Performance Metrics in OFDM Wireless Networks with Restricted Accessibility

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***Abstract*— In this paper we study the downlink of an Orthogonal Frequency Division Multiplexing (OFDM) based cell that accommodates calls from different service-classes with different resource requirements. We assume that calls arrive in the system according to a Poisson process while the call admission is based on restricted accessibility. In a restricted accessibility system, a new call may be blocked even if available resources do exist at the time of its arrival. To determine, via recursive formulas, the most important performance metrics, i.e., Call Blocking Probabilities (CBP) and resource utilization in this system, we model the system as a multirate loss model. The accuracy of the proposed formulas is verified via simulation and found to be quite satisfactory.**

Keywords—ofdm, recursive, restricted, call blocking, Poisson

I. INTRODUCTION

The teletraffic modelling is important in the call-level performance evaluation of contemporary Orthogonal Frequency Division Multiplexing (OFDM) wireless networks that service calls from different service-classes with different Quality of Service (QoS) requirements. In such a multidimensional call-level traffic environment, it is required to have resource sharing policies capable of treating differently some calls (of a particular service-class) from other calls (of another service-class).

The most common resource sharing policy is the Complete Sharing (CS) policy. In the CS policy, a new call is accepted in the system whenever the available system resources are greater than (or at least equal to) the call's required resources. The CS policy is an easy to apply policy and can be taken as the default policy, but it can lead to an unfair resource allocation among service-classes [1].

In [2]-[4], the CS policy is considered for the determination of Call Blocking Probabilities (CBP) in OFDM wireless networks. More specifically, in [2], Paik and Suh (P-S) consider the downlink of an OFDM-based cell that accommodates Poisson arriving calls generated by multiservice classes. The system is described via a multirate loss model, i.e., new calls are not allowed to wait in a queue if their required resources are not available. Instead, in case of resource unavailability, calls are blocked and lost. Contrary to [3] and [4], where the acceptance of a new call in the cell depends only on the availability of subcarriers (i.e., the subcarriers are the only system resource), in the P-S model both the subcarriers and power are modelled as system resources and thus participate in call admission.

The P-S model is significant since power is a limited resource in OFDM wireless networks and should be taken into consideration in call admission. In addition, the steady-state probabilities in the P-S model can be described via a Product Form Solution (PFS). The latter is quite important in teletraffic modelling since it usually results in efficient formulas for the determination of the various performance measures. However, the calculation of CBP and resource utilization in the P-S model is based on highly complexed formulas which are not attractive for network planning and dimensioning procedures. To solve this problem, a recursive formula which reduces substantially the complexity of the calculations of the P-S model is proposed in [5].

In this paper, we incorporate the notion of restricted accessibility in the P-S model. In a restricted accessibility system, a new call may be blocked even if available subcarriers do exist at the time of its arrival. Under the general term "restricted accessibility" one may include the case where each particular state of the system is associated with a blocking probability [6]. This means that a new call may be blocked and lost with a certain blocking probability when the system is in a particular state at the time of its arrival. To this end, we provide recursive formulas for the CBP determination in the proposed model which are verified via simulation.

This paper is organized as follows: In Section II, we provide a review of the P-S model. In Section III, we present the recursive formula for the determination of CBP and resource utilization in the P-S model. In Section IV, we propose the P-S model with restricted accessibility. In Section V, we compare the analytical with simulation results for the P-S model and the proposed model. The comparison verifies the accuracy of the proposed formulas. We conclude in Section VI.

II. THE P-S MODEL

To describe the P-S model of [2], consider the downlink of an OFDM-based cell that has M subcarriers and let R, P and B be the average data rate per subcarrier, the available

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power in the cell and the system's bandwidth, respectively. We assume that the entire range of channel gains or signal to noise ratios per unit power is partitioned into K consecutive and non-overlapping intervals and denote as γ_k, k=1,…,K the average channel gain of the kth interval. By further assuming L subcarrier requirements and K average channel gains, the cell accommodates a total of LK service-classes. A new call of service-class (k,l) call (k=1,…,K and l=1,…,L) requires b_l subcarriers in order to be accepted in the cell. This means that the call has a data rate requirement b_lR. In addition, it has an average channel gain γ_k. If these subcarriers are not available at the time of the call's arrival, then call blocking occurs. Otherwise, the call remains in the cell for a generally distributed service time with mean μ^{-1}. To calculate the power p_k required to achieve the data rate R of a subcarrier assigned to a call whose average channel gain is γ_k, we use the Shannon theorem: $R = (B/M)\log_2(1+\gamma_k p_k)$.

