Bit Error and Outage Probability of Serial Relaying Communication Systems

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***Abstract*—In this paper, a study on the end-to-end performance of multihop non-regenerative relaying over independent and identical generalized-gamma fading channels is presented. Novel closed-form bounds for the cumulative distribution function, the moments, and the moment generating function (MGF) of the end-to-end signal-to-noise ratio are presented. Using the MGF-based approach, closed-form expressions for the amounf-of-fading, the outage probability, and the average bit error probability for binary differential phase shift keying are derived. In order to validate the accuracy of the proposed mathematical analysis, various numerical and computer simulation results are presented and compared to the analytical ones.**

***Index Terms*—Multi-hop relaying, generalized-gamma fading, outage probability, average bit error probability, amplify-and-forward.**

I. Introduction

Recently, multihop networks technology has attracted great interest as it is a promising solution for the high data rate coverage required in future cellular, wireless local area and hybrid networks [1]–[7]. In a multihop system, the mobile terminal relays a signal between the base station and a nearby mobile terminal when the direct link between the base station and the original mobile terminal is in deep fade. As a result, signals from the source to the destination propagate through different hops/links.

A versatile fading channel model is the generalized-gamma (GG) distribution [8]. The GG distribution includes the Rayleigh, the Nakagami-m and the Weibull distribution as special cases and the lognormal distribution as a limiting case. Furthermore, it is considered to be mathematically tractable, as compared to lognormal-based models, and recently has gained increased interest in the field of digital communications over fading channels.

In [9], the end-to-end outage probability as well as the average error rate for multihop wireless systems with non regenerative relaying operating over Weibull fading channels were evaluated. In [10]–[16], the end-to-end outage probability as well as the average error rate for dual-hop wireless systems with non regenerative relaying operating over Rayleigh and Nakagami-m fading channels were presented. In [17], [18], performance bounds for multihop relayed transmissions with fixed-gain relays over Rice, Hoyt, and Nakagami-m fading channels using the moments-based approach were given. Moreover, in [19], an extensive performance analysis for dual-hop relaying communication systems over GG fading was presented.

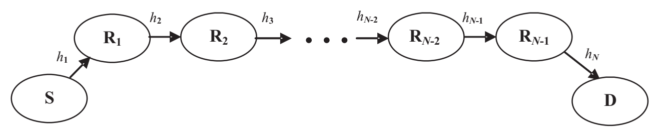

Fig. 1. The multihop communication system under consideration.

Motivated by all the above, in this paper, we analyze the performance of non regenerative (amplify-and-forward) multihop systems operating over i.i.d GG fading channels. In order to derive exact closed-form expressions for the end-to-end signal-to-noise ratio (SNR) statistics, a tight upper bound for the end-to-end SNR is used. Closed-form expressions for the statistics of the proposed bound, namely the probability density function (PDF), the moments and the moment generating function (MGF) are presented. Using the MGF approach, lower bounds for the outage probability (OP) and the average bit error probability (ABEP) of binary differential phase shift keying (BDPSK) are also given in closed form. Computer simulation results are also presented which demonstrate the tightness of the proposed bound especially at medium and high SNR.

II. System and Channel Model

We consider a multihop system as shown in Fig. 1, with a source node communicating with a destination node with $N-1$ relay nodes. The fading channel coefficients between source-to-relays and relays-to-destination are independent GG random variables. The overall SNR at the receiving end can be written as [11]:

$$\gamma_{equ} = \left[\prod_{n=1}^{N}\left(1+\frac{1}{\gamma_n}\right)-1\right]^{-1}. \tag{1}$$

