the output SNR needed in (23), can be derived in closed-form for arbitrary number of correlated nonidentical branches, using (12). Moreover, for EGC the AoF can be evaluated in closed-form for independent, nonidentical input paths, using (15) and for dual diversity with correlative fading, using (17).

C. Spectral Efficiency (SE)

The AoF can be used to study the SE of a flat-fading channel in the very noise (low power) region [21]. In such a region, the minimum bit energy E_b per noise level, required for reliable communication is $(E_b/N_0)_{\min} = -1.59$ dB and the slope of the SE curve versus E_b/N_0 bit/s/Hz per 3 dB, at $(E_b/N_0)_{\min}$ is [22]

$$S_0 = \frac{2E^2\langle r^2\rangle}{E\langle r^4\rangle} = \frac{2\bar{\gamma}_{out}^2}{E\left\langle \gamma_{out}^2\right\rangle} \tag{24}$$

with r being the combiner's output envelope. Using (23), a useful expression for the slope of the SE in the very noise region can be obtained as

$$S_0 = \frac{2}{A_F + 1}. \tag{25}$$

IV. Error Performance and Outage Probability

Using the well-known mgf approach, the ASEP $\bar{P}_{\mathrm{se}}$ for several coherent (e.g., M-AM, binary phase shift keying (BFSK), M-PAM, M-PSK and M-QAM) and noncoherent (e.g., noncoherent BFSK (NBFSK) and M-DPSK) modulation schemes and the outage probability P_{out}, are studied.

A. Average Symbol Error Probability (ASEP)

1) MRC: Using (2), (4), and $\gamma_\ell = a_\ell^2 E_s/N_0$, the mgf of the output SNR of an MRC receiver, $\mathcal{M}_{\gamma_{\mathrm{MRC}}}(s) = \prod_{i=1}^{L} \mathcal{M}_{\gamma_i}(s)$, where $\mathcal{M}_{\gamma_\ell}(s)$ is the mgf of the SNR of the ℓth input path, operating over independent Weibull fading channels is

$$\mathcal{M}_{\gamma_{\mathrm{MRC}}}(s) = \prod_{i=1}^{L} \frac{\beta}{2(a\bar{\gamma}_i)^{\frac{\beta}{2}}} \times \int_0^\infty \gamma_i^{\frac{\beta}{2}-1} \exp\left[-s\gamma_i - \left(\frac{\gamma_i}{a\bar{\gamma}_i}\right)^{\frac{\beta}{2}}\right] d\gamma_i. \tag{26}$$

This integral can be evaluated in closed-form as follows. By expressing the exponential function as a Meijer's G-function [23, eq. (9.301)], i.e., $\exp[-g(x)] = \mathrm{G}_{0,1}^{1,0}[g(x)\,|\,_{0}^{-}]$ ([24, eq. (11)], where $g(\cdot)$ is an arbitrary function, (26) can be written as

$$\mathcal{M}_{\gamma_{\mathrm{MRC}}}(s) = \prod_{i=1}^{L} \frac{\beta}{2(a\bar{\gamma}_i)^{\frac{\beta}{2}}} \times \int_0^\infty \gamma_i^{\frac{\beta}{2}-1} \mathrm{G}_{0,1}^{1,0}\left[s\gamma_i \,\Big|\, {}_{0}^{-}\right] \mathrm{G}_{0,1}^{1,0}\left[\left(\frac{\gamma_i}{a\bar{\gamma}_i}\right)^{\frac{\beta}{2}} \Big|\, {}_{0}^{-}\right] d\gamma_i. \tag{27}$$