Assuming that service-class (k,l) calls follow a Poisson process with rate λ_{kl} and that n_{kl} is the number of in-service calls, then the system can be described as a multirate loss model whose steady-state probabilities $\pi(\boldsymbol{n})$ have the following PFS [2]:

$$\pi(\boldsymbol{n}) = G^{-1}\left(\prod_{k=1}^{K}\prod_{l=1}^{L} p_{kl}^{n_{kl}} / n_{kl}!\right) \tag{1}$$

where: $\boldsymbol{n}=(n_{11},...,n_{k1},...,n_{K1},...,n_{1L},...,n_{kL},...,n_{KL})$, G is the normalization constant, $G=\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}}\left(\prod_{k=1}^{K}\prod_{l=1}^{L} p_{kl}^{n_{kl}} / n_{kl}!\right)$, $\boldsymbol{\Omega}$ is the state space of the system denoted as $\boldsymbol{\Omega}=\{\boldsymbol{n}: 0 \le \sum_{k=1}^{K}\sum_{l=1}^{L} n_{kl}b_l \le M,$ $0 \le \sum_{k=1}^{K}\sum_{l=1}^{L} p_k n_{kl} b_l \le P\}$ and $p_{kl} = \lambda_{kl}/\mu$ is the offered traffic-load (in erlang) of service-class (k,l) calls.

The derivation of the PFS requires that the available power in the cell, P, and the power p_k (required to achieve the data rate R of a subcarrier) are integers. This can be achieved by multiplying both P and p_k by a constant in order to have an equivalent representation of the constraint $\sum_{k=1}^{K}\sum_{l=1}^{L} p_k^{'} n_{kl} b_l \le P^{'}$ (that appears in the definition of $\boldsymbol{\Omega}$), where $P^{'}$ and $p_k^{'}$ are integers [2]. Thus, without loss of generality, it is assumed that P and p_k are integers.

According to [2], all performance metrics (such as CBP) are based on the calculation of $\pi(\boldsymbol{n})$'s via (1). As an example, the CBP $B(k,l)$ of service-class (k,l) calls is given by:

$$B(k,l) = 1 - G(P - p_k b_l, M - b_l)/G(\boldsymbol{\Omega}) \tag{2}$$

However, since the cardinality of $\boldsymbol{\Omega}$ grows as $(MP)^{KL}$, the applicability of (1) is limited to systems of moderate size and therefore is not recommended for network planning and dimensioning procedures.

In [2], Paik and Suh propose the algorithms of [7] and [8] for the CBP determination. The algorithms of [7] and [8] are proposed in the literature for the CBP determination in circuit-switched networks. The algorithms of [7] are based on z-transforms and mean-value analysis. On the other hand, the algorithm of [8] is based on numerical inversion of generating functions which is a quite complex approach [9]. Both algorithms: i) are applied to loss models whose steady-state probabilities have a PFS and ii) are less general than the classical Kaufman-Roberts (K-R) recursive formula ([10], [11]). The latter provides an efficient way for the CBP determination in a multirate loss system that accommodates Poisson arriving calls. Due to the effectiveness of the K-R formula, there exist an extensive list of applications not only in PFS but also in non-PFS models (e.g., [12]-[20]).