The equivalent SNR expression in (1) is not easily tractable due to the complexity in finding the statistics associated with it. Fortunately, this form can be bounded by [19]

$$\gamma_{equ} < \gamma_b = \min(\gamma_1, \dots, \gamma_N). \tag{2}$$

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Since the fading envelopes β_i follows the GG distribution, the PDF of γ_i can be written as

$$f_{\gamma_i}(\gamma) = \frac{\beta_i \gamma^{\frac{m_i \beta_i}{2}-1}}{2(m_i-1)!(\tau_i\,\overline{\gamma}_i)^{\frac{m_i\beta_i}{2}}}\ \exp\left[-\left(\frac{\gamma}{\tau_i\overline{\gamma}_i}\right)^{\frac{\beta_i}{2}}\right]. \quad (3)$$

where $\beta_i > 0$ and $m_i = 1, 2, \cdots$ are parameters related to fading severity, $\overline{\gamma}_i = \mathbb{E}\langle\gamma_i\rangle$ with $\mathbb{E}\langle\cdot\rangle$ denoting expectation and $\tau_i = (m_i-1)!/\Gamma(m_i+2/\beta_i)$ where $\Gamma(\cdot)$ is the gamma function. For $\beta_i = 2$, (3) reduces to the Nakagami-m fading distribution whereas for $m = 1$ the Weibull distribution is obtained. The corresponding cumulative distribution function of γ_i is given by

$$F_{\gamma_i}(\gamma) = 1-\exp\left[-\left(\frac{\gamma}{\tau_i\,\overline{\gamma}_i}\right)^{\frac{\beta_i}{2}}\right]\sum_{i=0}^{m_i-1}\frac{1}{i!}\left(\frac{\gamma}{\tau_i\,\overline{\gamma}_i}\right)^{\frac{i\beta_i}{2}}. \quad (4)$$

III. Statistics of the End-to-End SNR

A. Cumulative Distribution Function

Using (2) and (4) the CDF of γ_b for independent and identically distributed (i.i.d) hops ($m_i = m, \beta_i = \beta, \tau_i = \tau$, and $\overline{\gamma}_i = \overline{\gamma}\ \forall i$), and for integers values of m_i can be expressed as

$$F_{\gamma_b}(\gamma) = 1-[1-F_\gamma(\gamma)]^N = 1-\exp\left[-N\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\frac{\beta}{2}}\right] \times \left[\sum_{i=0}^{m-1}\frac{1}{i!}\left(\frac{\gamma}{\tau\,\overline{\gamma}}\right)^{\frac{i\beta}{2}}\right]^N. \quad (5)$$

Using the multinomial identity [20], (5) can be reexpressed as

$$F_{\gamma_b}(\gamma) = 1-N!\exp\left[-N\left(\frac{\gamma}{\tau\,\overline{\gamma}}\right)^{\frac{\beta}{2}}\right] \times \sum_{\substack{n_0,n_1,\ldots,n_{m-1}=0\\ n_0+n_1+\cdots+n_{m-1}=N}}^{N} A_{n_0,n_1,\ldots,n_{m-1}}\ \gamma^{\beta\sum_{i=1}^{m-1}\frac{in_i}{2}} \quad (6)$$

where

$$A_{n_0,n_1,\ldots,n_{m-1}} = \prod_{i=0}^{m-1}\frac{1}{(i!)^{n_i}n_i!\,(\tau\,\overline{\gamma})^{\frac{\beta\,i\,n_i}{2}}}. \quad (7)$$

B. PDF and the Moments

The PDF of γ_b can be found by taking the derivative of (6) with respect to γ. After some straightforward manipulations, the PDF can be expressed as

$$\begin{aligned} f_{\gamma_b}(\gamma) =& \frac{\beta N}{2\,(\tau\,\overline{\gamma})^{\frac{\beta}{2}}}\ \exp\left[-N\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\frac{\beta}{2}}\right]\gamma^{\frac{\beta}{2}-1}N! \\ &\times \sum_{\substack{n_0,n_1,\ldots,n_{m-1}=0\\ n_0+n_1+\cdots+n_{m-1}=N}}^{N} A_{n_0,n_1,\ldots,n_{m-1}} \\ &-\gamma^{\beta\sum_{i=1}^{m-1}\frac{in_i}{2}}\exp\left[-N\left(\frac{\gamma}{\tau\overline{\gamma}}\right)^{\frac{\beta}{2}}\right]\frac{\beta N!}{2} \\ &\times \sum_{\substack{n_0,n_1,\ldots,n_{m-1}=0\\ n_0+n_1+\cdots+n_{m-1}=N}}^{N} A_{n_0,n_1,\ldots,n_{m-1}} \\ &\times \gamma^{-1+\beta\sum_{i=1}^{m-1}\frac{in_i}{2}}\sum_{i=1}^{m-1} i\,n_i. \end{aligned} \quad (8)$$