Using [24, eq. (21)], (27) can be expressed in closed-form as

$$\mathcal{M}_{\gamma_{\mathrm{MRC}}}(s) = \prod_{i=1}^{L} \frac{\beta}{2(a\bar{\gamma}_i)^{\frac{\beta}{2}}} \frac{\left(\frac{k}{l}\right)^{\frac{1}{2}}\left(\frac{l}{s}\right)^{\frac{\beta}{2}}}{(2\pi)^{\frac{k+l}{2}-1}} \times \mathrm{G}_{l,k}^{k,l}\left[\frac{(a\bar{\gamma}_i)^{-\frac{k\beta}{2}}}{s^l}\frac{l^l}{k^k} \,\middle|\, \begin{matrix} \frac{1-\frac{\beta}{2}}{l}, \frac{2-\frac{\beta}{2}}{l}, \ldots, \frac{l-\frac{\beta}{2}}{l} \\ 0, \frac{1}{k}, \frac{2}{k}, \ldots, \frac{k-1}{k} \end{matrix}\right] \tag{28}$$

with

$$\frac{l}{k} = \frac{\beta}{2} \tag{29}$$

where k and l are positive integers. Depending upon the value of β, a set with minimum values of k and l can be properly chosen in order (29) to be valid (e.g., for $\beta = 2.5$ we have to choose $k = 4$ and $l = 5$). Note, that for the special case of β integer, $k = 2$ and $l = \beta$. To the best of the authors' knowledge, (28) is novel.

2) EGC: Unfortunately, there is not readily available any analytical expression for the mgf of the output SNR for EGC receivers operating in Weibull fading. Therefore, we propose the use of the Padé approximants [16] as an alternative and simple way to approximate this mgf and consequently to evaluate the ASEP for this kind of diversity receivers. By definition, the mgf is given by

$$\mathcal{M}_{\gamma_{\mathrm{EGC}}}(s) \triangleq E\langle \exp(s\gamma_{\mathrm{EGC}})\rangle \tag{30}$$

and it can be represented as a formal power series (e.g., Taylor) as

$$\mathcal{M}_{\gamma_{\mathrm{EGC}}}(s) = \sum_{n=0}^{\infty} \frac{1}{n!} E\left\langle \gamma_{\mathrm{EGC}}^n \right\rangle s^n. \tag{31}$$

Although the moments of all orders $E\langle\gamma_{\mathrm{EGC}}^n\rangle$ for the L-branch EGC can be evaluated in closed-forms using the analysis of Section III, in practice, only a finite number N can be used, truncating the series in (31). A Padé approximant to the mgf is that rational function of a specified order B for the denominator and A for the nominator, whose power series expansion agrees with the Nth-order ($N = A + B$) power expansion of $\mathcal{M}_{\gamma_{\mathrm{EGC}}}(s)$, i.e.,

$$R_{[A/B]}(s) = \frac{\sum_{i=0}^{A} c_i s^i}{1 + \sum_{i=1}^{B} b_i s^i} = \sum_{n=0}^{A+B} \frac{E\left\langle \gamma_{\mathrm{EGC}}^n\right\rangle}{n!} s^n + O(s^{N+1}) \tag{32}$$

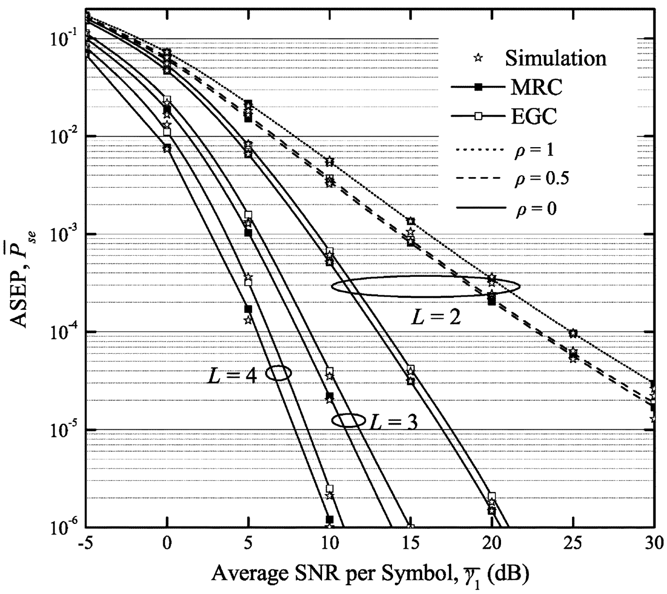

Fig. 2. ASEP of BPSK versus average SNR of the first branch for EGC and MRC with $\beta = 2.5$.