III. RECURSIVE FORMULAS IN THE P-S MODEL

To circumvent the complexity problem of (1), a recursive yet efficient formula that resembles the K-R formula is proposed in [5]. To present this formula, the following notation is necessary: let $j_1=\sum_{k=1}^{K}\sum_{l=1}^{L} n_{kl}b_l$ be the occupied subcarriers, i.e., $j_1 = 0,\ldots,M$ and $j_2 = \sum_{k=1}^{K}\sum_{l=1}^{L} p_k n_{kl} b_l$ the occupied power in the cell, i.e., $j_2 = 0,\ldots,P$. Also, let $q(\vec{j}) = q(j_1, j_2)$ be the occupancy distribution, given by:

$$q(\vec{j}) = q(j_1, j_2) = \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}_{\vec{j}}} \pi(\boldsymbol{n}) \tag{3}$$

where: $\boldsymbol{\Omega}_{\vec{j}}$ is the set of states in which the occupied subcarriers and the occupied power in the cell is j_1 and j_2, respectively.

The recursive determination of the unnormalized values of $q(j_1, j_2)$'s is based on the following formula [5]:

$$q(j_1, j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \frac{1}{j_1}\sum_{k=1}^{K}\sum_{l=1}^{L} p_{kl} b_l q(j_1 - b_l, j_2 - p_k b_l) \\ \text{for } j_1 = 1,\ldots,M \text{ and } j_2 = 1,\ldots,P \end{cases} \tag{4}$$

The recursive form of (4) and its lower computational complexity, in the order of $O(MPLK)$, makes (4) attractive for network planning and dimensioning procedures.

Having obtained $q(j_1, j_2)$ we can calculate $B(k,l)$ via:

$$B(k,l) = \sum_{\{(j_1+b_l>M)\cup(j_2+p_kb_l>P)\}} G^{-1} q(j_1, j_2) \tag{5}$$

and the mean number of in-service calls of service-class (k,l), $E(k,l)$, via the formula:

$$E(k,l) = p_{kl}(1 - B(k,l)) \tag{6}$$

where: G is the normalization constant, determined via the formula $G = \sum_{j_1=0}^{M}\sum_{j_2=0}^{P} q(j_1, j_2)$.

Having determined the values of $E(k,l)$, we can calculate the entire system Blocking Probability (BP), the Subcarrier Utilization (SU) and the Power Utilization (PU), via:

$$BP = \sum_{k=1}^{K}\sum_{l=1}^{L} B(k,l)\lambda_{k,l} \Big/ \Lambda, \quad \Lambda = \sum_{k=1}^{K}\sum_{l=1}^{L} \lambda_{k,l} \tag{7}$$

$$SU = \sum_{k=1}^{K}\sum_{l=1}^{L} E(k,l) b_l \Big/ M \tag{8}$$

$$PU = \sum_{k=1}^{K}\sum_{l=1}^{L} p_k E(k,l) b_l \Big/ P \tag{9}$$

IV. The Proposed Model

We consider again the P-S model of [2] and apply the notion of restricted accessibility. To incorporate the notion of restricted accessibility in the proposed model, we assume that each state j_1, except from the initial state where the system is empty (i.e., when $j_1 = 0$) is associated with a blocking probability, $pb_{k,l}(j_1)$ which can be different for each service-class (k,l). Clearly, when there are no available subcarriers for calls of service-class (k,l) (i.e., when $j_1 \geq M - b_l + 1$), then $pb_{k,l}(j_1) = 1$. On the same hand, when the system is empty, then $pb_{k,l}(0) = 0$.

The call admission mechanism for a new service-class (k,l) call in the proposed model is as follows: a) if $(M - j_1 \geq b_l) \cap (j_2 + p_k b_l \leq P)$ then the service-class (k, l) call is accepted in the cell with probability $1 - pb_{k,l}(j_1)$ and remains for a generally distributed service-time with mean μ^{-1}, b) if $(M - j_1 < b_l) \cup (j_2 + p_k b_l > P)$ then the service-class (k, l) call is blocked and lost due to subcarriers' unavailability.