The $\nu-th$ moment of γ_b is defined as

$$\mu_{\gamma_b}(\nu) \triangleq \int_0^\infty \gamma^\nu f_{\gamma_b}(\gamma)d\gamma. \quad (9)$$

By making a change of variables $t = N\,(\gamma/\tau\,\overline{\gamma})^{\beta/2}$ and using the definition of the gamma function, the ν-th moment of γ_b can be expressed in closed form as

$$\begin{aligned} \mu_{\gamma_b}(\nu) = N! &\sum_{\substack{n_0,n_1,\ldots,n_{m-1}=0\\ n_0+n_1+\cdots+n_{m-1}=N}}^{N} A_{n_0,n_1,\ldots,n_{m-1}}N^{1-\frac{2\eta_1}{\beta}} \\ &\times(\tau\overline{\gamma})^{\frac{\eta_1\beta}{2}}\ \Gamma\left[\frac{2\eta_1}{\beta}\right]-N!\sum_{\substack{n_0,n_1,\ldots,n_{m-1}=0\\ n_0+n_1+\cdots+n_{m-1}=N}}^{N} A_{n_0,n_1,\ldots,n_{m-1}} \\ &\times\frac{(\tau\overline{\gamma})^{\eta_2}}{N^{\frac{2\eta_2}{\beta}}}\Gamma\left[\frac{2\eta_1}{\beta}\right]\sum_{i=1}^{m-1} i\,n_i \end{aligned} \quad (10)$$

where

$$\eta_1 = \nu+\frac{\beta}{2}\sum_{i=1}^{m-1} i\,n_i+\frac{\beta}{2} \quad (11a)$$

and

$$\eta_2 = \eta_1-\frac{\beta}{2}. \quad (11b)$$

C. Moment Generating Function

The MGF of γ_b, defined as

$$\mathcal{M}_{\gamma_b}(s) \triangleq \mathbb{E}\left\langle\exp\left(-s\,\gamma_b\right)\right\rangle \quad (12)$$

can be easily evaluated given the CDF of γ_b as

$$\mathcal{M}_{\gamma_b}(s) = s\,\mathcal{L}\{F_{\gamma_b}(\gamma); s\} \quad (13)$$

where $\mathcal{L}\{\cdot\}$ denotes the Laplace transform. Using [21, Eq. (2.2.1.22)], the MGF of γ_b can be expressed in closed form as

$$\mathcal{M}_{\gamma_b}(s) = 1 - N! \sum_{\substack{n_0, n_1, \ldots, n_{m-1}=0 \\ n_0+n_1+\cdots+n_{m-1}=N}}^{N} A_{n_0,n_1,\ldots,n_{m-1}} \times \frac{\sqrt{k}\, l^{\nu_1+\frac{1}{2}} s^{-\nu_1}}{(2\pi)^{\frac{k+l}{2}-1}} G_{l,k}^{k,l}\left[\frac{N^k l^l}{(\tau\,\overline{\gamma})^{\frac{\beta k}{2}} k^k \mathrm{s}^l} \left| \begin{matrix} \Delta(l,-\nu_1) \\ \Delta(k,0) \end{matrix} \right.\right] \tag{14}$$

where $G_{l,k}^{k,l}[\cdot]$ is the Meijer's G-function [20, eq. (9.301)], k and l are the minimum integers that satisfy $\beta = 2l/k$, $\Delta(k,\alpha) = \frac{\alpha}{k}, \frac{\alpha+1}{k}, \ldots, \frac{\alpha+k-1}{k}$ and

$$\nu_1 = \frac{\beta}{2}\sum_{i=1}^{m-1} i n_i. \tag{15}$$

IV. Performance Analysis

Using the previously derived formulas, lower bounds for the AoF, the OP, as well as the ABEP for BDPSK modulation are derived for the end-to-end performance of the serial communication system under consideration.