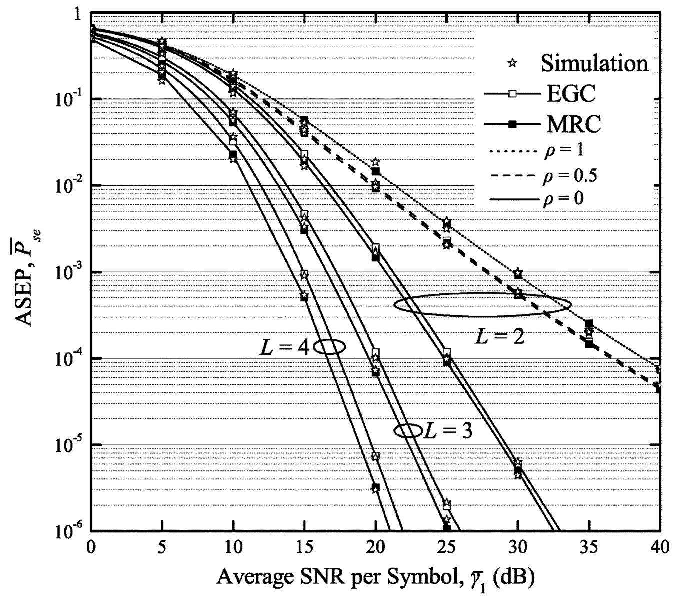

Fig. 3. ASEP of 16-QAM versus average SNR of the first branch for EGC and MRC with $\beta = 2.5$.

with $O(s^{N+1})$ being the remainder after the truncation and b_i and c_i real constant values [25]. Hence, the first $(A + B)$ moments are need to be evaluated in order to construct the approximant $R_{[A/B]}(s)$. In our analysis, $\mathcal{M}_{\gamma_{\rm EGC}}(s)$ is approximated using subdiagonals $(R_{[A/A+1]}(s))$ Padé approximants $(B = A+1)$, since it is only for such order of approximants that the convergence rate and the uniqueness can be assured [16], [25].

Using the mgf expressions, either in closed-form using (28), for MRC, or with the aid of Padé approximants, for EGC and dual correlated MRC using (15) and (17), respectively, the ASEP can be directly calculated for noncoherent and differential binary signaling [e.g., NBFSK and binary DPSK (BDPSK)], since for all other cases (e.g., BPSK, M-PSK, M-QAM, M-AM, and M-DPSK), single integrals with finite limits and integrands composed of elementary (exponential and trigonometric) functions, have to be readily evaluated via numerical integration. Some numerical results for the ASEP are presented to illustrate the proposed mathematical analysis. Figs. 2 and 3 plot the ASEP of BPSK and 16-QAM, respectively, of EGC and MRC, versus the average SNR of the first branch, for i.d. input paths with $\beta = 2.5$ and for several values of ρ and L. Figs. 2 and 3 show that the error performance of MRC is always better than that of EGC, while the diversity gain decreases for both combiners with the increase of the correlation, as expected. In the same figures, computer simulations results are also plotted for comparison purposes, in order to check the accuracy of the proposed Padé approximants approach. As it is clear an excellent match between computer simulations and analytical results is observed. To the best of the author' knowledge such curves, for the ASEP, are presented for the first time in the literature. Note, that although the normalized average output SNR of the EGC increases with ρ, as mentioned in Section III, the ASEP deteriorates. Thus, the average output SNR is not suitable performance criterion to study the performance of EGC and MRC in correlative fading.