The recursive determination of the unnormalized values of $q(j_1, j_2)$'s can be based on the following formula:

$$q(j_1,j_2) = \begin{cases} 1, & \text{for } j_1 = j_2 = 0 \\ \dfrac{1}{j_1}\sum_{k=1}^{K}\sum_{l=1}^{L} p_{kl} b_l \left(1 - pb_{k,l}(j_1 - b_l)\right) q(j_1 - b_l, j_2 - p_k b_l) \\ \text{for } j_1 = 1,...,M \text{ and } j_2 = 1,...,P \end{cases} \tag{10}$$

Having obtained $q(j_1, j_2)$ we can calculate $B(k,l)$ via the formula:

$$B(k,l) = \sum_{\{(j_1 + b_l > M) \cup (j_2 + p_k b_l > P)\}} G^{-1} q(j_1, j_2) pb_{k,l}(j_1) \tag{11}$$

while the values of $E(k,l)$, BP, SU and PU can be determined via (6), (7), (8) and (9), respectively.

A proper selection of the values of $pb_{k,l}(j_1)$ results in the classical Bandwidth Reservation (BR) policy. In the BR policy, a new service-class (k,l) call requests b_l subcarriers and has a reservation parameter t_l that expresses the number of subcarriers reserved to benefit calls of all other service-classes except for (k,l). The BR policy allows the reservation of subcarriers in order to favor calls of high subcarrier requirements. In that sense the BR policy can provide a certain QoS to calls of certain service-classes.

The call admission mechanism in the case of the BR policy (P-S/BR model) consists of the following two cases: a) if $(M - j_1 - t_l \geq b_l) \cap (j_2 + p_k b_l \leq P)$ then the service-class (k, l) call is accepted in the cell, b) if $(M - j_1 - t_l < b_l) \cup (j_2 + p_k b_l > P)$ then the service-class (k, l) call is blocked and lost.

By assuming that $pb_{k,l}(j_1) = 0$ when $j_1 \leq M - b_l - t_l$ and $pb_{k,l}(j_1) = 1$ when $j_1 > M - b_l - t_l$, then the BR policy is incorporated in the proposed model.

V. Evaluation

We consider the downlink of an OFDM-based cell and provide analytical and simulation CBP results for the P-S and the P-S/BR models. The input parameters for both models are: B = 20 MHz, P = 25 Watt, M = 256, R = 329.6 kbps, L = 64, $b_l = l$, l= 1, …, 64, and the values of b_l are uniformly distributed. In addition, let K = 3 which results in LK = 192 service-classes. Let the integer representations of p_k (k=1, 2, 3) and P be [2]: $p_1' = 6, p_2' = 10, p_3' = 16, P' = 2500$. The values of p_k' require: $p_1 \approx 0.06, p_2 \approx 0.01, p_3 \approx 0.16$ achieved by $\gamma_1 = 24.679$ dB, $\gamma_2 = 22.460$ dB, $\gamma_3 = 20.419$ dB. We further assume that the probability an arriving call has an average channel gain γ_k is given by the set: a_k = 1/3 (k=1, 2, 3). Also, let $\lambda_{kl} = \Lambda a_k / L$ be the arrival rate of Poisson arriving service-class (k,l) calls, where Λ is the total arrival rate in the cell, $\Lambda = pM\mu / \hat{g}$, p is the traffic intensity of the cell, μ = 0.00625 and $\hat{g}$ =32.5 is the average subcarrier requirement of a new call. Note that the value of $\hat{g}$ =32.5 is based on the fact that b_1=1, b_{64} = 64 subcarriers and the values of b_l are uniformly distributed. As far as the values of the BR parameters are concerned, let t_l = 64-l, l=1,…,64, so that $b_1 + t_1 = … = b_{64} + t_{64}$.

In the x-axis of Figs. 1-2, the value of p increases from 0.2 to 1.0. Simulation CBP results, based on Simscript III [21], are mean values of 7 runs, while each run is based on the generation of 10 million calls. To account for a warm-up period, the blocking events of the first 3% of these generated calls are not considered in the results.