A. Amount of Fading (AoF)

The amount of fading (AoF) is a unified measure of the severity of the fading defined by the ratio of the variance of the received energy to the square of the average received energy. For the considered system, using (10), AoF can be expressed in closed form as follows

$$A_F = \frac{\mu_{\gamma_b}(2)}{\mu_{\gamma_b}(1)^2} - 1. \tag{16}$$

B. Outage Probability

The outage probability is defined as the probability that the end-to-end output SNR, falls below a specified threshold γ_{th}. For the considered multihop system the use of upper bound γ_b leads to lower bounds for the OP at the destination terminal **D** expressed as $P_{out}(\gamma_{th}) \geq F_b(\gamma_{th})$. The OP of the considered system can be obtained based on (5) as

$$P_{out}(\gamma_{th}) = F_{\gamma_b}(\gamma_{th}). \tag{17}$$

C. Average Bit Error Probability

The ABEP of BDPSK can be expressed using the MGF expression, given by (14), as

$$P_{be} = \frac{1}{2}\mathcal{M}_{\gamma_b}(1) \tag{18}$$

V. Numerical and Computer Simulation Results

In this section, numerical and computer simulation results of the ABEP and the OP are presented. We assume that $\overline{\gamma}_1 = \overline{\gamma}_2 = \cdots = \overline{\gamma}_N$. In Fig. 2 lower bounds for the ABEP of BDPSK are plotted as a function of the average input SNR ($\overline{\gamma}$) for various values of N with m, β be constant. It is obvious that ABEP improves with a decrease in N. Also, in Fig. 3 the ABEP of BDPSK is plotted as a function of the average input SNR ($\overline{\gamma}$) for various values of β with m, N be constant. As expected, ABEP improves with an increase in β. For both

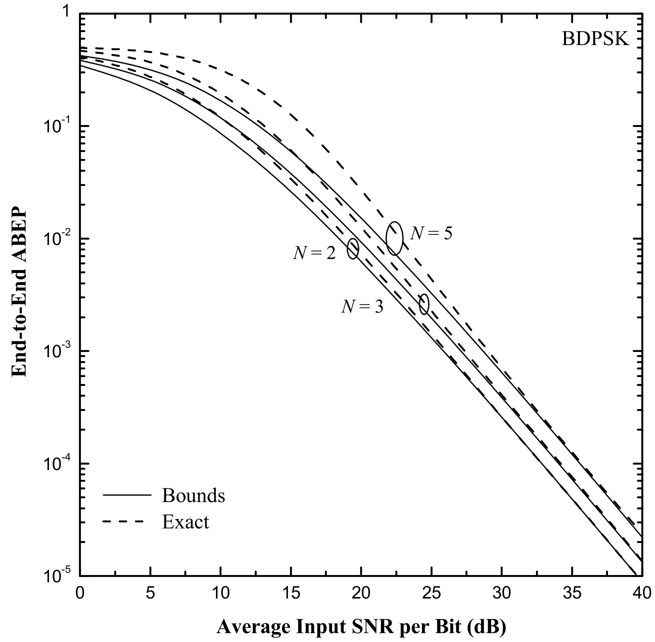

Fig. 2. ABEP of multihop wireless communication system operating over identical (i.i.d.) GG fading channels with BDPSK modulation as a function of the average input SNR per bit for $m = 3$, $\beta = 3$ and different number of relays.