B. Outage Probability

If γ_{th} is a certain specified threshold, then the outage probability is defined as the probability that the combiner's output SNR, γ_{out}, falls below γ_{th} and is given by [1]

$$P_{out}(\gamma_{th}) = F\gamma_{out}(\gamma_{th}) = \mathcal{L}^{-1}\left[\frac{\mathcal{M}_{\gamma_{out}}(s)}{s}\right]\Bigg|_{\gamma_{out}=\gamma_{th}} \quad (33)$$

where $F\gamma_{out}(\gamma_{out})$ is the cdf of the combiner's output SNR, $\mathcal{L}^{-1}(\cdot)$ denotes the inverse Laplace transform and $\mathcal{M}_{\gamma_{out}}(s)$ is either $\mathcal{M}_{\gamma_{\rm EGC}}(s)$ for EGC, or $\mathcal{M}_{\gamma_{\rm MRC}}(s)$ for MRC. For EGC, the Padé rational form of $\mathcal{M}_{\gamma_{\rm EGC}}(s)$ is

$$M_{\gamma_{\rm EGC}}(s) \cong \frac{\sum_{i=0}^{A} c_i s^i}{1 + \sum_{i=1}^{B} b_i s^i} = \sum_{i=1}^{B} \frac{\lambda_i}{s + p_i} \quad (34)$$

where $\{p_i\}$ are the poles of the Padé approximants to the mgf, which must have negative real part and $\{\lambda_i\}$ are the residues. Using the residue inversion formula, the outage probability can be easily evaluated from (33) in closed-form as

$$P_{out}(\gamma_{th}) = \sum_{i=1}^{B} \frac{\lambda_i}{p_i} \exp(-p_i \gamma_{th}). \quad (35)$$

For MRC, due to the complicated form of $\mathcal{M}_{\gamma_{\rm MRC}}(s)$, the outage probability can be evaluated using an accurate algorithm for numerically inverting Laplace transforms of cdfs, which is summarized in [26].

In Fig. 4, the outage probability for EGC and MRC is plotted versus the normalized threshold $\gamma_{th}/\overline{\gamma}_1$, for i.d. input paths and for several values of ρ and L. Similar with the ASEP, the outage performance deteriorates with an increase of the correlation between the two diversity paths (higher values of ρ), while the

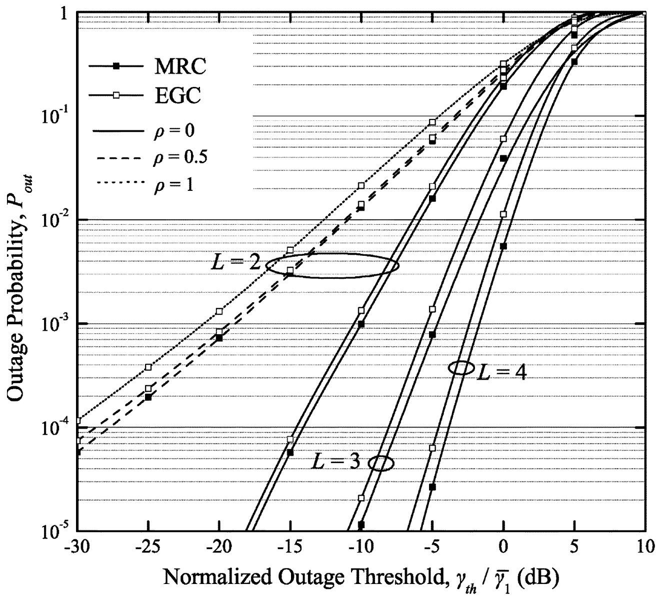

Fig. 4. Outage probability $\gamma_{th}/\bar{\gamma}_1$ versus of EGC and MRC with $\beta = 2.5$.

outage performance improves with an increase of the number of the combiners branches.

V. Conclusion

A performance analysis of the L-branch MRC and EGC receivers operating over Weibull fading channels, was presented. Approximated expressions were derived for the mgf of the output SNR for EGC utilizing the Padé approximants theory, while a closed-form expression for the corresponding mgf of the MRC was obtained. For both EGC and MRC receivers the moments of the output SNR were obtained in closed-form. Furthermore, significant performance criteria, such as average output SNR, AoF, SE at the low power regime, outage probability and ASEP were studied. We have observed, that although an increase of the correlation between the diversity branch leads to an increase of the normalized average output SNR, the outage probability and the ASEP deteriorate. Thus, the normalized average output SNR is not the appropriate metric to study the EGC performance in correlative fading.

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