Figure 1 shows the analytical and simulation CBP of service-classes (3, 64), (2, 64) and (1, 64) which require the highest number of subcarriers (l=64). We see that the BR policy reduces the CBP of these service-classes compared to the values of the P-S model. Figure 2 shows the analytical and simulation CBP of service-classes (3, 48), (2, 48) and (1, 48). We see that, in most of the cases, the BR policy increases the CBP of these service-classes compared to the values of the P-S model. The same behavior (CBP increase) appears in most of the service-classes whose calls require less than 64 subcarriers.

VI. Conclusion

We propose recursive formulas for the determination of performance measures in the downlink of an OFDM cell with restricted accessibility that accommodates multirate traffic of Poisson arrivals. The proposed formulas are quite accurate compared to simulation and can be used in network dimensioning procedures for the CBP and resource utilization calculation. As a future work we intend to study the case of restricted accessibility under call arrival processes that are more peaked and "bursty" than the Poisson process, such as the batched Poisson process, where calls arrive in batches and the batch arrival process is Poisson [22].

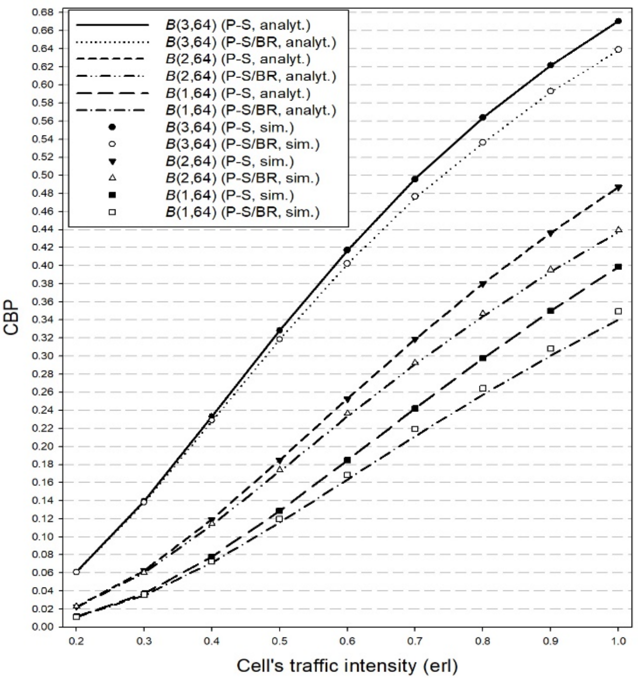

Figure 1. CBP of service-classes (1, 64), (2, 64) and (3, 64).

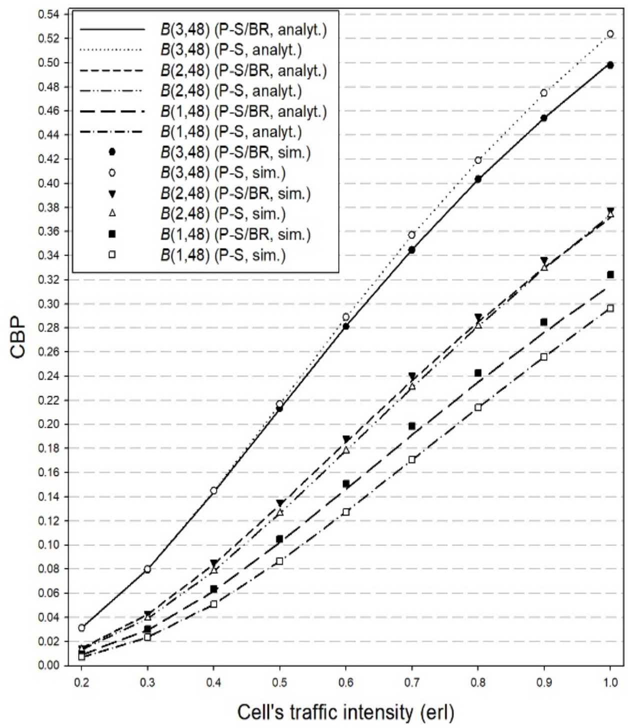

Figure 2. CBP of service-classes (1, 48), (2, 48) and (3, 48).

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