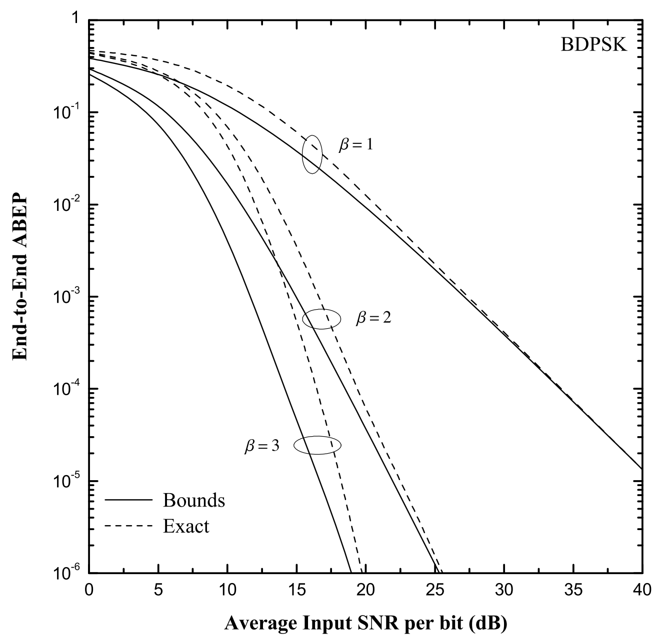

Fig. 3. ABEP of multihop wireless communication system operating over identical (i.i.d.) GG fading channels with BDPSK modulation as a function of the average input SNR per bit for $m = 3$, $N = 3$ and various values of β .

test cases, curves for the exact error performance, obtained via Monte Carlo simulations and assuming end-to-end SNR given by (1) are also depicted for comparison purposes. As it is evident, the tightness of the error performance increase

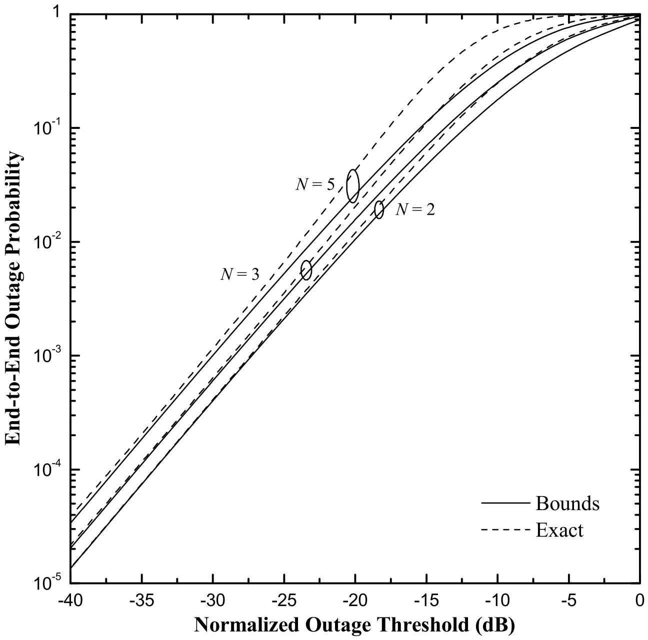

Fig. 4. Outage probability of multihop wireless communication system operating over identical (i.i.d.) GG fading channels as a function of the inverse normalized outage threshold for $m = 3$, $\beta = 3$ and different number of relays.

as SNR increased; however, the proposed bounds lose their tightness in the low and medium SNR region as N increases. Finally, in Fig. 4 lower bounds for the OP are plotted as a function of the inverse normalized outage threshold $\overline{\gamma}/\gamma_{th}$ and for different values of N. As expected, OP improves as $\overline{\gamma}/\gamma_{th}$ and/or N, decreases. Curves for the exact OP, obtained via Monte Carlo simulations and assuming end-to-end SNR given by (1) are also included for comparison purposes. It is obvious that the difference between the exact and the obtained bound gets tighter with the increase of the SNR. Also, it is obvious that the outage bound gets tighter as N gets smaller.

VI. Conclusions

In this paper, we provided performance bounds for multihop transmissions with non-regenerative relays in series, operating over i.i.d GG fading channels. The end-to-end SNR is determinate and upper bounded and novel closed-form expressions for the MGF, PDF, and CDF of this upper bounded SNR were derived. Additionally, tight lower bounds for the OP, the ABEP, and AoF were presented. The obtained results show that the obtained bounds gets tighter with the increase of the SNR. Also, it is obvious that the bounds gets tighter as N gets smaller. Finally numerical results were provided that demonstrated the accuracy of the proposed mathematical approach. Computer simulation results were also included that verified the accuracy and the correctness of the proposed analysis.

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